6.

NOTE ON A POINT OF NOTATION.

[*Philosophical Magazine*, VII. (1854), pp. 50—51.]

It frequently becomes important in algebraical investigations, and in the representation of results, to have a means of expressing that the sign + or − is to be affixed to an algebraical expression, according as certain indices $\theta_1, \theta_2, \theta_3 \ldots \theta_n$ which occur therein, and which represent the natural numbers from 1 to n in some regular or irregular order, can be derived from the fundamental arrangement $1, 2, 3 \ldots n$ by an even or by an odd number of interchanges. An example of this occurred in my short paper in the last Number of the *Philosophical Magazine*, on the extension of Lagrange's Rule of Interpolation [Vol. I., p. 646], where I used to denote that such a choice of signs was to be made, the awkward and unsuggestive symbol "?" There exists, however, a very simple algebraical mode of denoting the presence of the factor +1 or −1, according to the order of the natural numbers in the scale $\theta_1, \theta_2, \theta_3 \ldots \theta_n$.

ζ has been always consecrated by me to the purpose of signifying that the product of the squared differences is to be taken of the elements with which it is in regimen; and in the paper adverted to I introduced the highly convenient new symbol $\zeta^{\frac{1}{2}}$ to denote that the product is to be taken of the simple differences obtained by subtracting from each element in regimen therewith every subsequent element in the arrangement of the elements as set down. By aid of this new symbol $\zeta^{\frac{1}{2}}$, the positive or negative character of any permutation, as $\theta_1, \theta_2 \ldots \theta_n$, can be completely expressed; for

$$\zeta^{\frac{1}{2}}(\theta_1, \theta_2, \theta_3 \ldots \theta_n) \div \zeta^{\frac{1}{2}}(1, 2, 3 \ldots n)$$

will be +1 or −1 according as $1, 2, 3 \ldots n$ and $\theta_1, \theta_2, \theta_3 \ldots \theta_n$ belong to the same group, or to opposite groups in the natural dichotomous separation of the permutations of the n symbols in question, and thereby the desired object of giving a functional representation of the ambiguous sign is perfectly attained.