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Topographically induced Rossby waves

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IT IS well known that submarine topographies play an important role in the deflection of such major ocean current systems as the Antarctic Circumpolar Current (ACC). A recent analysis of oceanographic data by Gordon and Bye (1972), for example, gives evidence of the existence of standing wave patterns (Rossby waves) in the lee of a number of topographic features in the path of the ACC. The authors have previously reported on a laboratory experiment and accompanying theoretical models which could be used to examine the effect of bottom topography on rotating flows (Vaziri and Boyer 1971). These studies did not, however, include a variable Coriolis parameter which may be important in the determination of the characteristics of ocean currents (the Coriolis parameter is the vertical component of the earth's rotation rate and thus varies with latitude). In this paper we show how laboratory experiment and theory can be modified to incorporate a variable Coriolis parameter. The flow over a step topography is given as an example; the laboratory experiments conducted are found to be in good agreement with the theory advanced. One aspect of this flow is the existence of standing Rossby waves downstream of the step. These are similar in character to those observed by Gordon and Bye. A numerical solution for the flow past a topographic feature approximating the Campbell Plateau, which lies in the path of the ACC, is also presented.

Dobrze wiadomo, że topografia obszarów podmorskich gra ważną rolę w odchylaniu wielkich prądów oceanicznych, takich jak np. Antarktyczny Prąd Okołobiegunowy (APO). Niedawno dokonane analizy danych oceanograficznych (Gordon i Bye, 1972) dowodzą istnienia układów fal stojących (fale Rossby'ego) na wielu elementach topograficznych leżących na drodze APO. Autorzy donosili uprzednio o doświadczeniu laboratoryjnym i odpowiednim modelu teoretycznym, które mogły służyć do analizy wpływu topografii dna na przepływy rotacyjne (Vaziri i Boyer, 1971). Badania te nie uwzględniały jednak zmiennego parametru Coriolisa, który może okazać się istotny w określeniu charakterystyki prądów oceanicznych; parametr Coriolisa określa pionową składową prędkości obrotowej ziemi i dzięki temu zmienia się z szerokością geograficzną. W niniejszej pracy pokazano w jaki sposób można zmodyfikować wspomniane doświadczenia i teorię, uwzględniając wpływ zmiennego parametru Coriolisa. Jako przykład omówiono przepływ nad uskokiem topograficznym. Stwierdzono dobrą zgodność wyników doświadczenia z zaproponowaną teorią. Cechą takiego przepływu jest pojawianie się fal stojących Rossby'ego w obszarze za uskokiem. Ich charakter jest podobny do charakteru fal zaobserwowanych przez Gordona i Bye'a. Przedstawiono rozwiązanie numeryczne dla przepływu nad elementem topograficznym modelującym w sposób przybliżony Plateau Campbella leżące na drodze APO.

Хорошо известно, что топография подморских областей играет важную роль в отклонениях великих океанических течений, таких как например Антарктическое Околополюсное Течение. Недавно проведенные анализы океанографических данных (Гордон и Бай, 1972) доказывают существование систем стоячих волн (волны Россби) на многих топографических элементах, лежащих на пути АОТ. Авторы сообщали раньше о лабораторном эксперименте и соответствующей теоретической модели, которые могли послужить для анализа влияния топографии дна на ротационные течения (Вазири и Боер, 1971). Эти исследования не учитывали однако переменного параметра Кориолиса, который может оказаться существенным в определении характеристики океанических течений; параметр Кориолиса определяет вертикальную составляющую скорости вращения Земли и благодаря тому изменяется с географической широтой. В настоящей работе показываем каким образом можно модифицировать упомянутые эксперимент и теорию, учитывая влияние переменного параметра Кориолиса. Как пример обсуждаем течение над топографическим уступом. Констатировано хорошее совпадение результатов эксперимента с предложенной теорией. Характерной чертой такого течения является появление стоячих волн Россби в области за уступом. Их характер аналогичен характеру волн наблюдаемых Гордоном и Бай. Представлено численное решение для течения над топографическим элементом, моделирующим приближенным образом Плато Кэмпбелла лежащее на пути АОТ.

1. Introduction

IN THE PAPER published by one of us (Boyer, 1971a) was decribed a laboratory facility which has been utilized to examine the effects of bottom topography on rotating flows (e.g. Boyer, 1971b and Vaziri and Boyer, 1971). Theoretical and numerical results for a number of topographic features (long ridges and cones) were reviewed and these were shown to be in good agreement with laboratory experiments.

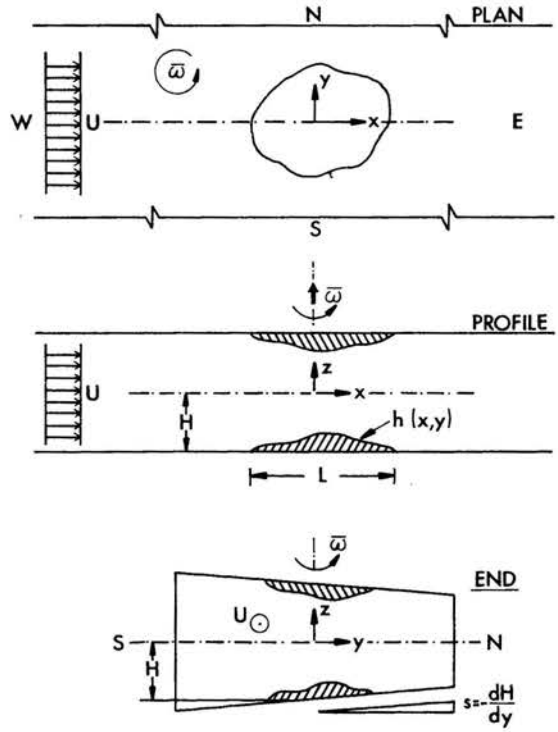

FIG. 1. The physical system.

The physical system considered above is the same as that given in Fig. 1, except that the bounding planes in the vertical were horizontal (i.e., see end view). A homogeneous incompressible fluid is confined between two horizontal plane surfaces. Identical topographic features are mounted symmetrically on the bounding planes and the entire system rotates uniformly about a vertical axis. Relative to a rotating observer the flow for upstream of the topographic features is uniform and rectilinear.

Using the following parametric restrictions, an equation for the lowest order motion can be derived (for more details see VAZIRI and BOYER, 1971):

(a) $E = \nu/2\omega L^2 \ll 1$,

(b) $\mathrm{Ro} = U/2\omega L = kE^{1/2}$ where k is of order unity,

(c) H/L is of order unity,

(d) $h(x, y)/L = h_0(x, y)E^{1/2}$ where $h_0(x, y)$ is of order unity,

(e) h_{0x} and h_{0y} are of order unity.

In the above E is the Ekman number, ν the kinematic viscosity, ω the rotation rate, L the horizontal dimension of the topographic feature, Ro the Rossby number, U the free stream speed, H half the separation distance of the planes, $h(x, y)$ the height function of the topographic feature.

Under the above, the lowest order flow consists of Ekman layers along the bounding surfaces and an interior geostrophic flow. The horizontal motion in the geostrophic region is given by

$$\frac{D\zeta_0}{Dt} = \frac{a}{\sqrt{2}}\zeta_0 - aJ(\psi, h_0), \tag{1.1}$$

where $\zeta = \nabla^2\psi$ is the vertical component of vorticity relative to a rotating observer, ψ is the stream function defined by $U_0 = -\psi_y$, $V_0 = \psi_x$, (U_0, V_0) are the lowest order velocity components in the (x, y) directions, and $a = L/kH$. Relation (1.1) is a transport equation for the relative vorticity ζ_0. The first term on the right hand side is the dissipation caused by Ekman suction, while the second is the contribution of the topographic features.

One of motivating factors in considering the above physical system is its possible relation to geophysical phenomena, in particular the deflection of ocean currents by submarine topographies. In this regard the lower half (i.e. $z \leqslant 0$) of the system sketched in Fig. 1 can be shown to represent the flow over a submerged topography above which there is a horizontal free surface on which there are no shearing stresses. In the following we restrict to $z \leqslant 0$ and take positive x to be "east" and "Y" to be north (since the rotation is vertically upward, this is a "Northern Hemisphere" model).

Two factors, not incorporated in (1.1), which may play substantial roles in altering the vertical component of relative vorticity are the wind shear stress and the so-called β-effect. The fact that wind shear may be important is obvious since horizontal gradients in the shear stress will tend to "spin-up" or "spin-down" the moving fluid columns. The inclusion of a wind shear would add a term of the form $\gamma\ \mathrm{curl}_z\, \bar{\tau}$ to the right hand side of (1.1) where γ is a positive constant, and $\bar{\tau}$ is a dimensionless shear stress. It is not clear how this term could be modeled in the laboratory and thus it is not considered further in this discussion.

If a fluid column is advected across lines of constant latitude it experiences a change in the local value of the vertical component of the earth's rotation rate, and thus an attendant change in its relative vorticity. A column moving to the north in either hemisphere thus tends to have its relative vorticity reduced in this regard (note that the vertical component of the earth's rotation is negative in the Southern Hemisphere). This phenomena is commonly called the β-effect, β being the coefficient of the first term of a Taylor series expansion of the vertical component of the earth's rotation about a given location on the earth's surface; i.e.

$$f = f_0 + \beta y + \ldots,$$

where $f = 2\Omega\sin\phi_0$ is twice the vertical component of the earth's rotation, $f_0 = 2\Omega\sin\phi_0$ is the value of f at the origin of the local coordinate system in question, and $\beta = 2\Omega\cos\phi_0$ (ϕ is the latitude).

The rate of change of vertical vorticity for a fluid column crossing lines of constant latitude is proportional to the velocity component in the direction, i.e. $-cx$, where c is a positive constant. Referring to (1.1), one notes that such a contribution can be included in the laboratory model by merely tilting the plane surfaces with respect to the y-axis as indicated in the end view of Fig. 1 (i.e., the spacing between the planes being smaller toward the north or increasing Y). Thus introduce a new h_0 function defined by

$$h_0 = s_0 y + h_0^*(x, y)$$

where $s_0 = sE^{-1/2}$, s is the slope of the plane surfaces and $h_0^*(x, y)$ is the height function for the topographic feature to be considered. Thus (1.1) becomes

$$\frac{D\zeta_0}{Dt} = \frac{a}{\sqrt{2}}\zeta_0 - s_0 a\psi_x - aJ(\psi, h_0^*). \tag{1.2}$$

Let us now consider as an example the flow over a long step topographic feature.

2. Flow over a step

If one considers a topographic feature of constant cross-section and of infinite length and assumes a steady state solution, relation (1.2) simplifies to a linear ordinary differential equation with constant coefficients. Let us consider the flow over a step-topography oriented along the y-axis for example. The analytical solution along with some experimental results for this problem for the case in which the β-term is identically zero (i.e. $s = 0$) was given in BOYER (1971b). Figure 2 is an example of the flow over a step for the case in which $s = 0$. It is presented here for comparison with the flow in which $s \neq 0$, as below.

The solution for $s \neq 0$ is obtained in the same manner as that for $s = 0$ (see GUALA, 1971, for example). The geostrophic solution is obtained by solving (1.1) in three regions: upstream, downstream, and above the step. Since the flow is assumed independent of y, the stream function in each of these regions can be written as

$$\psi = -y + f(x), \tag{2.1}$$

where $f(x)$ is to be determined in each region. Substituting (2.1) into (1.2) and integrating once, one obtains

$$f_{xx} + \frac{a}{\sqrt{2}} f_x + af = A, \tag{2.2}$$

where A is a constant (different in each region).

Vertical shear layers located at the edges of the step separate these geostrophic regions. The analysis of these layers is identical to that given in BOYER (1971b); i.e. one determines that there is a balance between inertial and Coriolis effects. The analysis provides the boundary conditions for the determination of $f(x)$ in the three regions, i.e.

$$\left.\begin{aligned} f(-{}^1/_2)^- &= f(-{}^1/_2)^+, \\ f_x(-{}^1/_2)^- &= f_x(-{}^1/_2)^+, \\ f_{xx}(-{}^1/_2)^- - f_{xx}(-{}^1/_2)^+ &= h_0 a, \end{aligned}\right\} \quad \text{leading edge;}$$

$$\left.\begin{aligned} f(^1/_2)^- &= f(^1/_2)^+, \\ f_x(^1/_2)^- &= f_x(^1/_2)^+, \\ f_{xx}(^1/_2)^- - f_{xx}(^1/_2)^+ &= h_0 a, \end{aligned}\right\} \quad \text{trailing edge;}$$

where h_0 is the step height and the $-$ and $+$ signs represent regions to the left and right, respectively, of the shear layer in question.

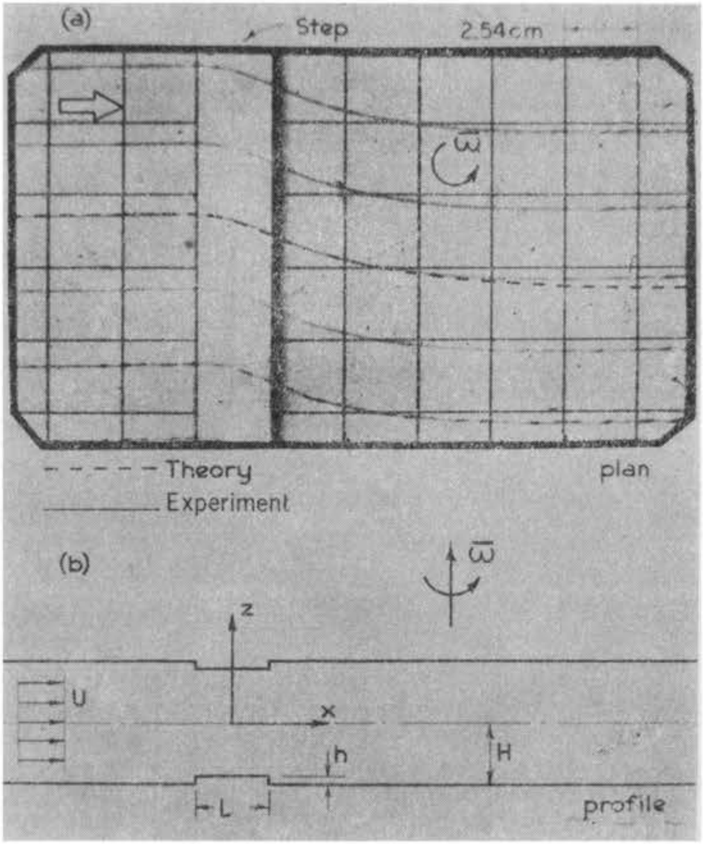

FIG. 2. Experimental and theoretical (dashed line) streamline patterns for flow over a step topography without β-effect. $E = 1.78\times10^{-3}$, Ro $= 5.05\times10^{-2}$, $H/L = 0.75$, $h/L = 0.033$ and $s = 0$. The flow is from left to right and the rotation is counterclockwise.

For upstream we require the flow to be a uniform rectilinear free stream; i.e.

$$f(x \to -\infty) = 0.$$

Downstream the lateral velocity component vanishes; i.e.

$$f_x(x \to \infty) = 0.$$

The solution of (2.2) subject to the above boundary conditions is straightforward, is given in GUALA (1971), and is thus not reproduced here. Relation (2.2) is the classic damped harmonic oscillator equation and one thus obtains "damped", "critically damped" and "overdamped" streamline patterns. The parameter range which can be investigated in the laboratory restricts to the "damped" regime. One parameter of interest is the wavelength λ of the oscillation which is determined as

$$\lambda = \frac{4\sqrt{2}\pi}{\sqrt{8as_0 - a^2}}.$$

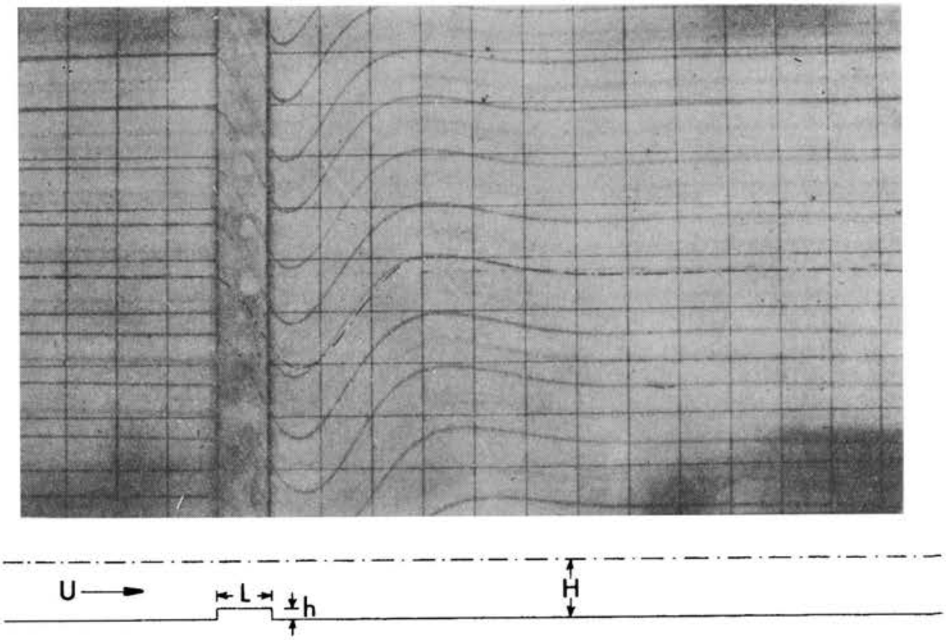

FIG. 3. Experimental and theoretical (dashed line) streamline patterns for flow over a step topography. $E = 6.5\times10^{-4}$, Ro $= 2.36\times10^{-2}$, $H/L = 0.69$, $h/L = 6.25\times10^{-2}$ and $s = 2.43\times10^{-2}$. The flow is from left to right and the rotation is counterclockwise.

Figure 3 is an experimental run for the flow over a step topography. The parameters for the flow are given in Table 1.

Table 1

$L = 2.54$ cm,	$E = 6.5\times10^{-4}$,
$H = 1.76$ gm,	$R_0 = 2.36\times10^{-2}$,
$h = 1.67\times10^{-1}$ cm,	$h/L = 6.57\times10^{-2}$,
$\nu = 8.4\times10^{-3}$ cm^2/sec,	$H/L = 0.69$,
$\omega = 1.0$ rad/sec,	$s = 2.43\times10^{-2}$.
$U = 1.2\times10^{-1}$ cm/sec,	

The experimental streamlines are made visible by introducing a neutrally buoyant tracer from a series of hypodermic needles upstream of the step and in the mid-plane of the water tunnel (for more details see BOYER 1971b). The flow in the central portion of the tunnel (i.e. away from the lateral walls) is approximately independent of the lateral coordinate (y) and thus to a good approximation represents flow over an infinite step. The dashed line is the analytical solution of (2.2) for the parameters in Table 1 and subject to the boundary conditions above.

The most striking differences between the non-β flow (Fig. 2) and the flow with β-effects (Fig. 3) are the Rossby wave patterns in the latter and the displacement of the streamlines to the right of the upstream positions in the former.

It should also be noted that theoretical considerations as above for an easterly flow (i.e. the one toward negative x) leads to an unstable flow with $f(x)$ growing without limit downstream of the topography. Experiments were conducted for such a situation. The experimental flow patterns were qualitatively similar to those shown in Fig. 2 in which the β-effect was not included. Since the side walls of the channel do not allow unbounded motions to occur, it is not surprising that unstable downstream flows are not obtained.

The above theoretical results are qualitatively similar to those obtained by Porter and Rattray (1964). The above formulation, however, is of a considerably simpler form than that of Porter and Rattray.

3. On the existence of standing Rossby waves in the Antarctic Circumpolar Current

No direct ocean current measurements (i.e., with current meters) are available which indicate the existence of Rossby wave patterns downstream of topographic features on the ocean floor. A recent analysis by Gordon and Bye (1972), however, does provide some evidence of the existence of such waves in the Antarctic Circumpolar Current.

Figure 4 is a plot from their paper of the anomaly of the sea surface dynamic height relative to the 2500-db isobaric surface (reproduced here with permission of the authors). The dynamic topography is obtained from direct measurements of the temperature and salinity. From the given temperature and salinity distributions one calculates the attendant density field. Then assuming that a particular horizontal surface is an isobaric one (i.e. 2500 meters in the Gordon-Bye calculation), the pressure field relative to that isobaric surface can be calculated. Figure 4 is a plot of the dynamic topography so-obtained at the sea surface, with the numbers given in terms of dynamic meters (see Sverdrup, Johnson and Fleming 1942).

Assuming that frictional effects are negligible and that there is a balance between pressure and Coriolis forces, these lines of equal dynamic height are parallel to streamlines with the direction of the flow being such that the higher dynamic topography is to the left of the velocity in the Southern Hemisphere; i.e. eastward flow in Fig. 4.

The above considerations provide an estimate of the flow field relative to that occurring at the 2500 meter level. To obtain the absolute surface velocity one would then have to add the flow field occurring at 2500 meters. Since no direct measurements of the flow at that depth (or any other) are available one can but suggest that the surface flow and that at 2500 meters are qualitatively similar (e.g. any Rossby waves in the implicit calculation would also occur in the flow at 2500 meters).

One must also emphasize that calculations such as those made to obtain Fig. 4 generally utilize data obtained over periods of many years and thus at most flow patterns so deduced

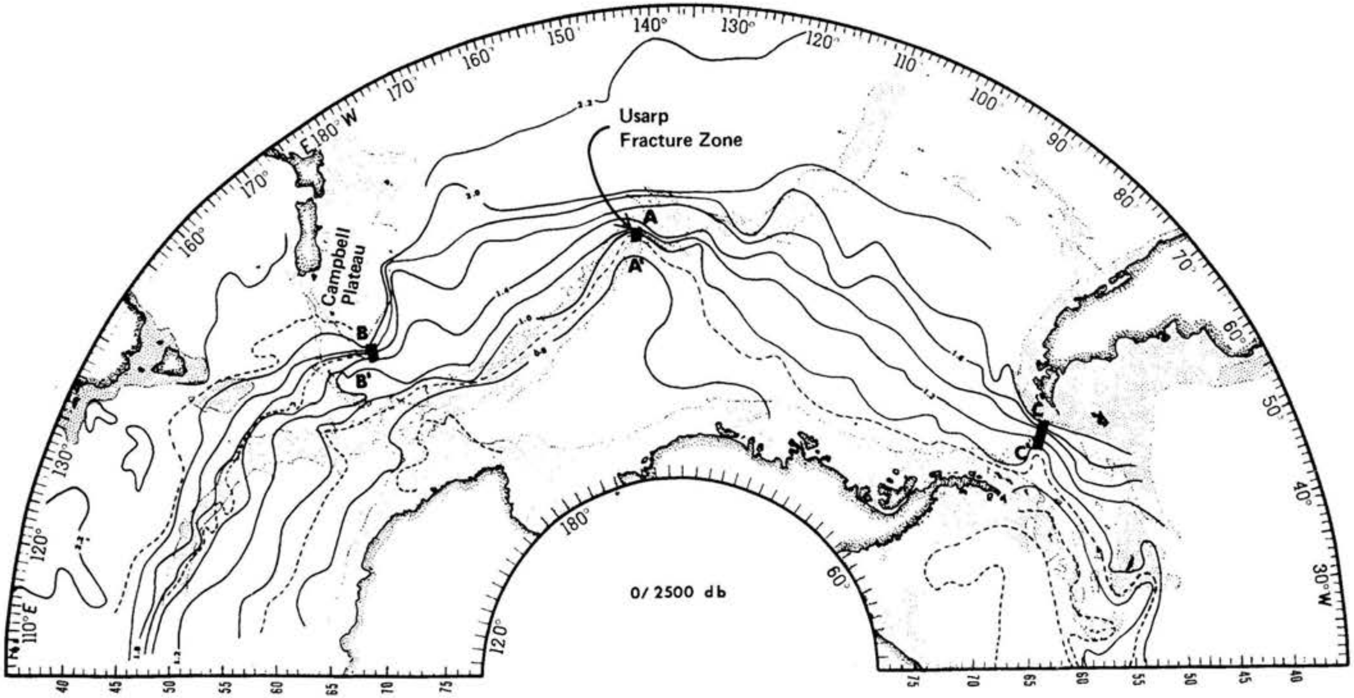

FIG. 4. Sea surface dynamic height anomaly relative to the 2500-db level. The depths less than 3000 meters are shaded. Dynamic-height isobars are given in dynamic meters.

must be considered as due to "average conditions." While implicit calculations are thus not entirely satisfactory, such measurements are the only ones presently available for providing a measure of the synoptic state of the oceans. They must thus be used, but with caution.

Returning to Fig. 4 one notes two regions in which a damped Rossby wave-like pattern occurs. The first is to the east (i.e. downstream) of the Campbell Plateau (170° W) and the other to the east of the USARP Fracture Zone (near 120° W).

In very general terms the topography near the USARP Fracture Zone may be approximated by a long ridge of constant cross-section such as the step considered above. One should note that the step flow in Fig. 3 is for positive upward or Northern Hemisphere rotation. In order to obtain the Southern Hemisphere equivalent, one transforms $y \to -y$ in the photograph. Our comparison here is meant to point out the qualitative similarities of the laboratory and oceanographic flows and, in particular, the damped Rossby wave pattern downstream of the topographic features.

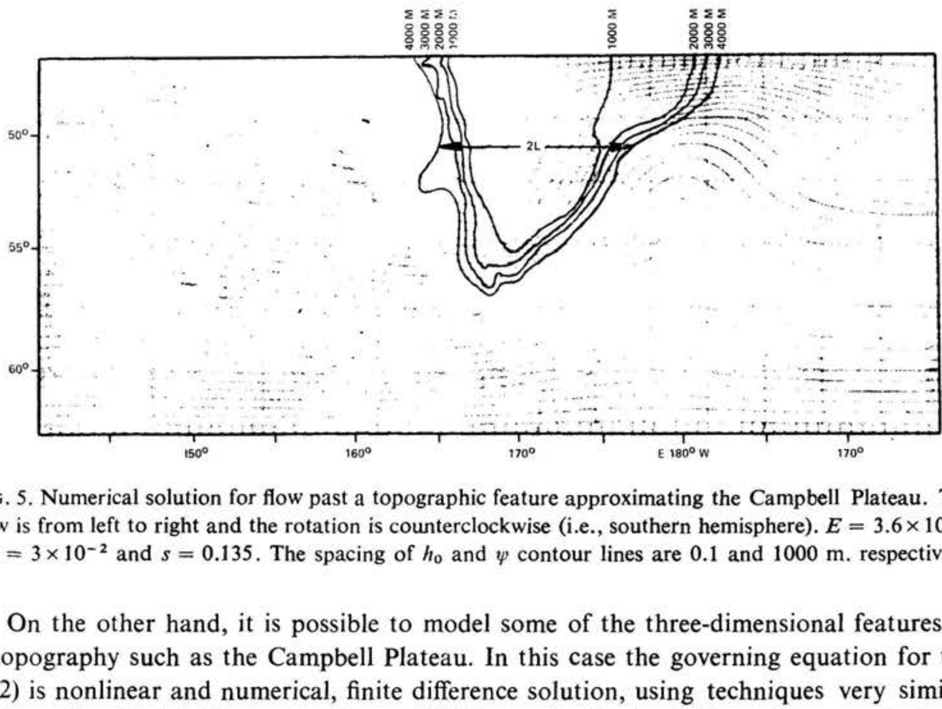

FIG. 5. Numerical solution for flow past a topographic feature approximating the Campbell Plateau. The flow is from left to right and the rotation is counterclockwise (i.e., southern hemisphere). $E = 3.6 \times 10^{-3}$, $Ro = 3 \times 10^{-2}$ and $s = 0.135$. The spacing of h_0 and ψ contour lines are 0.1 and 1000 m. respectively.

On the other hand, it is possible to model some of the three-dimensional features of a topography such as the Campbell Plateau. In this case the governing equation for the (1.2) is nonlinear and numerical, finite difference solution, using techniques very similar to those described in VAZIRI and BOYER (1971), may be obtained. Figure 5 is a preliminary numerical experiment for the flow past a topographic feature approximating the Campbell Plateau. Again one notes qualitative similarity of the flows in the lee of the topographic feature and those calculated by Gordon and Bye (Fig. 4). More work on the numerical modeling of these topographies is in progress.

In summary, these results suggest that simple laboratory and finite difference numerical models can be used to demonstrate some of the large scale features of the ocean current systems.

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