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Constitutive modelling for brittle dynamic fracture in dissipative solids

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THE PAPER aims at the description of brittle dynamic fracture in dissipative solids by means of an internal state variable constitutive structure. Experimental results for Armco iron obtained in the plate-impact configuration system have been discussed. Physical mechanisms of brittle dynamic fracture are considered. An elastic-viscoplastic model of damaged solids is proposed. The main role in this model is played by two internal state variables, namely, the density of microcracks and the average size of microcrack. Different forms of the evolution equations for these two internal state variables have been investigated. Some particular cases of the evolution equations have been discussed. Generalization to three-dimensional state of stress is proposed. The porosity parameter is described by two internal state variables introduced. This is achieved by postulating an additional equation of state. The criterion of brittle dynamic fracture is postulated. Application of the theory to the dynamic fragmentation process is shown.

Celem pracy jest opis kruchego dynamicznego zniszczenia w ciałach dysypatywnych za pomocą konstytutywnej struktury z wewnętrznymi zmiennymi stanu. Przedyskutowano rezultaty badań doświadczalnych dla żelaza Armco otrzymane w układzie konfiguracyjnym dla płyt uderzających. Rozważono fizykalne mechanizmy kruchego dynamicznego zniszczenia. Zaproponowano model ciała sprężysto-lepkoplastycznego z uszkodzeniami. Główną rolę w zaproponowanym modelu odgrywają dwie zmienne stanu wewnętrznego, mianowicie gęstość mikroszczelin oraz średni wymiar mikroszczeliny. Szczegółowo zbadano różne postacie równań ewolucji dla obydwu parametrów. Przedyskutowano również ich szczególne przypadki, które mają znaczenie dla zastosowań praktycznych. Zaproponowano uogólnienie do trójwymiarowego stanu naprężenia. Parametr porowatości jest opisany przez dwie wprowadzone zmienne wewnętrzne za pomocą dodatkowego równania stanu. Wprowadzono kryterium kruchego dynamicznego zniszczenia. Pokazano zastosowanie teorii do opisu procesu dynamicznej fragmentacji.

Целью работы является описание хрупкого динамического разрушения в диссипативных телах при помощи определяющей структуры с внутренними переменными состояния. Обсуждены результаты экспериментальных исследований для железа Армко, полученные в кофигурационной системе для ударяющихся плит. Рассмотрены физические механизмы хрупкого динамического разрушения. Предложена модель упруго-вязкопластического тела с повреждениями. Главную роль в предложенной модели играют две переменные внутреннего состояния, а именно плотность микротрещин, а также средний размер микротрещины. Подробно исследованы разные виды уравнений эволюции для обоих параметров. Обсуждены тоже их частные случаи, которые имеют значение для практических применений. Предложено обобщение к трехмерному напряженному состоянию. Параметр пористости описан двумя введенными внутренними переменными при помощи дополнительного уравнения состояния. Введен критерий хрупкого динамического разрушения. Показано применение теории к описанию процесса динамичеекой фрагментацнн.

1. Introduction

THE MAIN objective of the present paper is the description of brittle fracture in dynamic processes in dissipative solids. Analysis of experimental results for Armco iron obtained by means of the plate-impact configuration system is given.

Heuristic consideration of physical mechanisms of brittle dynamic fracture is presented. Brittle dynamic fracture is described as a sequence of the nucleation, growth and coalescence of microcracks.

An elastic-viscoplastic model of damaged solid is proposed. The theory is developed within the framework of an internal state variable structure. The density of microcracks and the average size of a microcrack are introduced as internal state variables. These two internal state variables are suggested by experimental observations and by analysis of physical mechanisms. Assuming that for dynamical processes the thermally-activated mechanism of the nucleation of microcracks is the most important one and that for brittle solids the growth mechanism of a microcrack is governed by the viscoplastic flow process, the evolution equations for the internal state variables are proposed.

Generalization to three-dimensional state of stress is proposed.

Taking advantage of the porosity parameter, the theory of viscoplasticity for demaged solids is developed. The fracture criterion is introduced and the softening of the material implied by the micro-damage process generated by the nucleation, growth and coalescence of microcracks is described.

Application of the theory proposed to the dynamic fragmentation process for brittle solids is shown.

2. Experimental and physical motivations

2.1. Discussion of experimental results

The most popular dynamical experimental investigation of the fracture phenomena in metals is a plate-impact configuration system. It offers a unique opportunity for studying microvoid and microcrack kinetics under conditions of extremely high tensile stress. By varying the impact velocity and target/impactor geometry it provides to change amplitude and duration of stress impulse over the range of approximately 0.1 to 10 GPa and 0.01 to 10 μs, respectively (cf. CURRAN, SEAMAN and SHOCKEY [3]).

An example of brittle fracture for Armco iron is presented in Fig. 1 (cf. CURRAN, SEAMAN and SHOCKEY [2]). It shows the polished cross section through the plate impact specimen with very clearly visible cleavage (penny shaped) microcracks. The damage, which appears as randomly oriented planar microcraks, depends on the impact velocity as well as on the duration of the tensile wave. The second property is directly observed from the results presented in Fig. 2 (cf. CURRAN, SEAMAN and HOCKEYS [2]). Use of a tapered flyer results in longer tensile impulses at the thicker end. As it is shown in Fig. 2, these longer pulses lead to greater damage in the Armco iron target (the inset gives the approximate durations of the tensile pulses).

The damage observed in this experiment is termed brittle, although the microcrack growth is much slower than the elastic crack velocities, indicating considerable plastic flow at microcrack tips.

For a thorough discussion of the experimental and theoretical works in the field of dynamic fracture and spalling of metals please consult the review paper by MEYERS and AIMONE [9].

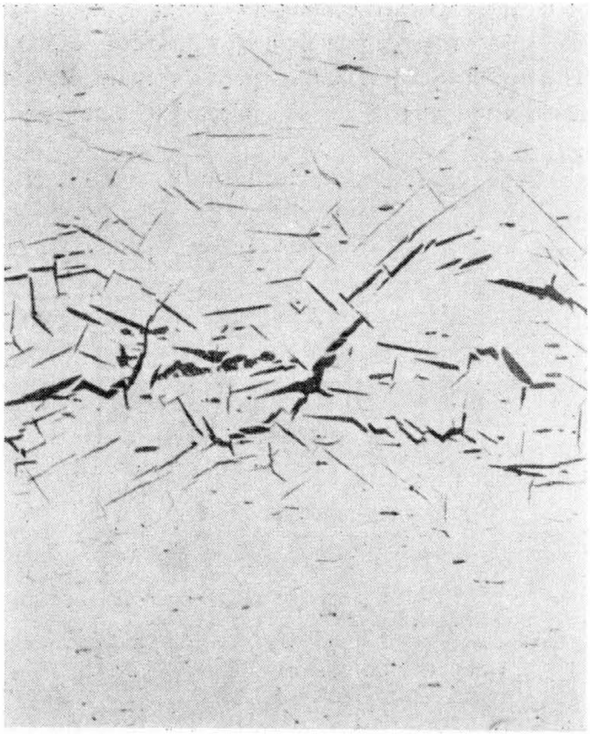

FIG. 1. Internal cleavage (penny-shaped) microcracks caused by shock loading in the polished cross-section of an Armco iron specimen (After CURRAN, SEAMAN and SHOCKEY [2]).

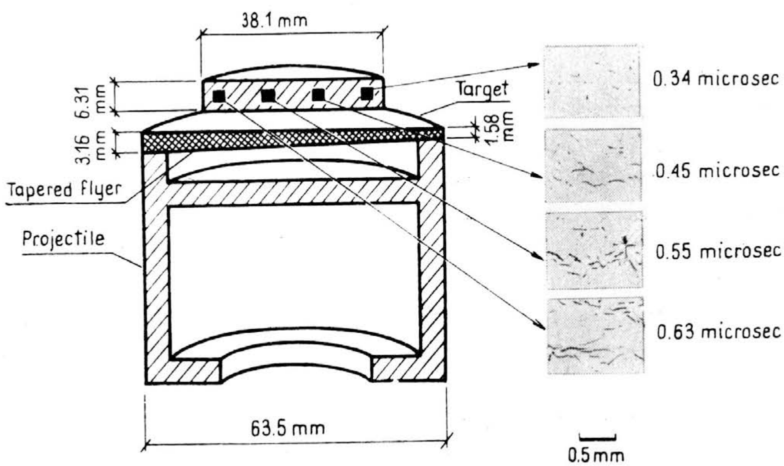

FIG. 2. Tapered flyer impact experimental results for the Armco iron target (After CURRAN, SEAMAN and SHOCKEY [2]).

2.2. Physical mechanism of brittle dynamic fracture

When subjected to high rate loads from impact, Armco iron undergoes relatively brittle fracture from nucleation, growth and coalescence of planar microcracks.

To understand better the physical mechanism of brittle dynamic fracture, let us consider the variation of tensile stress with specific volume (or porosity), Fig. 3 (cf. CURAN, SEAMAN and SHOCKEY [2]). The trajectory tensile stress-specific volume represents the real dynamic process in the Armco iron target (specimen) subjected to a constant strain rate of $1.3\times$

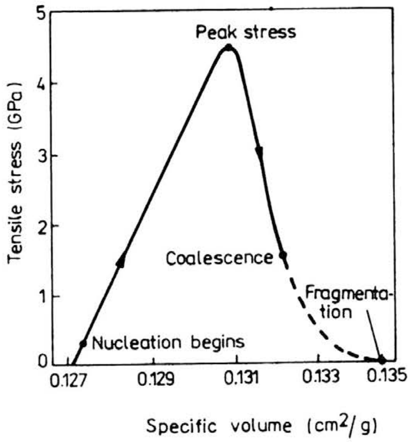

FIG. 3. Stress-specific volume trajectory of Armco iron loaded to fragmentation at constant strain reate (After CURRAN, SEAMAN and SHOCKEY [2]).

$\times 10^5$ s^{-1}. From this trajectory we can follow the events in the order in which things naturally happen during the dynamic process. The process starts at the initial specific volume of about 0.1272, and when the tensile stress reaches the threshold value for nucleation the nucleation process begins. The process goes on, the tensile stress peaks up at the value of specific volume 0.1310 and dramatically breaks down to attain at 0.1323 the point at which the coalescence of microcracks begins.

If no stress relaxation were allowed, the tensile stress-specific volume trajectory would follow that determined by the constitutive laws of elastic-plastic flow theory, and the stress would increase indefinitely. However, the microcrack nucleation and growth processes cause the stress to peak up and decay.

The segment of the dynamic process marked by the grey line represents the mechanism of brittle fracture (or fragmentation process) by microcrack coalescence which ends at zero tensile stress; at that point the specific volume reaches the value 0.135. As it has been suggested by SEAMAN, CURRAN and MURRI [22] the physical process of coalescence occurs when the planar microcracks become so large that they begin to intersect other microcracks. They may intersect in the same plane, thus forming larger microcracks, and they may intersect at right angles, forming corners of fragments. Also, microcracks of the same orientation, but on different planes, may coalesce by developing crack extensions out of the plane to join the nearby microcracks. Thus a family of microcracks in one orientation can coalesce and form a rough, multifaceted spall plane.

SHOCKEY, SEAMAN and CURRAN [25] have recently investigated the coalescence process for the XAR 30 armor steel under plate impact loading conditions. Their experimental results show two parallel but nonplanar macrofractures in the process of coalescing.

A profusion of tiny microfractures has formed in a path linking the tips of macrocracks, suggesting that coalescence is a nucleation and growth process on a smaller scale.

From this analysis of the dynamic deformation process in the Armco iron target and from the analysis of the previously discussed experimental results one can see that the main cooperative phenomena which are most important for proper description of brittle dynamic fracture are as follows:

(i) The inelastic deformation wave phenomena. The propagation of the inelastic shock wave induced by the impact process produces significant structural changes and affect the mechanical properties. In general, one observes an increase in the flow stress with a corresponding decrease of ductility.

(ii) The nucleation and growth processes of microcracks. Damage in brittle metals such as Armco iron depends on the amplitude of tensile stress as well as on the duration of stress impulse. As the damage occurs, the stiffness of the material decreases. This softening of the material is mainly due to nucleation and growth of microcracks. The nucleation and growth processes may be accompanied by thermal effects.

(iii) The coalescence of microcracks which leads to fragmentation process. As the number and sizes of microcracks increase, fragments are formed until the entire material disintegrates into fragments.

(iv) Full separation as a result of the propagation of a macrocrack through heavily damaged material.

3. Internal state variables describing brittle fracture

3.1. Physical justification

For practical purposes it will be sufficient to introduce two internal state variables, namely the density of microcracks (the number of microcracks per unit volume) $N(x, t)$ and the average size of a microcrack $R(x, t)$. These two internal state variables are suggested by experimental observations, by analysis of physical mechanisms as well as by previous theoretical descriptions of ductile and brittle fracture of metals.

We shall prove that the simple model of an elastic-viscoplastic damaged solid with two internal state variables N and R can represent the following phenomena:

(i) Nucleation of microcracks or microvoids as a function of stress level and stress impulse duration.

(ii) Growth of microcracks or microvoids during the dynamic process considered.

(iii) A range of microcrack or microvoid sizes at all stages of fracture process.

(iv) Coalescence of microcracks or microvoids to form fragments.

(v) A range of fragment sizes.

(vi) A transition from no damage through fracture to full separation.

These phenomena and properties provide the main features noted previously in observations of dynamic brittle failure (cf. SEAMAN, CURRAN and SHOCKEY [23, 24]).

3.2. Evolution equations for one-dimensional state of stress

For dynamical processes the thermally-activated mechanism of the nucleation of microcracks is most important(1) (cf. discussion presented by CURRAN, SEAMAN and SHOCKEY [3], and RAJ and ASHBY [21]).

For brittle solids the growth mechanism of a microcrack is assumed to be governed by the elastic-viscoplastic flow process.

Thus, we shall postulate for N and R the evolution equations in the form as follows:

$$\tag{3.1} \begin{aligned} \dot{N} &= \dot{N}_0 \left[\exp \frac{m(\sigma-\sigma_N)}{k\vartheta} - 1\right], \\ \dot{R} &= A(N, R)\dot{\sigma} + \frac{B(N, R)}{\tau}[\sigma-\sigma_{eq}(N, R)], \end{aligned}$$

where $\dot{N}_0$, m are the material constants, σ denotes the stress, σ_N is the threshold stress for nucleation (in general $\sigma_N = \sigma_N(N, R)$), k is the Boltzmann constant, ϑ denotes the actual temperature, $A(N, R)$ is the material function that describes the elastic (reversible) change in microcrack size, $B(N, R)$ is the material function which describes the viscoplastic (irreversible) change in microcrack size, τ denotes the relaxation time related to viscoplastic flow, $\sigma_{eq}(N, R)$ is the equilibrium stress corresponding to the instantaneous value of R and depends on N and R due to the interaction of microvoids and dot denotes the differentiation with respect to time t.

For simplicity we assume

$$\tag{3.2} \begin{aligned} A(N, R) &= A(R), \\ B(N, R) &= B(R) = MR, \qquad M = \text{const}, \\ \sigma_{eq} &= \text{const}, \qquad \sigma_N = \text{const}, \end{aligned}$$

and introduce the notation

$$\tag{3.3} \frac{M}{\tau} = \frac{1}{\eta},$$

where η is called the viscosity coefficient.

Then the evolution equations (3.1) take the form

$$\tag{3.4} \begin{aligned} \dot{N} &= \dot{N}_0 \left[\exp \frac{m(\sigma-\sigma_N)}{k\vartheta} - 1\right], \\ \dot{R} &= A(R)\dot{\sigma} + \frac{1}{\eta}(\sigma-\sigma_{eq})R. \end{aligned}$$

CARROL and HOLT [1] have proved that the elastic compressibility does not significantly affect the result for an elastic-perfectly plastic material. Therefore we may introduce very useful simplification(2) (cf. also the analysis presented by JOHNSON [7]).

(1) To prove this conjecture it is sufficient to investigate incubation times for different nucleation mechanisms considered and to compare them with the time duration of the impact process.

(2) It is noteworthy that this simplification is fully justified for ductile fracture mode only. In the case of brittle fracture it may happen that the growth mechanism can be influenced by the elastic properties of particular materials.

When dealing with crack growth in materials of very low initial porosities, we can neglect the initial elastic and elastic-viscoplastic phases of the process and consider the case of fully viscoplastic response of the material around the microcrack.

Thus, we can neglect the elastic change in microcrack size by assuming $A(R) = 0$. Finally we have

$$\dot{R} = \frac{1}{\eta}(\sigma - \sigma_{eq})R. \tag{3.5}$$

The linearized form of the viscoplastic response assumed by the evolution equation (3.5) can be justified by the analysis of the contribution to the result superposed by the nonlinear terms. It has been proved (for ductile solids) that the contribution of nonlinear terms to the growth mechanism, when compared with the result given by the linear term, is small (cf. Ref. [19]).

3.3. Generalization to three-dimensional state

In the case when the state of stress is three-dimensional we define the stress intensity invariant for nucleation I_n and the stress intensity invariant for growth I_g as follows

$$\begin{aligned} I_n &= a_1 J_1 + a_2 \sqrt{J_2'} + a_3 (J_3')^{\frac{1}{3}}, \\ I_g &= b_1 J_1 + b_2 \sqrt{J_2'} + b_3 (J_3')^{\frac{1}{3}}, \end{aligned} \tag{3.6}$$

where a_i and b_i ($i = 1, 2, 3$) are the material constants, and by J_1 we denote the first invariant of the Cauchy stress tensor $\boldsymbol{\sigma}$, J_2' and J_3' are the second and third invariants of the stress deviator, respectively.

We postulate the evolution equations for N and R in the form

$$\begin{aligned} \dot{N} &= \dot{N}_0 \left\{ \exp \frac{m[I_n - \sigma_N(N, R)]}{k\vartheta} - 1 \right\}, \\ \dot{R} &= \frac{B(N, R)}{\tau} [I_g - \sigma_{eq}(N, R)] R, \end{aligned} \tag{3.7}$$

where σ_N (N, R) and σ_{eq} (N, R) denote the threshold values of stress intensity invariant for nucleation and for growth, respectively.

3.4. Phenomenological description of porosity effect

The main idea of this paper is to take advantage of the porosity parameter (the void volume fraction) $\xi(x, t)$ in the constitutive modelling of damaged solids. To do this let us postulate the fundamental equation of state

$$\xi = \Xi N R^3, \qquad \xi \in [0, 1], \tag{3.8}$$

where Ξ is the material constant.

The evolution of ξ during the dynamic process is determined by the differential equation

$$\dot{\xi} = \xi\left(\frac{\dot{N}}{N} + 3\frac{\dot{R}}{R}\right). \tag{3.9}$$

For ductile solids, when the microcracks have spherical shapes

$$\xi = \Xi N R^3 = \frac{4}{3}\pi R^3 N, \tag{3.10}$$

and the material constant Ξ is directly determined

$$\Xi = \frac{4}{3}\pi = 4.18\,87\,90. \tag{3.11}$$

For brittle solids, when microcracks can be assumed as penny-shaped, we have

$$\xi = \Xi N R^3 = \pi R^2 \cdot 0.1 R N = 0.1\pi N R^3 \tag{3.12}$$

and again the material constant is determined

$$\Xi = 0.1\pi = 0.314159. \tag{3.13}$$

3.5. Internal equilibrium state

The internal equilibrium state is defined as follows:

$$\dot{N} = 0, \quad \dot{R} = 0, \tag{3.14}$$

then

$$\dot{\xi} = \xi\left(\frac{\dot{N}}{N} + 3\frac{\dot{R}}{R}\right) = 0, \quad \xi = \text{const}. \tag{3.15}$$

The condition of the internal equilibrium (3.14) gives

$$\begin{aligned} a_1 J_1 + a_2\sqrt{J_2'} + a_3 (J_3')^{\frac{1}{3}} &= \sigma_N(N, R), \\ b_1 J_1 + b_2\sqrt{J_2'} + b_3 (J_3')^{\frac{1}{3}} &= \sigma_{\mathrm{eq}}(N, R). \end{aligned} \tag{3.16}$$

The last result has a very important physical interpretation. When

$$\begin{aligned} I_n &= \sigma_N(N, R), \\ I_q &= \sigma_{\mathrm{eq}}(N, R), \end{aligned} \tag{3.17}$$

then the state of stress corresponds to the instantaneous values of N and R.

Equations (3.16) can help in the determination of the material constants a_i and b_i ($i = 1, 2, 3$).

4. Elastic-viscoplastic model of damaged solids

4.1. Constitutive and evolution equations

In the previous papers [10–18] of the author a simple model of an elastic-viscoplastic solid with internal imperfections was proposed. The model was developed within the framework of the internal state variable structure. The model proposed was applied to the description of quasi-static plastic flow processes as well as to the description of dynamic fracture in impact processes for ductile solids.

In the present paper we shall propose the alternative model of an elastic-viscoplastic damaged solid with the aim of its application to the description of brittle dynamic fracture.

The damage that occurs in solids during dynamic process is presumed to affect both the yield strength and the effective shear and bulk moduli of the material.

As the measure of damage we shall use the porosity parameter ξ.

Thus, for dynamic processes not only viscoplastic properties but also elastic properties of a material depend on the porosity parameter ξ. The shear and bulk moduli are assumed to be degraded by the presence of microcracks according to a model suggested by MACKENZIE [8] (cf. also SEAMAN, CURRAN and SHOCKEY [23] and JOHNSON [7]):

$$\tag{4.1} \begin{aligned} \overline{G} &= G_0(1-\xi)\left(1-\frac{6K_0+12G_0}{9K_0+8G_0}\,\xi\right),\\ \overline{K} &= 4G_0K_0(1-\xi)/(4G_0+3K_0\xi), \end{aligned}$$

where G_0 and K_0 are the elastic moduli of undamaged material.

As a result the Poison ratio $\bar{\nu}$ is determined by the relation

$$\tag{4.2} \bar{\nu} = \frac{1}{2}\,\frac{3\overline{K}-2\overline{G}}{3\overline{K}+2\overline{G}}.$$

Let us denote the symmetric rate of the deformation tensor by $\mathbf{D}$ and postulate

$$\tag{4.3} \begin{aligned} \mathbf{D}^e &= \mathbf{D}-\mathbf{D}^p,\\ \mathbf{D}^e &= \frac{1}{2\overline{G}}\left[\overset{\nabla}{\boldsymbol{\sigma}}-\frac{\bar{\nu}}{1+\bar{\nu}}\,\mathrm{tr}\overset{\nabla}{\boldsymbol{\sigma}}\mathbf{I}\right],\\ \mathbf{D}^p &= \frac{\gamma}{\psi}\left\langle\Phi\left[\frac{f(\cdot)}{\varkappa}-1\right]\right\rangle\partial_{\boldsymbol{\sigma}}f, \end{aligned}$$

where $\overset{\nabla}{\boldsymbol{\sigma}}$ denotes the symmetric Zaremba–Jaumann rate of change of the Cauchy stress tensor $\boldsymbol{\sigma}$, $\mathbf{I}$ denotes the unit tensor, γ is the viscosity constant, ψ is introduced as the control function and is assumed to depend on $(I_2/I_2^s)-1$, where I_2 is the second invariant of the rate of the deformation tensor and I_2^s is its static value, Φ denotes the viscoplastic overstress function, f is the quasi-static yield function which is postulated to depend on the first invariant of the Cauchy stress tensor J_1, on the second and third invariants of the stress deviation J_2' and J_3' and on the porosity parameter ξ, $\varkappa = \hat{\varkappa}(\in^p, \xi)$ is the material work-hardening-softening function, the symbol $\langle\,[\,]\,\rangle$ is understood according to the definition

$$\tag{4.4} \langle[\;]\rangle = \begin{cases} 0 & \text{if} \quad f \leqslant \hat{\varkappa}(\in^p, \xi),\\ [\;] & \text{if} \quad f > \hat{\varkappa}(\in^p, \xi). \end{cases}$$

We postulate the particular form of the yield function

$$\tag{4.5} f(\cdot) = J_2'\left[1-(n_1+\xi n_2)\frac{J_3'^2}{J_2'^3}+n_3\xi\frac{J_1^2}{J_2'}\right],$$

where n_1, n_2 and n_3 denote the material constant, and the work-hardening-softening material function

$$(4.6)\qquad \varkappa = \hat{\varkappa}(\overline{\in}^p, \xi) = (\varkappa_0 + H'\overline{\in}^p)^2 \left(1 - n_4 \xi^{\frac{1}{2}}\right)^2,$$

where $\varkappa_0$ denotes the yield stress, H' is the work-hardening coefficient and n_4 is the material constant.

We also postulate

$$(4.7)\qquad \Phi[\cdot] = \left[\frac{f(\cdot)}{\varkappa} - 1\right]^n, \quad n = 1, 3, 5, \ldots,$$

It is noteworthy that the theory proposed describes inherently the dilatational effects generated by the nucleation, growth and coalescence of microcracks. To prove this, it is sufficient to compute

$$(4.8)\qquad \operatorname{tr}\mathbf{D}^p = \frac{2\gamma}{\psi}\left\langle \Phi\left\{\frac{J_2'\left[1 - (n_1 + \xi n_2)\dfrac{J_3'^2}{J_2'^3} n_3 \xi \dfrac{J_1^2}{J_2'}\right.}{(\varkappa_0 + H'\overline{\in}^p)^2 \left(1 - n_4 \xi^{\frac{1}{2}}\right)^2} - 1\right\}\right\rangle \times \frac{1}{\varkappa_0}\left\{3n_3 \xi J_1 + 2(n_1 + \xi n_2)\frac{J_3'}{J_2'}\right\}.$$

4.2. Determination of the material functions and constans

To determine the material functions and constants involved in the theory we have to base on the mechanical test data as well as on the metallurgical observation results. There is no general method concerning the identification procedure. We shall proceed in a similar way as it has been shown in Refs. [12–18].

5. Phenomenon of dynamic brittle fracture

5.1. Criterion of dynamic brittle fracture

During the dynamic inelastic flow process it is very difficult to control plastic deformation for different stages. It seems natural to base a criterion of dynamic brittle fracture (similarly to ductile fracture, cf. Refs. [15–18]) on the control of the porosity parameter ξ.

It is postulated that during the dynamic inelastic flow process the brittle fracture phenomenon occurs when

$$(5.1)\qquad \xi = \xi^F \Rightarrow \overline{\in}^p = \overline{\in}_F^p(\mathbf{D}^p, \vartheta),$$

what leads to the condition

$$(5.2)\qquad \varkappa = \hat{\varkappa}(\overline{\in}^p, \xi)\Big|_{\substack{\xi = \xi^F \\ \overline{\in}^p = \overline{\in}_F^p}} = 0.$$

The condition (5.2) expresses the fact that fracture means a catastrophe or intrinsic failure when the material loses its stress carrying capacity. In other words, at $\xi = \xi^F$ one can observe full separation of the material.

It is noteworthy that for $N = \text{const}$ the criterion of dynamic brittle fracture (5.1) and the fundamental equation of state (3.8) lead to the condition

$$R^F = \left(\frac{\xi^F}{\Xi N}\right)^{\frac{1}{3}}, \tag{5.3}$$

which defines the critical average size of a microcrack at fracture.

This result has very important practical implications. It means that the criterion of dynamic brittle fracture (5.1) has a dimensional nature. This fact has been broadly suggested by experimental observations.

5.2. Determination of the softening constant

The condition (5.2) leads directly to the particular relation

$$\hat{\varkappa}(\bar{\epsilon}^p, \xi)\Big|_{\substack{\xi=\xi^F \\ \bar{\epsilon}^p=\bar{\epsilon}^p_F}} = (\varkappa_0 + H'\bar{\epsilon}^p_F)^2\left[1 - n_4(\xi^F)^{\frac{1}{2}}\right]^2 = 0. \tag{5.4}$$

As a result of this relation we have the material constant

$$n_4 = (\xi^F)^{-\frac{1}{2}}, \tag{5.5}$$

which is crucial for the description of softening of the material implied by the micro-damage process generated by the nucleation, growth and coalescence of microcracks.

6. Application to dynamic fragmentation process

6.1. General description of dynamic fragmentation

One consequence of intense impulsive loading of a solid can be the fragmentation of the body into discrete parts.

An approach to a global description of the dynamic fragmentation process of a solid body has been recently developed by GRADY [4]. A conception presented by GRADY [4] is utilized whereby the surface or interface area created in the fragmentation process is governed by an equilibrium balance of the surface or interface energy and a local inertial or kinetic energy. He introduced a hypothesis which claims that during the fragmentation process the forces seek to minimize the energy with respect to the fracture surface area.

6.2. Brittle fracture mode

For the tensile loading process and fragmentation of a brittle solid, GRADY [4] obtained an expression for the nominal fragment diameter:

$$d = \left[\frac{(20)^{\frac{1}{2}} K_{IC}}{\varrho c \dot{\epsilon}}\right]^{2/3}, \tag{6.1}$$

where K_{IC} is the material fracture toughness, ϱ denotes the actual density, c the velocity of fragment and $\dot{\epsilon}$ denotes the strain rate.

The condition for the propagation of a microcrack is as follows (cf. IRWIN [6])

$$R = \frac{1}{\pi}\left(\frac{K_{IC}}{\sigma}\right)^2, \tag{6.2}$$

hence

$$K_{IC} = \sigma(\pi R)^{\frac{1}{2}}. \tag{6.3}$$

For R we have the evolution equation (3.5) which, at σ = const, gives the result

$$R = R_0 \exp\left(\frac{\sigma - \sigma_{eq}}{\eta} \Delta t\right) \tag{6.4}$$

and

$$K_{IC} = \sigma \left\{\pi R_0 \exp\left[\frac{\sigma - \sigma_{eq}}{\eta} \Delta t\right]\right\}^{1/2}. \tag{6.5}$$

Finally we have

$$d = \left\{\frac{(20)^{1/2}\sigma \left[\pi R_0 \exp\left(\frac{\sigma - \sigma_{eq}}{\eta} \Delta t\right)\right]^{1/2}}{\varrho_s(1-\xi)c\dot{\epsilon}}\right\}^{2/3}, \tag{6.6}$$

where

$$\xi = \Xi N R^3 = \xi^F. \tag{6.7}$$

6.3. Comparison of theoretical predictions with eperimental data

The numerical computations are obtained for oil shale. It is assumed

$\varrho_s = 2300$ kg/m³, $\quad R_0 = 2.0 \times 10^{-6}$m,

$c = 4000$ m/s, $\quad \eta = 17.0$ Ns/m²,

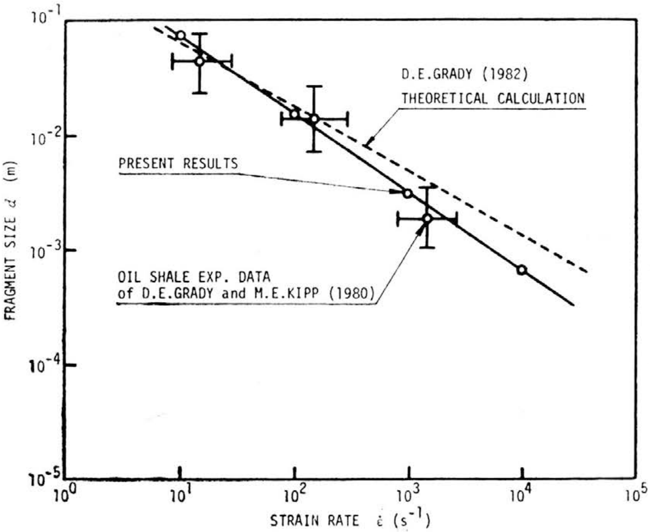

FIG. 4. The fragment size versus the strain rate plots for oil shale.

$\sigma = 4.0\times10^{7}$ N/m^2, $\sigma_{eq} = 0.9\times10$ N/m^2,

$\Delta t = 0.25\times10^{-5}$ s,

$\xi^F = 0.35$ (porosity at fracture, critical porosity).

The results for the fragment size versus the strain rate are plotted in Fig. 4. These results are compared with theoretical calculations obtained by GRADY [4] and with experimental data presented by GRADY and KIPP [5].

7. Final comments

The main idea of the description of brittle fracture is based on two internal state variables, namely the density of microcracks N and the average size of microcrack R. The evolution equations proposed for these two internal state variables describe directly the nucleation and growth mechanisms of microvoids. By postulating the additional equation of state we have taken advantage of the porosity parameter in the constitutive modelling of damaged solids.

It is noteworthy that the coalescence process of microcracks is also described because, as it has been suggested by SHOCKEY, SEAMAN and CURRAN [25], the linkage process in the case of brittle fracture is mostly generated by a nucleation and growth mechanism on a smaller scale.

It seems that the important result of the paper is a new model of an elastic-viscoplastic damaged material.

Taking advantage of the physical metallurgical observation results, a criterion of dynamic brittle fracture is proposed. This criterion is inherently time-dependent and it describes fracture as a final stage of the entire inelastic flow process, so that the evolution of the constitutive structure of a material is taken into consideration.

As an example, the application of the theory proposed to the dynamic fragmentation process has been shown. The comparison of the numerical results for the fragment size versus the strain rate obtained for oil shale with the theoretical calculations of GRADY [4] as well as with the experimental data presented by GRADY and KIPP [5] has proved the usefulness of the theory proposed.

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