62.

ON THE DEGREE AND WEIGHT OF THE RESULTANT OF A MULTIPARTITE SYSTEM OF EQUATIONS.

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LET there be $(1+n)$ equations each homogeneous in any number of sets of variables, and suppose that the degrees of the several equations in respect to these sets are respectively

$$\begin{array}{l} a,\ b,\ c,\ \ldots,\ l, \\ a_1,\ b_1,\ c_1,\ \ldots,\ l_1, \\ a_2,\ b_2,\ c_2,\ \ldots,\ l_2, \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ a_n,\ b_n,\ c_n,\ \ldots,\ l_n, \end{array}$$

where the a, b, c, &c. are any positive integers, zero not excluded.

Let the number of variables in the several sets be respectively $1+\alpha$, $1+\beta$, $1+\gamma$, ... $1+\lambda$, then in order that the system may have a resultant, since the number of ratios to be eliminated is $\alpha+\beta+\gamma+\ldots+\lambda$, this sum must be equal to n.

Let $$a_i\rho+b_i\sigma+c_i\tau+\ldots+l_i\omega=L_i,$$

and let $$LL_1L_2\ldots L_n=P,$$

then 1st, the degree of the resultant in question in regard to the coefficients of the rth equation will be the coefficient of $\rho^\alpha.\sigma^\beta.\tau^\gamma\ldots\omega^\lambda$ in $\dfrac{P}{L_r}$.

2nd. As regards weight. By the weight of any letter in respect to any given variable is to be understood the exponent of that variable in the term affected with the coefficient; and by the weight of any term of the resultant in respect to such variable, the sum of the weights of its several simple factors;

each term in the resultant in respect to any given variable has the same weight; and this weight may also be proved to be alike for each variable in the same set, and may be taken as the weight of the resultant in respect to such set. This being premised, we have the following theorem :—

The value of the weight of the resultant in respect to any particular set of the variables, for example, the $(1+\alpha)$ set, will be the coefficient of

$$\rho^{1+\alpha} . \sigma^{\beta} . \tau^{\gamma} \ldots \omega^{\lambda} \text{ in } P.$$

In the particular case where $\alpha = \beta = \gamma \ldots = \lambda$, the above expressions for the degree and weight evidently become polynomial coefficients. Thus, for example, if we suppose each equation *linear* in respect to the variables of each set, the degree of the resultant in respect to the coefficients of any equation will be

$$\frac{(\alpha+\beta+\gamma+\ldots+\lambda)!}{\alpha!\,\beta!\,\gamma!\ldots\lambda!},$$

and its weight in respect to the $(1+\alpha)$ set will be

$$\frac{(1+\alpha+\beta+\ldots+\lambda)!}{(1+\alpha)!\,\beta!\,\gamma!\ldots\lambda!}.$$

In particular if each set is binary, so that $\alpha = \beta = \gamma \ldots = \lambda = 1$, the degree becomes $n!$, and the weight $\frac{1}{2}(n+1)!$.

The above theorems are, I believe, altogether new.

It may just be noticed (as a passing remark) that the total degree in the general case is the coefficient of

$$\rho^{\alpha} . \sigma^{\beta} . \tau^{\gamma} \ldots \omega^{\lambda} \text{ in } P\left\{\frac{1}{L} + \frac{1}{L_1} + \ldots + \frac{1}{L_n}\right\},$$

and the *total* weight the coefficient of the same argument in

$$P\left\{\frac{1}{\rho} + \frac{1}{\sigma} + \ldots + \frac{1}{\omega}\right\}.$$