46.

NOTE ON A SYSTEM OF IMAGINARIES.

[From the *Philosophical Magazine*, vol. xxx. (1847), pp. 257—258.]

THE octuple system of imaginary quantities $i_1, i_2, i_3, i_4, i_5, i_6, i_7$, which I mentioned in a former paper [21], (and the conditions for the combination of which are contained in the symbols

$$123, \quad 246, \quad 374, \quad 145, \quad 275, \quad 365, \quad 167,$$

i.e. in the formulæ

$$i_2i_3 = i_1, \quad i_3i_1 = i_2, \quad i_1i_2 = i_3, \quad i_3i_2 = -i_1, \quad i_1i_3 = -i_2, \quad i_2i_1 = -i_3,$$

with corresponding formulæ for the other triplets $i_2i_4i_6$, &c.,) possesses the following property; namely, if i_α, i_β, i_γ be any three of the seven quantities which do *not* form a triplet, then

$$(i_\alpha i_\beta) \cdot i_\gamma = -i_\alpha \cdot (i_\beta i_\gamma).$$

Thus, for instance,

$$(i_3i_4) \cdot i_5 = -i_7 \cdot i_5 = -i_2;$$

but

$$i_3 \cdot (i_4i_5) = \quad i_3 \cdot i_1 = \quad i_2,$$

and similarly for any other such combination. When i_α, i_β, i_γ form a triplet, the two products are equal, and reduce themselves each to -1, or each to $+1$, according to the order of the three quantities forming the triplet. Hence in the octuple system in question neither the commutative nor the distributive law holds, which is a still wider departure from the laws of ordinary algebra than that which is presented by Sir W. Hamilton's quaternions.

I may mention, that a system of coefficients, which I have obtained for the rectangular transformation of coordinates in n dimensions (Crelle, t. XXXII. [1846] "*Sur quelques propriétés des Déterminans gauches*" [52]), does not appear to be at all connected with any system of imaginary quantities, though coinciding in the case of $n=3$ with those mentioned in my paper "On Certain Results relating to Quaternions," *Phil. Mag.* Feb. 1845, [20].