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ON A SPECIAL CLASS OF QUESTIONS ON THE THEORY OF PROBABILITIES.

[*Birmingham British Association Report* (1865), p. 8.]

AFTER referring to the nature of geometrical or local probability in general, the author of the paper drew attention to a particular class of questions partaking of that character in which the condition whose probability is to be ascertained is one of pure form. The chance of three points within a circle or sphere being apices of an acute or obtuse-angled triangle, or of the quadrilateral formed by joining four points, taken arbitrarily within any assigned boundary, constituting a reentrant or convex quadrilateral, will serve as types of the class of questions in view. The general problem is that of determining the chance that a system of points, each with its own specific range, shall satisfy any prescribed condition of form. For instance, we may suppose two pairs of points to be limited respectively to segments of the same indefinite straight line: the chance of their anharmonic ratio being under or over any prescribed limit will belong to this category of questions, to which, provisionally, the author proposed to attach the name of form-probability. In questions of form-probability, in which all the ranges are either collinear segments or coplanar areas, or defined portions of space, rules may be given for transforming the data, so as to make the required probability depend on one or more probabilities of a simpler kind, leading to summations of an order inferior by two degrees to those required by the methods in ordinary use. Thus Mr Woolhouse's question relating to the chance of a triangle within a circle or sphere being acute can be made to depend upon an easy simple integration, the solutions heretofore given of this problem involving complicated triple integrals. It was shown, as a further illustration, that the form-probability of a group of points all ranging over the same triangle remains unaltered when the range of one of them is limited to any side of the triangle chosen at will, and, again, (for convenience of expression distinguishing the contour into a base and two sides) will be the mean of the two probabilities resulting from limiting one point to range over either side with uniform probability, and simultaneously therewith a second point of the group over the base, with a probability varying as its distance from that end of the base in which it is met by the side. An analogous rule can be given for transforming the form-probability of a group limited to any the same parallelogram. So again for a group of points ranging over a plane figure bounded by any curvilinear contour. The problem may be transformed by supposing two of the points of the group to range on the contour itself, according to a law which may be expressed by saying that the probability of their being found on any arc shall vary as the product of the segment included between the arc and its chord, multiplied by the time of describing the arc about any centre of force arbitrarily chosen within or upon the contour,—a theorem which, accepting the idea of negative probability, admits also of extension to the case of a centre of force exterior to the contour.

Among other problems which the author readily resolves by aid of his principle of transformation, may be mentioned that of determining the mean value of a triangle whose angles are taken at random anywhere within a given triangle, parallelogram, ellipse, or ellipsoid. In this description of questions a peculiar difficulty arises, from the fact that the figure which is to be integrated in order to determine the numerator of the fraction which gives its mean value must always be taken positive, whereas its algebraical expression will repeatedly change its sign, according to a more or less complicated law. This quality of the analytical exponent of the arithmetical value of the figure constitutes, in fact, a sort of polarization which has to be got rid of; and the depolarizing process is effected with great ease by virtue of the simplified form impressed upon the data by the method set forth in the paper.

The author further took occasion briefly to allude to the form in which his own problem of four and Mr Woolhouse's problem of three points were originally proposed, viz. in each case without a specified boundary, and to express his opinion that the principle which had been applied to them, and in which he had formerly acquiesced, was erroneous, as it could be made to lead to contradictory conclusions, and must be abandoned. He was strongly inclined to believe that, under their original form, these questions do not admit of a determinate solution.