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THE COLLECTED MATHEMATICAL PAPERS OF JAMES JOSEPH SYLVESTER

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PREFATORY NOTE.

THE object aimed at in this volume has been to present a faithful record of the course of the author's thought, without such additions as recent developments of the subjects treated of might have afforded, and without any alterations other than that considerable number involved in the attempt to make the algebraical symbols read as the writer intended. While, for the reader's convenience, the author's references to his own papers have been accompanied by cross references to the pages of this volume, placed in square brackets.

By far the longest paper in the volume is No. 57, "On the Theory of the Syzygetic Relations of two Rational Integral Functions, comprising an application to the Theory of Sturm's Functions," and to this many of the shorter papers in the volume are contributory.

The volume contains also Sylvester's dialytic method of elimination (No. 9, etc.), his Essay on Canonical Forms (No. 34), and early investigations in the Theory of Invariants (Nos. 42, 43, etc.).

It contains also celebrated theorems as to Determinants (Nos. 37, 39, 48, etc.) and investigations as to the Transformation of Quadratic Forms (the *Law of Inertia*, No. 47, and the recognition of the Invariant factors of a matrix, Nos. 22, 24, 36).

A full table of contents is prefixed.

H. F. BAKER.

St John's College, Cambridge.
April, 1904.

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