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Delayed yield effect in dynamic flow of elastic/visco-perfectly plastic material

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THE PURPOSE of the paper is to analyze the influence of the delayed yield phenomenon on the dynamic stress process in the framework of the theory of ideally elastic/viscoplastic flow. The phenomenon is manifested by the increase of the initial dynamic yield stress in comparison with the static value. A numerical integration of the P. Perzyna's constitutive differential equations is done with the initial condition as the dynamic yield stress determined according to the J. D. Campbell's criterion. The perturbation of the initial condition for viscoplastic dynamic flow affects the very beginning of the stress process. The analysis performed indicates the direct relation between the plastic softening and the delayed yield phenomena. In the case of uniaxial tension, theoretical prediction of appearance of plastic softening is given. The comparison with an experiment is carried out.

Celem pracy jest analiza wpływu zjawiska opóźnienia plastycznego na proces dynamicznego naprężania materiału w ramach teorii płynięcia sprężysto/lepkoplastycznego. Zjawisko to objawia się wzrostem początkowej dynamicznej granicy plastyczności w porównaniu do wartości statycznej. Różniczkowe równania konstytutywne P. Perzyny scałkowano z warunkiem początkowym wyznaczonym zgodnie z kryterium dynamicznego uplastycznienia wg J. D. Campbell'a. Zaburzenie warunku początkowego dla procesu płynięcia lepkoplastycznego wpływa na charakter początkowego procesu naprężania materiału. Analiza wskazuje na ścisły związek zjawiska początkowego mięknienia plastycznego ze zjawiskiem opóźnienia. W przypadku jednoosiowego stanu naprężania i odkształcania podano prognozę teoretyczną występowania zjawiska mięknienia plastycznego. Przeprowadzono porównanie z eksperymentem.

Целью работы является анализ влияния явления пластического запаздывания на процесс динамического напряжения материала в рамках теории упруго-вязкопластического течения. Это явление проявляется ростом начального динамического предела пластичности по сравнению со статическим значением. Дифференциальные определяющие уравнения П. Пэжина проинтегрированы с начальным условием, определенным согласно критерию динамического перехода в пластическое состояние по Дж. Д. Кампбеллу. Возмущение начального условия, для процесса вязкопластического течения, влияет на характер начального процесса напряжения материала. Анализ указывает на тесную связь явления начального пластического мягчения с явлением запаздывания. В случае одноосных напряженного и деформационного состояний приведен теоретический прогноз выступания явления пластического мягчения. Проведены сравнения с экспериментом.

1. Introduction

RESULTS of experimental investigations show that most metals are strongly sensitive to dynamic straining, what is manifested by qualitative and quantitative differences between the stress-strain curves obtained in the static and dynamic tests.

The fundamental effect of this sensitivity occurring in the plastic range is known as the effect of material viscosity. The constitutive equations expressing the process of superposition of viscous and plastic properties are well known. The most general constitutive law is that proposed by P. PERZYNA [10]. In the case of perfectly plastic materials the law has the form

$$(1.1)\qquad \varepsilon_{ij} = \frac{\dot{s}_{ij}}{2\mu} + \frac{\dot{\sigma}}{3K}\delta_{ij} + \gamma\langle\Phi(F)\rangle\frac{s_{ij}}{\sqrt{J_2}}, \qquad F = \frac{\sqrt{J_2}}{k} - 1,$$

μ, K, γ, k denoting the Lamé constants, the viscosity coefficient and the static yield limit at pure shear, respectively; s_{ij} is the deviator of stress σ_{ij}. The second invariant of s_{ij} defines the dynamic yield surface for $F > 0$,

$$(1.2)\qquad \sqrt{J_2} = k\left[1 + \Phi^{-1}\left(\frac{\sqrt{I_p^2}}{\gamma}\right)\right],$$

where $I_p^2 = \frac{1}{2}\dot{\varepsilon}_{ij}^p\dot{\varepsilon}_{ij}^p$ is the second invariant of the viscoplastic strain rate tensor. Tensor $\dot{\varepsilon}_{ij}^p$ is determined by the third R.H.S. term of (1.1) and depends on the excess of the actual yield surface over the static surface $F = 0$. Symbol $\langle\Phi(F)\rangle$ denotes a function defined according to the value of the argument F,

$$(1.3)\qquad \langle\Phi(F)\rangle = \begin{cases} \Phi(F) & \text{if} \quad F > 0, \\ 0 & \text{if} \quad F \leqslant 0. \end{cases}$$

The conditions given by Eq. (1.3) express the assumption of disregarding the material sensitivity to the strain rate in the elastic range and determine, at the same time, the extent of the elastic range as identical with the static yield surface. Thus the definition of $\langle\Phi(F)\rangle$ according to Eq. (1.3) leads to the conclusion that the static yield surface constitutes the initial condition of evolution of the dynamic viscoplastic yield surface. The viscoplastic component of the stress response of the material develops in the process of loading from zero and represents a smooth, regular evolution after passing the elastic limit. The reaction may be determined by integrating the set of differential equations (1.1) for the assumed process of forced strains $\dot{\varepsilon}_{ij}(t)$. Behaviour of the material after yielding, for $F > 0$, is uniquely determined by the character of the forced straining process, according to the viscous properties assumed.

Another effect manifested during the dynamic processes of metal deformation is the increase of the initial yield limit with respect to the corresponding statical value. Such an effect is typical for such metals as pure iron, mild steel and is also observed in many other materials. The materials are capable of undergoing momentary overloading processes reaching beyond the yield limit σ_0 without loosing the elastic properties. This phenomenon is called the delayed yielding and it is experimentally found to result from the sensitivity of metals to the strain history in the elastic range. Obviously, the delayed yield affects the initial yielding of the material; the yielding process is usually irregular and may exhibit the plastic softening phenomena characterized by short-lived material instability following the instant of yielding.

Plastic softening takes place in perfectly plastic metals. This phenomenon is supposed to represent the spontaneous process of initial yielding with stresses changing from the so-called upper to lower yield points. Sometimes, however, softening is not observed at all or is less pronounced; non-repeatibility of this phenomenon or its variable intensity may be interpreted in different ways.

The influence of delayed yield on the initial dynamic yielding of an elastic/viscoplastic material will be studied in this paper. The method of the theoretical analysis applied preserves the basic concept of the constitutive relations (1.1) but makes use of a different initial condition for the viscoplastic yielding process. Certain modifications must be introduced to the conditions for $\langle \Phi(F) \rangle$ in Eq. (1.3). The dynamic yield limit defined according to the criterion proposed by J. D. CAMPBELL [3] will be assumed as the initial condition. The dynamic yield stress may considerably exceed its static value and, hence, the procedure proposed becomes equivalent to the assumption that the viscoplastic yielding process starts at a high cumulation of the elastic energy in the material. Certain portion of the energy may have the tendency to a violent transformation into the dissipation energy immediately after the dynamic yielding, what may produce a local instability of the material, its plastic softening. The analysis presented in the paper indicates that this instability is due to both the plastic viscosity and the forced strain intensity. The results obtained indicate that the law proposed by P. Perzyna is sufficiently accurate in describing the initial yielding process and the plastic delay effect.

2. Models of deformation with the delayed yield effect

2.1. Dynamic yielding criterion

The plastic yielding criterion, based on the dislocation theory by H. COTTRELL and B. A. BILBY, was proposed by J. D. CAMPBELL [3] in the form

$$\int_0^{t_d} \left[\frac{\sigma(t)}{\sigma_0} \right]^\alpha dt = t_0, \tag{2.1}$$

where σ_0 is the static yield stress, α and t_0 are material constants, and $\sigma(t)$ is the dynamic stress function of the material in the elastic range. Time t_d denotes the instant of dynamic yielding. The initial dynamic yield stress is then equal to

$$\sigma_{0d} = \sigma(t_d). \tag{2.2}$$

Constant t_0 has the dimension of time and a simple interpretation; namely, t_0 is the instant of time of dynamic yielding of the material subject to stresses $\sigma(t) = \sigma_0 = \text{const}$. Criterion (2.1) was proposed by J. D. CAMPBELL for mild steel under single uniaxial stressing. Author of the paper [3] was aware of the fact that dynamic yielding of materials was possible not only at loading but also at unloading. In the latter case the dynamic yield stress must not necessarily be the largest stress in the elastic stress history.

Criterion (2.1) was verified experimentally by numerous specialists; the problems of experimental investigation of the delayed yielding are extensively presented in the review paper [13]. Since the present paper is aimed at theoretical analysis of the phenomenon, let us briefly report the principal results of experimental data obtained thus far. The duration of delayed yielding was found to depend on the chemical constitution of steel, on the dimension of grains of its crystalline structure, on the methods of surface treatment and temperature. Experimental investigations were made to confirm the fact that constants α and t_0 were independent of the stress history $\sigma(t)$, [15]. In papers [11] and [4] it was shown, that the character of the plastic delay phenomenon is identical, in principle, in both the tension and compression tests. The material memory was investigated having in view the delayed yield taking place in two consecutive dynamic tests [12]; the delayed yielding could be observed in the first tests only. At the repeated loading the specimen behaved similarly as in the static case. According to J. KLEPACZKO [8], mild steel regains the ability to exhibit the delayed yielding effect after the time period of about $(\alpha+1)t_0$.

It should be noted that Campbell's criterion (2.1) does not make it possible to determine the accurate value of the static yield stress σ_0 as the limit of a sufficiently slow time-dependent loading process. It would be possible if the function appearing in brackets of Eq. (2.1) were assumed in the form of excess $(\sigma(t)/\sigma_0-1)$, [11, 13]. The time of delayed yielding should then be counted not from the beginning of the test $t = 0$ but from the instant $t = T$ for which $\sigma(T) = \sigma_0$.

From the point of view of possible applications in structural dynamics and wave propagation analysis it is important to decide upon the method of application of the criterion (2.1) in cyclic loading processes with consecutive amplitudes being higher and lower than the static yield stresses. The method of evaluation of the integral (2.1) is schematically shown in Fig. 1. Elimination of periods (t_1-t_2), (t_3-t_4) from the stress history ensures the possibility of determining the dynamic yield stress $\sigma_0 = \sigma(t) \geqslant \sigma_0$ from the criterion (2.1).

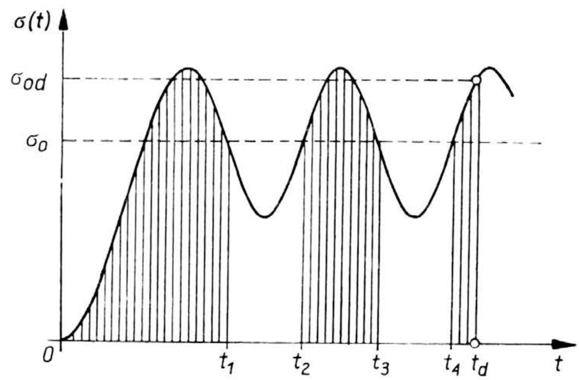


FIG. 1.

According to the author of paper [11], the question of application of the criterion (2.1) to the cases of alternating stresses (when the signs of stresses are different in consecutive overloading cycles) remains open. The problem of applying Campbell's criterion in the case of initial stresses is also questionable.

In the case of complex stress states Yu. N. RABOTNOV expressed the opinion [11] that application of the criterion (2.1) should necessitate the introduction of the stress intensity history $\sigma_i(t)$ insted of $\sigma(t)$,

$$\int_0^{t_d} \left[\frac{\sqrt{J_2(t)}}{k}\right]^{\alpha} dt = t_0. \tag{2.3}$$

Such a proposition is equivalent to the isotropic transformation of the static yield surface $F = 0$ into the initial dynamic yield surface.

The results following from the application of Campbell's criterion to the determination of delay time t_d and the initial dynamic yield stresses σ_{0d} are shown below. The corresponding relations for uniaxial stress states and various dynamic straining processes are:

$$(2.4)\qquad \begin{array}{l}\dot{\varepsilon}(t) = \dot{\varepsilon} = \text{const}\\ \sigma(t) = E\dot{\varepsilon}t\end{array} \left\{\begin{array}{l} t_d = \left[(\alpha+1)t_0\left(\dfrac{\sigma_0}{E\dot{\varepsilon}}\right)^{\alpha}\right]^{\frac{1}{\alpha+1}},\\ \sigma_{0d} = \sigma_0\left[(\alpha+1)t_0\dfrac{E\dot{\varepsilon}}{\sigma_0}\right]^{\frac{1}{\alpha+1}};\end{array}\right.$$

$$(2.5)\qquad \begin{array}{l}\ddot{\varepsilon}(t) = \ddot{\varepsilon} = \text{const}\\ \sigma(t) = \dfrac{1}{2}E\ddot{\varepsilon}t^2\end{array} \left\{\begin{array}{l} t_d = \left[(2\alpha+1)t_0\left(\dfrac{2\sigma_0}{E\ddot{\varepsilon}}\right)^{\alpha}\right]^{\frac{1}{2\alpha+1}},\\ \sigma_{0d} = \sigma_0\left\{[(2\alpha+1)t_0]^2\dfrac{E\ddot{\varepsilon}}{2\sigma_0}\right\}^{\frac{1}{2\alpha+1}};\end{array}\right.$$

$$(2.6)\qquad \begin{array}{l}\dot{\varepsilon}(t) = \dfrac{\sigma_m}{E}\omega\cos\omega t\\ \sigma(t) = \sigma_m\sin\omega t\end{array} \left\{\begin{array}{l} t_d = \dfrac{\tau_d}{\omega}, \text{ where } \tau_d \text{ according to } \dfrac{\omega t_0}{\left(\dfrac{\sigma_m}{\sigma_0}\right)^{\alpha}} = \displaystyle\int_0^{\tau_d}\sin^{\alpha}\tau\, d\tau,\\ \sigma_{0d} = \sigma_m\sin\tau_d;\end{array}\right.$$

$$(2.7)\qquad \begin{array}{l}\dot{\varepsilon}(t) = \dfrac{\sigma_m}{E}\omega\sin\omega t\\ \sigma(t) = \sigma_m(1-\cos\omega t)\end{array} \left\{\begin{array}{l} t_d = \dfrac{\tau_d}{\omega}, \text{ where } \tau_d \text{ according to } \dfrac{\omega t_0}{\left(\dfrac{\sigma_m}{\sigma_0}\right)^{\alpha}} = \displaystyle\int_0^{\tau_d}(1-\cos\tau)^{\alpha}d\tau,\\ \sigma_{0d} = \sigma_m(1-\cos\tau_d);\end{array}\right.$$

where E — modulus of elasticity, ω — strain rate frequency, σ_m — maximum elastic stress amplitude.

2.2. Theoretical concepts of application of the delayed yield effect

2.2.1. Delayed yield effect in the elastic-perfectly plastic deformation model. An approximate model of elastic-perfectly plastic, dynamic deformation was proposed in paper [11]. The elastic strain range in this model is determined by the upper dynamic yield stress σ_{0d} found from Eq. (2.1). Plastic softening is expressed by a rapid stress relaxation following the dynamic yielding at the static yield stress σ_0, Fig. 2. Further deformation of the material is determined by its ideal properties in the plastic range. Rabotnov's proposition was concerned with uniaxial stresses; according to the author of this proposition, the model could be refined by introducing the lower dynamic yield stress σ_{0l} instead of σ_0. To this end the knowledge of the relation between σ_{0l} and σ_{0d} would be required.

Application of the discussed model to the elastic-plastic structural dynamics necessitates the introduction of the elastic unloading law and certain generalizations to the cases of three-dimensional states of stress; this was done in papers [1, 2]. The rapid stress relaxation occurring at the instant of dynamic yielding t_d makes the state of strain $\varepsilon(t_d)$ di-

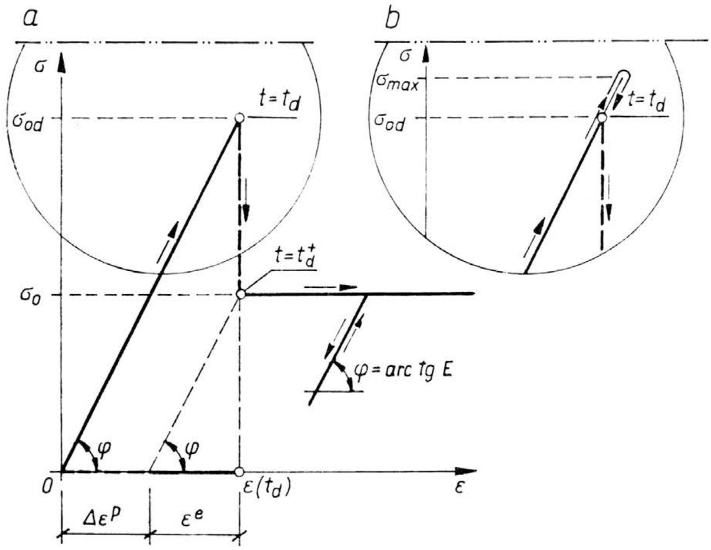


FIG. 2.

vide into the elastic ε^e and plastic components $\Delta\varepsilon^p$, Fig. 2. This division results from the elastic unloading law

$$\varepsilon(t_d) = e^e + \Delta\varepsilon^p,$$

(2.8)
$$\varepsilon^e = \frac{\sigma_0}{E}, \quad \Delta\varepsilon^p = \frac{\sigma_{0d}-\sigma_0}{E}.$$

Discontinuity of the deformation model at the interface of the strain ranges makes the plastic yielding process start at the finite value of plastic strains $\Delta\varepsilon^p$.

Due to Prandtl and Reuss, the theory of perfect plasticity in the case of complex states of stress is governed by the equations:

(a) In the primary elastic loading process, $t \leqslant t_d$,

(2.9)
$$\dot{\varepsilon}_{ij} = \frac{\dot{s}_{ij}}{2\mu} + \frac{\dot{\sigma}}{3K}\delta_{ij},$$

(b) In the process of active plastic yielding, $t \geqslant t_d$,

(2.10)
$$\dot{\varepsilon}_{ij} = \frac{\dot{s}_{ij}}{2\mu} + \frac{\dot{\sigma}}{3K}\delta_{ij} + \lambda s_{ij}, \quad \sqrt{J_2} = k,$$

(c) In the case of elastic unloading after the dynamic yielding, and under repeated loadings, it is assumed that conditions (2.9) hold true again.

In order to determine the stress relaxation immediately after dynamic yielding, let us consider the situation shown in Fig. 3. Components of the strain tensor ε_{ij} at instant t_d^+ are divided into the elastic ε_{ij}^e and plastic $\Delta\varepsilon_{ij}^p$ components.The stress tensor σ_{ij} satisfying the dynamic yielding criterion at time t_d is transformed into σ_{ij}^0. Components of the tensor σ_{ij}^0 determine the point at the static yield surface from which the perfect yielding process begins. They may be determined from the system of equations

(2.11)
$$\varepsilon_{ij}(t_d) = \varepsilon_{ij}^e + \Delta\varepsilon_{ij}^p,$$

(2.11) [cont.]

$$\varepsilon^e_{ij} = \frac{s^0_{ij}}{2\mu} + \frac{\sigma}{3K}\delta_{ij},$$

$$\Delta\varepsilon^p_{ij} = \lambda_0 \frac{\partial F}{\partial \sigma^0_{ij}},$$

$$F = \frac{1}{2} s^0_{ij} \cdot s^0_{ij} - k = 0.$$

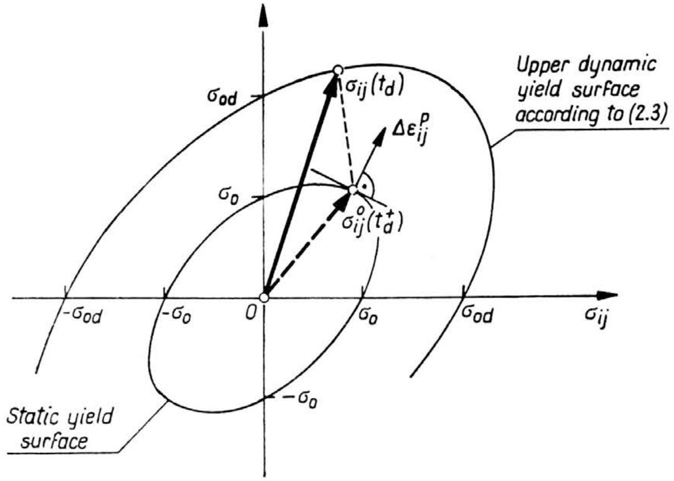


FIG. 3.

Equation $(2.11)_3$ expresses the associated plastic flow law, its validity being extended to the finite plastic strain increment represented by a vector normal to the static yield surface at the point σ^0_{ij}.

In papers [1, 2] the analyzed model was applied to the dynamics of circular symmetric, elastic-perfectly plastic plates in the range of small displacements. Numerical experiments showed that the instantaneous material instability at $t = t_d$ did not lead to the instability of motion of the plate within the period of time comprising 3 to 5 maximum displacement amplitudes. The volume of material subject to dynamic yielding was established mainly in the period preceding the appearance of the first amplitude. The observed accomodation of the form of motion of the plastically deformed plate was the reason why the initial plastic region remained practically unchanged by the consecutive stress pulses.

2.2.2. Delayed yield effect in elastic/viscoplastic strain model. Dynamic stress-strain curves for steel presented in papers [9] and [7] take into account the delayed yield effect and the viscosity effects appearing in the plastic range. A characteristic feature of the approximate result obtained in [9] was the theoretical dynamic curve exhibiting no softening effects and being similar to the corresponding experimental curve. Let us concentrate, however, upon the paper [7] in view of the method applied there. J. M. Kelly proposed to use the initial dynamic yield stress as the initial condition for integration of the L. E. Malvern–V. V. Sokolovskii constitutive equation for elastic/viscoplastic materials

(2.12)

$$E\dot{\varepsilon} = \dot{\sigma} \qquad \text{for} \quad t \leqslant t_d,$$

$$E\dot{\varepsilon} = \dot{\sigma} + \frac{\sigma_c}{\tau}\left\{\exp\left[\frac{\sigma - f(\varepsilon)}{\sigma_c}\right] - 1\right\} \qquad \text{for} \quad t > t_d.$$

Here σ_c, τ are the corresponding material constants, and $f(\varepsilon)$ determines the real static $\sigma-\varepsilon$ characteristic of the material. The dynamic stress-strain curves were determined for constant forced strain-rates $\dot{\varepsilon} = \alpha =$ const. Integration of Eq. $(2.12)_2$ with respect to time was performed under the initial condition $\sigma = \sigma_{0d}$ at $t = t_d$. Mild steels of various grain-site distributions were analyzed. The dynamic $\sigma-\varepsilon$ curves exhibit the regular softening effect after dynamic yielding. The softening effect increase with increasing forced strain-rates. Author of the paper [7] showed that the theoretical "moment-curvature" curves coincided well with the experimental results obtained for rectangular cross-sections subject to bending. It should be stressed that the process of rapid development of the softening effect observed in the uniaxial stress test remains intensive but becomes slower in the case of the "moment-curvature" curves.

3. Elastic/visco-perfectly plastic analysis with the delayed yield effect

3.1. Modifications of the conditions for $\langle\Phi(F)\rangle$

The delayed yield effect may be taken into account by changing the elastic deformation range [14], without violating the fundamental concept of idealisation of the P. Perzyna constitutive equations; they remain the same as those given by Eq. (1.1), with the same definition of argument F expressing the excess of the actual yield surface over the static yield surface. The conditions for $\langle\Phi(F)\rangle$ are, however, modified and assume the form

$$\langle\Phi(F)\rangle = \begin{cases} 0 & \text{for} \quad t \leqslant t_d, \\ \left.\begin{matrix}\Phi(F) & \text{if} & F>0 \\ 0 & \text{if} & F \leqslant 0\end{matrix}\right\} & \text{for} \quad t > t_d. \end{cases} \tag{3.1}$$

Function $\langle\Phi(F)\rangle$ defined in this manner exhibits a discontinuity at $t = t_d$ what affects the behaviour of the material after dynamic yielding. Response of the material to the prescribed forced straining process is determined by integrating the Eqs. (1.1) under the conditions (3.1). In the case of mild steel it is assumed that the relations

$$\Phi(F) = F^\delta, \quad \delta = 5 \tag{3.2}$$

coincide fairly well with the experimental results. Integration of Eq. (1.1) with conditions (3.2) requires, even in the case of uniaxial stress, the application of numerical methods. In view of the expected violent relaxation process following the dynamic yielding, the most suitable method of numerical analysis appears to be the implicit small time-step method.

3.2. Uniaxial stress

The corresponding constitutive relation has the following form:

$$\dot{\sigma}(t) - E\dot{\varepsilon}(t) = \begin{cases} 0 & \text{for} \quad t \leqslant t_d, \\ -E\gamma^*\left[\dfrac{\sigma(t)}{\sigma_0} - 1\right]^\delta & \text{for} \quad t > t_d, \quad \gamma^* = \dfrac{2}{\sqrt{3}}\gamma. \end{cases} \tag{3.3}$$

The initial condition for Eq. $(3.3)_2$ is the dynamic yield stress σ_{0d} (Eq. (2.2)). Discussion of the results of numerical integration of Eqs. (3.3) and comparison with the experimental data will be preceded by a qualitative analysis of Eq. $(3.3)_2$. The first step consists in the analysis of stability of Eq. $(3.3)_2$ with respect to the introduced initial condition variations. The second step of the analysis will be the investigation of the relation between the delayed yield effect and the material softening effect which follows the dynamic yielding.

3.2.1. Analysis of stability of the equation. The solution of the original problem of yielding of elastic/viscoplastic material described by Eqs. (1.1) and (1.3) is considered as the undisturbed solution $\sigma(t)$. It satisfies the initial condition

$$\sigma(t_p) = \sigma_0, \tag{3.4}$$

where t_p denotes the instant of time at which the static yield stress σ_0 is reached. Variation of the initial condition (2.2) produces the disturbance $x(t)$. The new, disturbed solution is denoted by

$$\bar{\sigma}(t) = \sigma(t)+x(t). \tag{3.5}$$

Let us consider the magnitude of disturbances for times $t > t_d$ $(t_d > t_p)$. Disturbance $x(t)$ satisfies the following differential equation:

$$\begin{gathered} \frac{\dot{x}(t)}{E\gamma^*} = -f(x, t), \\ f(x, t) = \left[\frac{\sigma(t)-x(t)}{\sigma_0} - 1\right]^\delta - \left[\frac{\sigma(t)}{\sigma_0} - 1\right]^\delta . \end{gathered} \tag{3.6}$$

Stability of the zero-order solution of this equation may be investigated by assuming the Lapunov function in the form

$$V(x, t) \equiv V(x) = \int_0^x f(\lambda) d\lambda. \tag{3.7}$$

After integration of Eq. (3.7), account being taken of Eq. $(3.6)_2$ for $\sigma = 5$, we obtain

$$V(x) = \frac{E\gamma^*}{6\sigma x_0^5} x^2(t)\{x^4(t)+6[\sigma(t)-\sigma_0]x^3(t)+15[\sigma(t)-\sigma_0]^2x^2(t)+20[\sigma(t)- -\sigma_0]^3x(t)+15[\sigma(t)-\sigma_0]^4\}. \tag{3.8}$$

It may easily be proved that $V(x)$ satisfies all assumptions of the Barbashin–Krasovskiĭ theorem on the global asymptotic stability of solution $\sigma(t)$ in Lapunov's sense [5].

3.2.2. Relation between the delayed yielding and the material softening effects after dynamic yielding. The analysis performed indicates that the changes in the initial condition of Eq. $(3.3)_2$ may produce only a local disturbance of $\sigma(t)$ in a small neighbourhood of the time instant t_d. Character of this disturbance may be determined by investigating the sign of the stress-rate $\dot{\sigma}(t)$ at $t = t_d^+$. To this end let us consider the algebraic equation

$$\dot{\sigma}(t_d^+) = -E\gamma^*\left(\frac{\sigma_{0d}}{\sigma_0} - 1\right)^\delta + E\dot{\varepsilon}(t_d^+) = 0, \qquad \delta = 5. \tag{3.9}$$

The number of roots of Eq. (3.9) depends on the viscosity coefficient γ^*. This relation is illustrated by the curves shown in Fig. 4 and constructed for the processes of constant

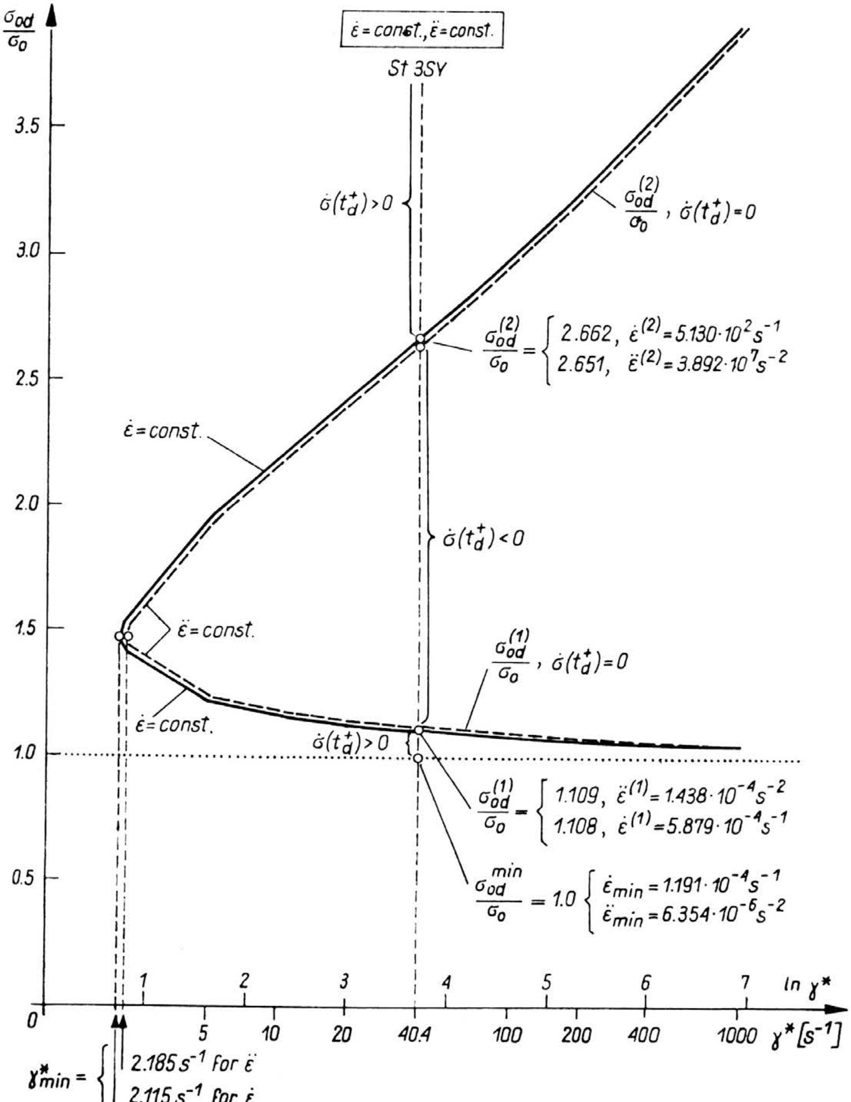


FIG. 4.

forced strain rates and accelerations. In numerical calculations the material constants for Polish mild steel St3SY are assumed:

(3.10) $\sigma_0 = 250$ MPa, $E = 210000$ MPa, $\alpha = 14.6$, $t_0 = 0.641$ s.

The values of α and t_0 are taken from paper [8].

From Fig. 4 it follows that for the viscosity coefficients $\gamma^* > \gamma^*_{\min}$ two levels of dynamic yielding exist, $\sigma_{0d}^{(1)}$ and $\sigma_{0d}^{(2)}$, from which the dynamic yielding begins as a perfect viscoplastic process. This character of the deformation process remains unchanged only in the case of constant forced strain rate processes, $\varepsilon^{(1)} = \text{const}$, $\dot{\varepsilon}^{(2)} = \text{const}$. If the dynamic yielding takes place for $\sigma_{0d} \leqslant \sigma_{0d}^{(1)}$ or $\sigma_{0d} \geqslant \sigma_{0d}^{(2)}$, then it is not followed by plastic softening. The plastic softening process which develops immediately after dynamic yielding may be observed for $\sigma_{0d}^{(1)} < \sigma_{0d} < \sigma_{0d}^{(2)}$. For the viscosity coefficients $\gamma^* \leqslant \gamma^*_{\min}$ the softening effect is not observed.

Results of a similar analysis of plastic softening phenomena under forced strains (2.6), (2.7) are qualitatively similar to those presented in Fig. 4.

3.2.3. Results of numerical analysis. Let us now present the stress-strain curves and the time-dependent stress history curves resulting from the forced straining processes given by Eqs. (2.4), (2.5), (2.6); mild St3SY steel is considered.

At constant strain rates the dynamic $\sigma-\varepsilon$ curves are shown in Fig. 5. Curves a and c do not exhibit any softening effects, in contrast to curve b. Dashed lines represent the curves obtained by integrating Eq. $(3.3)_2$ without the delayed yield effect. From the figure it follows that the delayed yield effect may be observed in the immediate vicinity of the dynamic yield limit only, and for typically dynamic strain rate values. The limiting values of dynamic yield stresses $\sigma_{0d}^{(1)}$, $\sigma_{0d}^{(2)}$ correspond to the strain rates equal to $\dot{\varepsilon}^{(1)} = 5.879 \times$

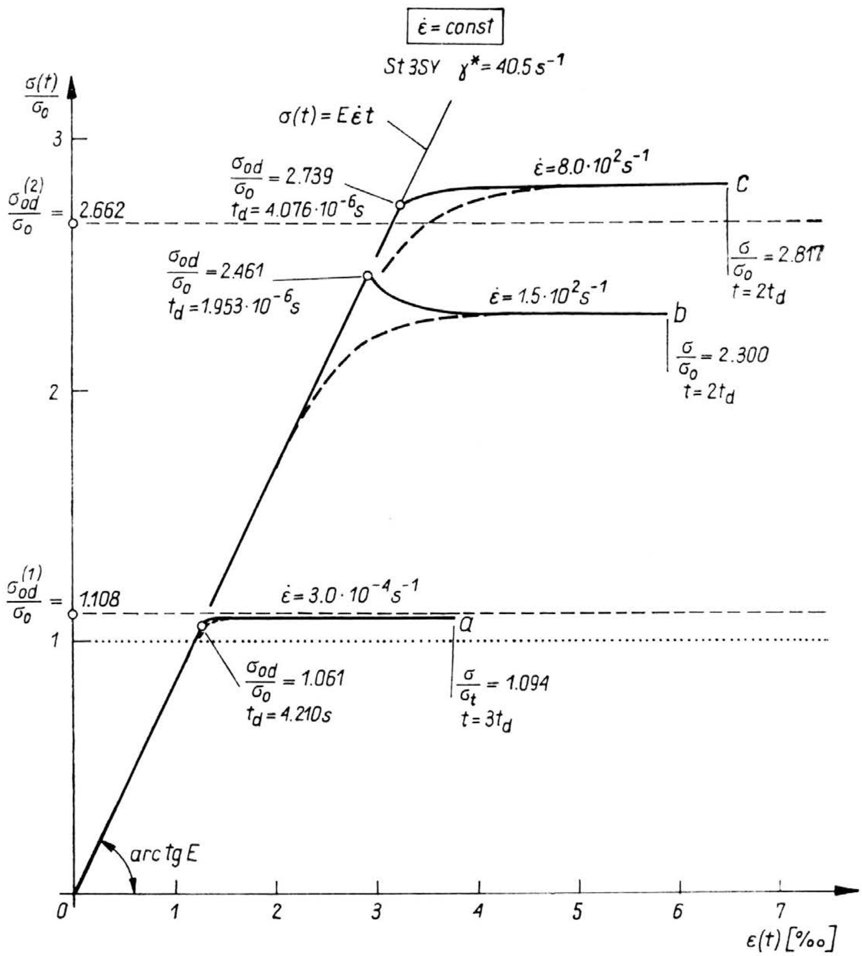


FIG. 5.

$\times 10^{-4}\ s^{-1}$ and $\dot{\varepsilon}^{(2)} = 513\ s^{-1}$, respectively. If the J. D. Campbell criterion were assumed to be valid for slow strain tests, we would be able to determine such a minimum value of strain rate $\dot{\varepsilon}_{\min} = 1.191 \cdot 10^{-4}\ s^{-1}$ for which $\sigma_{0d} = \sigma_0$. This strain rate would correspond to the delay time $t_d \approx 10$ s. For such large time values the accuracy of criterion (2.1) would be, however, not satisfactory.

In the case of constant forced strain acceleration, the corresponding dynamic $\sigma - \varepsilon$ curves are shown in Fig. 6. It is seen that for dynamic yielding developing in the interval $(\sigma_{0d}^{(1)}, \sigma_{0d}^{(2)})$ it is possible to indicate the so-called lower yield stress. The ratio of the upper and lower yield limits is variable; it reaches a maximum in the vicinity of the center of the interval and decreases at approaching its end points, $\sigma_{0d}^{(1)}$ and $\sigma_{0d}^{(2)}$. Like before, the dashed line denotes the solution corresponding to the constitutive equation of viscoplasticity without the delayed yield effect.

A similar phenomenon of the influence of delayed yield on the response of elastic/vis-

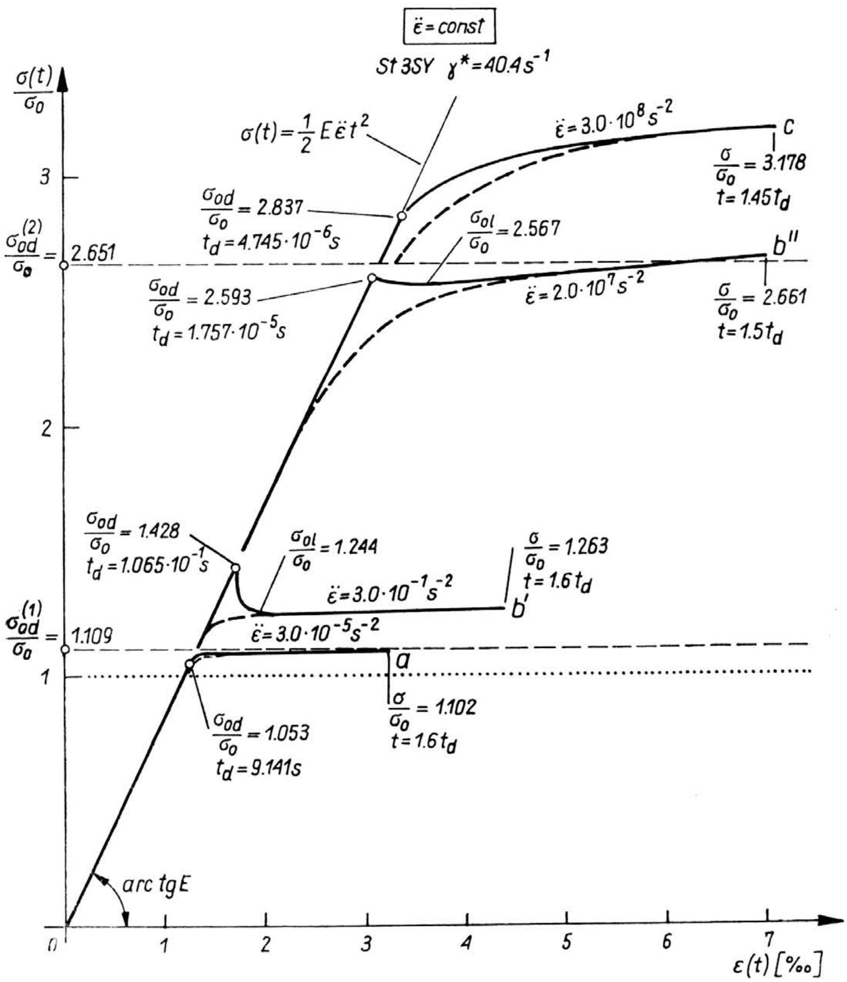


FIG. 6.

coplastic material is observed in the case of sinusoidal forced deformations, Eq. (2.7). Dynamic $\sigma-\varepsilon$ curves for $\sigma_m = 3\sigma_0$ and various strain rate frequencies ω are shown in Fig. 7. The values of ω decide on the various types of behaviour of the solutions of Eq. $(3.3)_2$ immediately after dynamic yielding. The influence of the viscosity coefficient and of the exponent δ in Eq. (3.2) on the time-dependent process of forced dynamic strains (Eq. (2.7)) was investigated for a fixed value of $\omega = 300\ s^{-1}$. Figure 8 illustrates the sensibility of the material stress response to the variations of γ^*. Increasing viscosity coefficients enhance the plastic softening effect. For the viscosity coefficient $\gamma^* \to \infty$ the solution of Eqs. (3.3) corresponds to the approximate deformation model proposed by Yu. N. Rabotnov, (2.9), (2.10).

In Fig. 9 are shown the solutions of Eq. $(3.3)_2$ for various exponents δ in Eq. (3.2) in the same case of dynamic yielding. It is seen that with $\delta = 1$ (linear viscosity) the variation of stresses in time after the dynamic yielding is similar to the solution corresponding to the approximate Rabotnov model. Increasing values of δ produces decreasing plastic

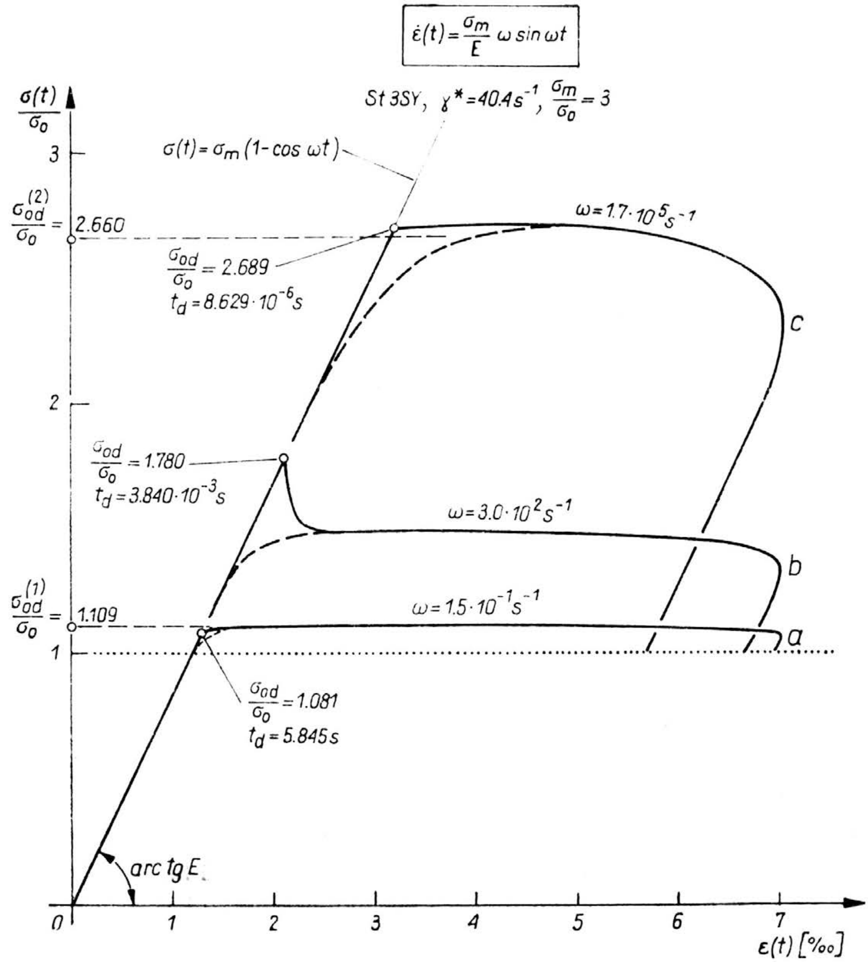


FIG. 7.

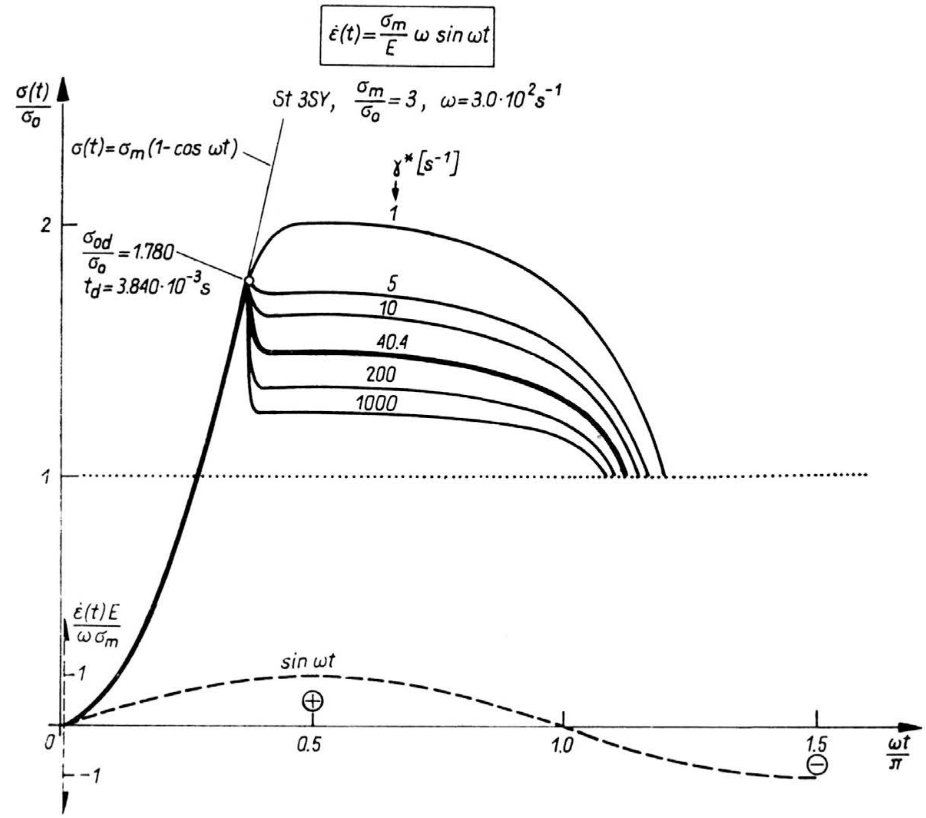
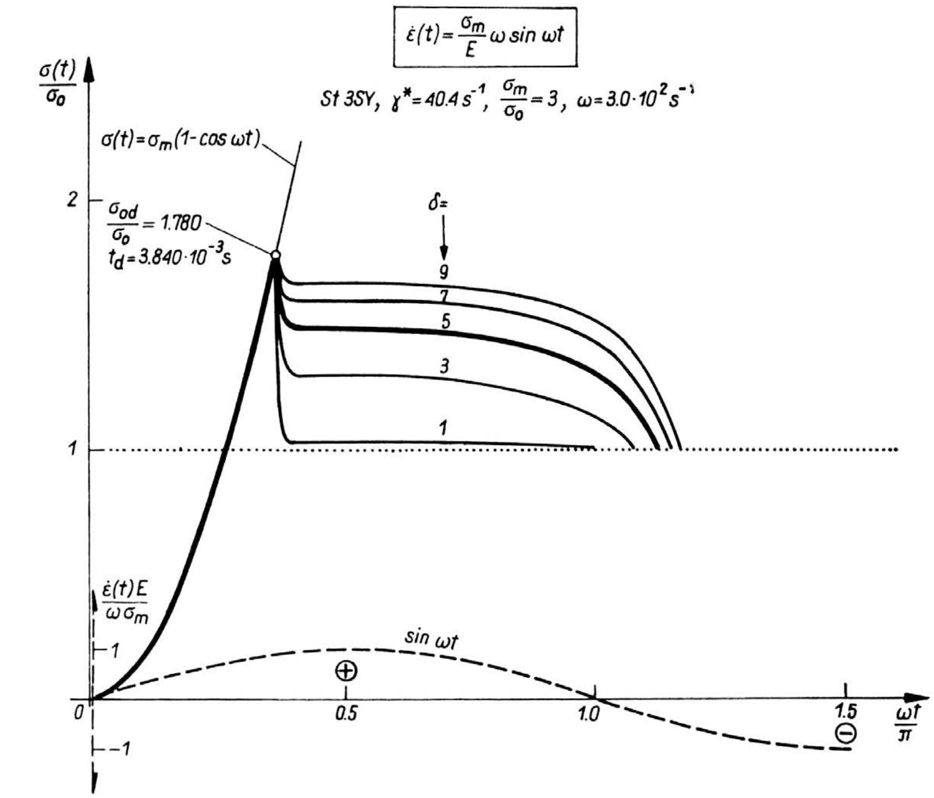


FIG. 8.

softening effects, which at $\delta \to \infty$ vanish completely. Similarly to Fig. 8, this figure also contains the strain rate diagram concerning the forced straining process analyzed here.

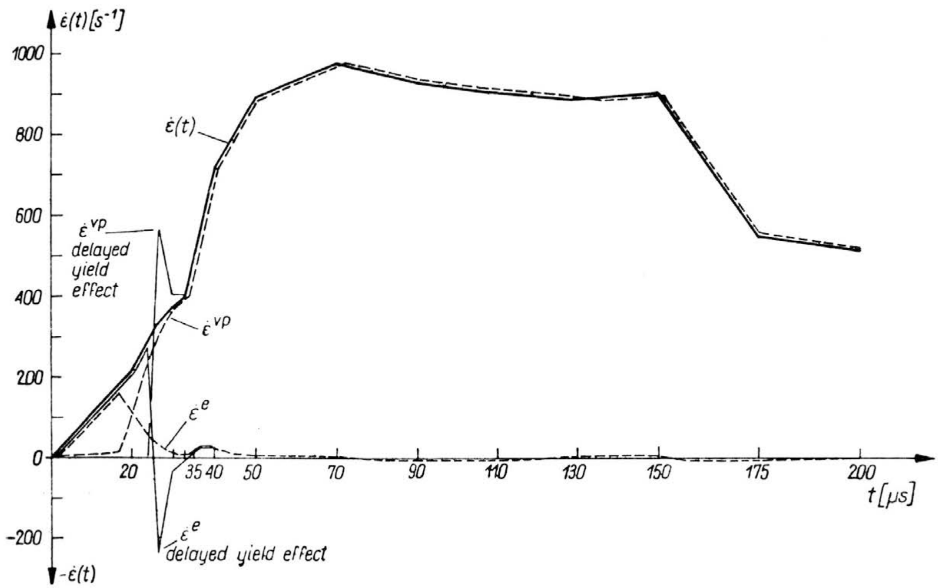
The analysis indicates that the delayed yielding has a local effect upon the material behaviour after dynamic yielding. The delayed yield effects depend on the forced strain rates, and their intensity may be modelled by both the viscosity coefficient γ^* and by the exponents δ.

3.2.4. Experimental verification. Let us now present the experimental verification of the modified method of integration of the elastic/viscoplastic constitutive equations which takes into account the delayed yield effect. The experimental dynamic curve for mild steel was derived in paper [6]. For the sake of comparison, let us analyze Eqs. (3.3) numerically by assuming the dynamic straining process to follow the $\dot{\varepsilon}(t)$ diagram marked by heavy line in Fig. 10. Determination of the curve $\dot{\varepsilon}(t)$ was based on the $\dot{\varepsilon}(t)-\varepsilon(t)$ and $\sigma(t)-\varepsilon(t)$ relations given in [6]. Figure 11 presents the time-dependence of stresses; heavy line denotes the experimental result. Theoretical solutions, with and without taking into account the delayed yield effect, are marked by thin and dashed lines, respectively. Once the delayed yield is taken into account in the integration of Eq. $(3.3)_2$ it becomes possible to construct the theoretical description of formation of physically nonlinear properties of the material immediately after its dynamic yielding. In the case considered the theoreti-



$\dot{\varepsilon}(t)=\frac{\sigma_m}{E}\omega\sin\omega t$
St 3SY, $\gamma^*=40.4\,s^{-1}$, $\frac{\sigma_m}{\sigma_0}=3$, $\omega=3.0\cdot10^2 s^{-1}$
$\frac{\sigma(t)}{\sigma_0}$
$\sigma(t)=\sigma_m(1-\cos\omega t)$
$\frac{\sigma_{od}}{\sigma_0}=1.780$
$t_d=3.840\cdot10^{-3}s$
δ=
9
7
5
3
1
$\frac{\dot{\varepsilon}(t)E}{\omega\sigma_m}$
sin ωt
⊕
⊖
0
0.5
1.0
1.5
$\frac{\omega t}{\pi}$
2
1
-1

FIG. 9.



$\dot{\varepsilon}(t)[s^{-1}]$
$\dot{\varepsilon}(t)$
$\dot{\varepsilon}^{vp}$ delayed yield effect
$\dot{\varepsilon}^{vp}$
$\dot{\varepsilon}^{e}$
$\dot{\varepsilon}^{e}$ delayed yield effect
$-\dot{\varepsilon}(t)$
1000
800
600
400
200
0
-200
20
35 40
50
70
90
110
130
150
175
200
t [μs]

FIG. 10.

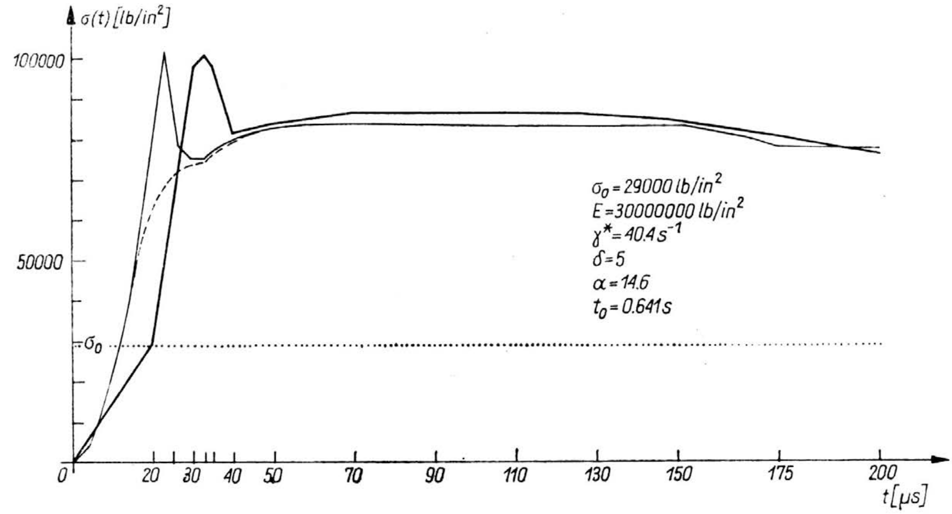


FIG. 11.

cal prediction of the upper and lower yield limits agrees very well with the corresponding experimental data. The quantitative differences are mainly due to inaccurate knowledge of the material constants of mild steel tested in [6]; the values assumed in the theoretical analysis are shown in Fig. 11. Delayed yielding has no effect on the description of developed viscoplastic yield processes. Elastic and viscoplastic components $\dot{\varepsilon}^e$ and $\dot{\varepsilon}^{vp}$ of $\dot{\varepsilon}(t)$ is shown in Fig. 10. It should be noted that the rapid stress relaxation following the dynamic yielding is associated with negative elastic strain rates, $\dot{\varepsilon}^e < 0$. Intensive growth of $\dot{\varepsilon}^{vp}$ is observed in this period. Without the delayed yield effect, integration of Eq. (3.3) leads to the solution according to which gentle decrease of $\dot{\varepsilon}^e$ and moderate increase of $\dot{\varepsilon}^{vp}$ should be observed after passing the yield stress σ_0.

3.3. Plane stress

Let us illustrate the behaviour of materials subject to complex states of stress and exhibiting the delayed yield effect. The solution corresponding to a plane state of stress produced by prescribed kinematic straining is written in the form

$$\varepsilon_{xx} = \varepsilon\left(1-\cos\frac{2\pi}{T}\,t\right),$$

$$\varepsilon_{yy} = -\nu\varepsilon_{xx}.$$

The process is characterized by $\varepsilon = 2\%$, $T = 0.1$ ms and $\nu = 0.3$. Material constants for St3SY steel are assumed. The stressing path is shown in Fig. 12 with the time instants $t = l\Delta t$. Isotropic shrinkage of the instantaneous dynamic yield surfaces is observed between $l = 19$ and $l = 22$, what indicates the plastic softening effect. Dashed line denotes the solution derived without the delayed yield effect.

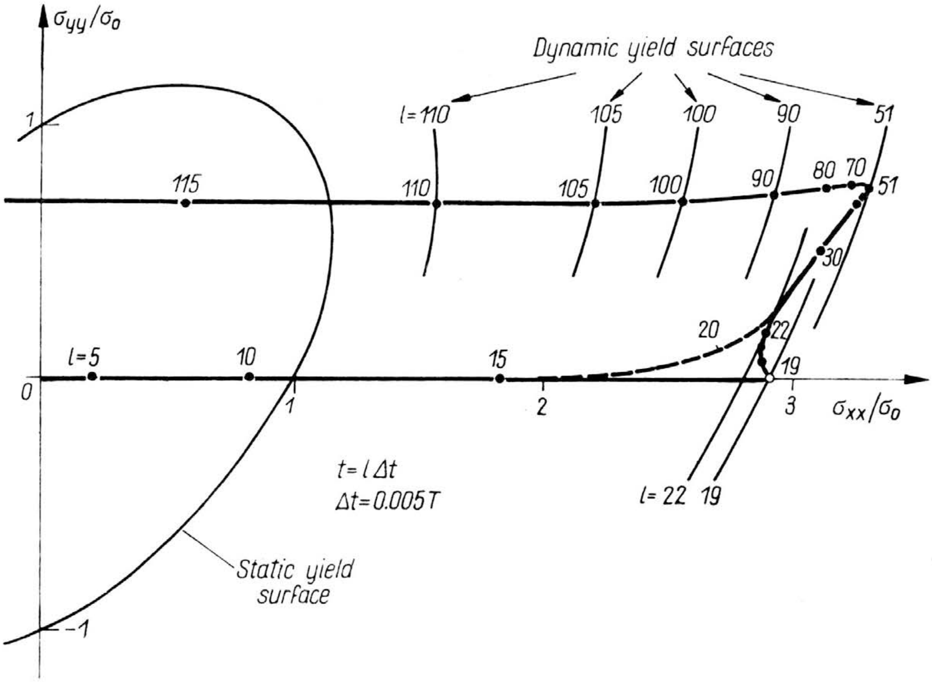


FIG. 12.

4. Conclusions

The principal aim of the paper consisted in investigating the effect of delayed yielding on the process of formation of the initial yielding of elastic/visco- perfectly plastic material, what required the analysis of solutions of the constitutive equations proposed by P. Perzyna with modified conditions for $\langle \Phi(F) \rangle$. This modification corresponds to the introduction of a new initial condition to the set of constitutive equations of viscoplasticity; this condition has the form of the dynamic yield surface determined according to J. D. Campbell's criterion, instead of the static surface. Such procedure is based on the paper by J. M. Kelly who was considering the Malvern-Sokolovskii law.

The analysis performed shows the solutions of constitutive equations (1.1) to be stable in the sense of Lapunov. Changes in the initial condition produce only local disturbances in the solution, their character depending on the forced strains prescribed. Intensity of the disturbances depends on both the strain history in the elastic range (deciding upon the dynamic yield limit), and on the viscous properties of the material characterized by its viscosity coefficient and exponent δ. The theoretical results obtained indicate the existence of a certain relation between the phenomena of delayed yielding and softening. Within the deformation model considered, the conditions are established under which plastic softening phenomena take place; this process should not be considered to be spontaneous. It is supposed that the so-called lower yield limit may not represent any physical magnitude.

No influence of delayed yield on the developed viscoplastic yielding process can be established. Hence, consideration of the delayed yield effect may be of certain importance in the analysis of dynamic problems within the framework of the infinitesimal strain theory; this problem will be dealt with in a separate paper.

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