55.

ON THE SOLUTION OF THE LINEAR EQUATION OF FINITE DIFFERENCES IN ITS MOST GENERAL FORM.

THE author exhibited (and illustrated with examples) a simple and readily applied method of obtaining the general term of, and consequently the complete, solution of an equation of finite differences with any number of independent variables, a question which, although touched upon by Libri and laboriously investigated by Binet, had hitherto, to the best of his knowledge, remained unsolved even in the case of an equation with but one independent variable with non-constant coefficients; when the coefficients are supposed constant, the well-known solution flows as an immediate corollary from the author's general form. Essentially the method depends upon the adoption of a natural principle of notation for the given coefficients, according to which each coefficient is to be denoted by a *twofold* group of indices, the number of the double indices in a group being equal to the number of independent variables in the given equation. Thus, supposing $u_{m, n, p \dots}$ to be expressible, by means of the given general equation, as a sum of u's with inferior indices, the coefficient of $u_{\mu, \nu, \pi \dots}$ in that sum must be denoted by the double index group

$$\begin{bmatrix} m, & n, & p \dots \\ \mu, & \nu, & \pi \dots \end{bmatrix}.$$

The process for obtaining the general term in $u_{x, y, z \dots}$ is then shown to be reducible virtually to the problem of effecting the simultaneous *decomposition* of the integer variables $x, y, z \dots$ into *parts* in every possible manner and order of relative arrangement, the magnitudes of such parts being limited by the degree or degrees of the given equation in respect of these variables. The collective value of the terms thus obtained constituting the complete solution may be termed, in the author's nomenclature, a hyper-cumulant, whose properties and their applications remain to be studied, as those of the elementary kinds of common cumulants have been, to a considerable extent, in the ordinary theory of continued fractions. The first stage in the process of constructing the terms of a general cumulant or general hyper-cumulant is almost identical with that of finding the coefficients in the expansion of a power of a polynomial function of one or several variables, differing from it indeed only in the circumstance that permutations which lead to repetitions in the latter case, represent distinct values in the former.

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