19.

NOTE ON A FORMAL PROPERTY OF A LATENT INTEGER.

[*Quarterly Journal of Mathematics*, I. (1857), p. 185.]

THE following was proposed some years ago by an author, whose name I do not recollect, among the mathematical questions in the *Educational Times**.

"Required to prove that the integer part of $(1+\sqrt{3})^{2m+1}$ contains 2^{m+1} as a factor."

The proof probably ran, or at all events might have run, as follows:

$(1-\sqrt{3})^{2m+1}$ being a negative fraction less than unity, the integer part of $(1+\sqrt{3})^{2m+1}$ is evidently

$$(1+\sqrt{3})^{2m+1}+(1-\sqrt{3})^{2m+1},$$

or is the sum of the $(2m+1)$th powers of the roots of the equation

$$x^2-2x-2=0,$$

from which the truth of the proposition is manifest.

We may add the remark that it may easily be shown in like manner that the integer next *above* the fraction $(1+\sqrt{3})^{2m}$, will also contain 2^{m+1} as a factor; and more generally, if we suppose that a is that integer congruent *quâ* the modulus 2 with n, which is next above or next below $\sqrt{n}$, then in the former case the two integers next above $(\sqrt{n}+a)^{2m+1}$ and $(\sqrt{n}+a)^{2m}$ respectively, and in the latter case the two integers next below the first and next above the second respectively, will each of them contain the factor 2^{m+1}.

The student is invited to ascertain whether any analogous theorem exists for latent integers expressed by means of higher surd forms.

* Questions of a similar nature, I am informed by Mr Ferrers, appeared in the Cambridge Senate House problems for the years 1847 and 1848.