59.

ON THE QUANTITY AND CENTRE OF GRAVITY OF FIGURES GIVEN IN PERSPECTIVE, OR HOMOGRAPHY.

[*Newcastle-on-Tyne British Association Report* (1863), p. 2.]

In the first instance, the author showed how to find the point in the perspective representation of a plane figure into which the centre of gravity of such figure is projected. For this purpose it is only necessary to be furnished with the direction of the vanishing-line corresponding to the plane of the object put into perspective. The rule for finding the point in question is the following: every element of the picture is to be charged with a density equal to the inverse fourth power of its distance from the vanishing-line; the centre of gravity of the figure so charged will be the point required, and may of course be found by the rules of the integral calculus.

Next, as to the area of the unknown object. To determine this another datum (but only one other) is required besides the direction of the vanishing-line, which may be termed the constant of perspective, being determined when the position of the eye and that of the object-plane in reference to the picture are given. This constant is the product of the eye's distance from the vanishing-line into the square of the distance of the intersections of the object- and picture-planes from the same line. If now every element of the picture be charged with a density equal to the constant of perspective divided by the *cube* of the element's distance from the vanishing-line, the mass of the figure so charged will be the area of the unknown object-figure.

The author then proceeded to show how the area and the perspective centre, by aid of the preceding principles, admit of being reduced to depend on one single integral, closely analogous to the *potential* used in the theory of attractions to which he gives the name of *polar potential*. The polar potential of a plane figure in respect to a given line is defined to be the sum

of the quotients of the elements by their respective distances from the line, and consequently the polar potential of the picture in respect to a vanishing-line in its plane becomes a function of the two parameters by which its position may be determined. The parameters which the author finds most convenient to employ are the distance of the vanishing-line from an arbitrary fixed point in the picture and the angle which it makes with a fixed line therein.

The author then supplied the formulæ (which are of a very simple character) for calculating the area of the object and the coordinates of its perspective centre of gravity, by means of differentiation processes performed upon the polar potential of the picture treated as a function of these parameters. He afterwards proceeded to extend the same method to figures, plane or solid, connected by the more general relation known under the name of homography, of which the relation between figures generated through the medium of perspective is only a particular kind. In the case of a solid figure, its polar potential in respect to a variable plane becomes a function of three parameters; and by means of differentiations performed upon it in respect to these parameters, the content and the coordinates of the point corresponding homographically to the centre of gravity of a solid figure may be expressed when its homograph and the position of a plane corresponding to the points at infinity in the otherwise unknown figure are given in addition (as regards the content) to a certain constant termed the homographic determinant.

Professor Rankine threw out a suggestion as to the possibility of a practical application of the preceding theory to the stability of structures standing to each other in a certain simple relation of homography.