49.

REMARK ON THE TACTIC OF NINE ELEMENTS.

[*Philosophical Magazine*, XXII. (1861), pp. 144—147.]

AT the end of my preceding paper in this *Magazine* for July [p. 284 above], I hazarded an opinion that any grouping of 28 synthemes comprising the 84 triads belonging to a system of 9 elements, might be regarded as made up of 1 syntheme of monomial triads, 18 synthemes of binomial triads, and 9 of trinomial triads, the denominations (monomial, binomial, and trinomial) having reference to a duly chosen distribution of the 9 elements into 3 nomes of 3 elements each. This conjecture is capable of being brought to a very significant, although not decisive test, by examining a peculiar and important distribution of the 28 synthemes into 7 sets of 4 synthemes each, the property of each set being that its 12 triads contain amongst them all the 36 *duads* appertaining to the 9 elements. I discovered this mode of distribution very many years ago; but it was first published independently by a mathematician whose name I forget, either in the *Philosophical Magazine* or in the *Cambridge and Dublin Mathematical Journal*, I think at some time between the years 1847–53. A similar mode of distribution exists for any system of elements of which the number is a power of 3. Without pausing to give the law of formation, I shall simply observe that for 9 elements we may take as a basic arrangement the square

$$\begin{array}{ccc} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{array}$$

and form from this, by a symmetrical method, the annexed six derived arrangements :—

$$\begin{array}{ccc} 7\,1\,2 & 7\,2\,3 & 9\,2\,3 \\ 3\,5\,6 & 1\,4\,5 & 1\,5\,6 \\ 4\,8\,9 & 6\,8\,9 & 4\,7\,8 \\ & & \\ 4\,3\,1 & 5\,2\,3 & 4\,2\,3 \\ 7\,5\,6 & 7\,6\,4 & 8\,5\,6 \\ 2\,8\,9 & 1\,8\,9 & 1\,9\,7 \end{array}$$

and reading off each of these squares in lines, in columns, and in right and left diagonal fashion, we obtain the 7 sets of 4 synthemes each referred to, namely

123 456 789
147 258 369
159 267 348
168 249 357

712 356 489	723 145 689	923 156 478
734 158 269	716 248 359	914 257 368
759 164 238	749 256 318	958 264 317
768 139 254	758 219 346	967 218 354

431 756 289	523 764 189	423 856 197
472 358 169	571 268 349	481 259 367
459 362 178	569 241 378	457 261 389
468 379 152	548 279 361	469 287 351

If, now, we take any distribution of the 9 elements into nomes other than 123, 456, 789, we shall find that some of the synthemes will contain trinomials, some binomials only, but others (in number either 9 or 18, according to the distribution chosen) will contain binomials and trinomials mixed; but if we adopt 123, 456, 789 as the nomes, then it will be found that the remaining 27 synthemes (after excluding the monomial syntheme 123, 456, 789) will consist of 18 purely binomial triads, and 9 purely trinomial triads. The former will consist of the first, second, and fourth synthemes of the 6 derived groups: the latter of the second, third and fourth of the basic group, and of the second synthemes of each of the 6 derived groups.

It may be remembered that there are two types or species of groupings of trinomial triad synthemes appertaining to 3 nomes of 3 elements; one of these species contains 4, the other 36 individual groupings. It may easily be ascertained that the grouping above indicated belongs to the first (the less numerous) of these species. Again, there are 3 types or species of groupings of binomial triad synthemes appertaining to the same system of nomes; one containing 12, one 24, and the third 108 groupings. The grouping with which we are here concerned will be found to belong to the *second* of these species,—that denoted by the symbols $\left|\begin{matrix}\alpha\\ \epsilon\end{matrix}\right|$ in my paper of last month. Hence, then, we derive a very considerable presumption in favour of the opinion which I advanced at the close of my preceding paper on Tactic, and derived, too, from a case apparently unfavourable to the verisimilitude of the conjecture; for a natural subdivision of 28 things into 7 sets of 4 each seems at first sight hardly compatible with another natural division into 3 sets of 1, 18, and 9 respectively. Notwithstanding this seeming incompatibility, we have found that the two methods of decomposition do coexist, owing essentially to the fact that the 7 sets (of 4 synthemes each) stand not in a relation of indifference set to set, but are to be considered as composed of a base and 6 derivatives indifferently related to the base and to each other. The theory of these 7 sets is extremely curious, and well worthy of being fully investigated by the student of tactic, but cannot be gone into within the limits suitable to the pages of a philosophical miscellany.

Before taking final leave of the subject (at all events for the present, and in the pages of this *Magazine*), as I have been questioned as to the meaning of the important word "syntheme," derived from *συν θημα*, I repeat that a "syntheme" is the general name for any consociation of the single or combined elements of a given system of elements in which each element is once and once only contained. A *nome,* from *νεμω* (*to divide*), means a consociation of a certain number out of a given system of elements; and a binomial, trinomial, or r-nomial combination of any specified sort, means a combination whose elements are dispersed between 2, 3, or r of the nomes between which the entire system of elements is supposed to have been divided.

P.S. I have found the date and place of the resolution into 7 sets referred to in the text; it is given in a paper by Mr Kirkman, Vol. v. p. 261 of the *Cambridge and Dublin Mathematical Journal* for 1850. His 7 squares, whose horizontal, vertical, and two diagonal readings (like mine) constitute the 7 sets in question, are substantially as follows:—

	1 2 3	
	4 5 6	
	7 8 9	
1 2 4	4 5 7	7 8 1
5 6 7	8 9 1	2 3 4
8 9 3	2 3 6	5 6 9
7 1 2	1 4 5	4 7 8
3 4 5	6 7 8	9 1 2
6 8 9	9 2 3	3 5 6*.

* By changing the positions of the lines and columns of the six derivative squares, which may be done without affecting the value of their *readings*, they may be represented under the form following, which will be seen to render much clearer their relation to the primitive square:—

412	623	423	239	129	127
756	745	956	451	563	453
389	189	178	786	784	896.

On assuming 123, 456, 789 as the three nomes, the 28 synthemes contained in the sets will be found to consist of purely monomial, binomial, and trinomial synthemes.

Thus there would be an additional presumption in favour of the supposed law of *homonomial resolubility*, provided that Mr Kirkman's solution were essentially distinct in type from my own; his binomial and trinomial systems, taken separately, coincide in type with those afforded by my solution, notwithstanding which it would not be lawful to assume (indeed I had at first some reasons for doubting) the identity of type of the total groupings of which these systems form part; all we could have positively inferred from that fact would have been, that these two groupings both belong to the same class or genus containing 26,880 individuals, the second of the six referred to at the close of my last paper; a comparison of the two solutions has, however, satisfied me that they are absolutely identical in form.