27.

ON A PORISMATIC PROPERTY OF TWO CONICS HAVING WITH ONE ANOTHER A CONTACT OF THE THIRD ORDER.

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IF two conics have with one another a contact of the third order, that is, if they intersect in four consecutive points, it will easily be seen that their *characteristics* referred to coordinate axes in the plane containing them must be of the relative forms x^2+yz, $k(y^2+x^2+yz)$ respectively, y characterizing their common tangent at the point of contact*.

Hence if we take planes of reference in space, and call t the characteristic of the plane of the conics, the equations to any two conoids drawn through them respectively will be of the relative forms

$$U=x^2+yz+tu=0,$$

$$V=y^2+x^2+yz+tv=0.$$

Using W to denote $V-U$, and (W) to denote what W becomes when ey is substituted for t, we see that W and (W) are of the respective forms y^2+tw and $y\theta$; showing that the former is the characteristic of a cone which will be cut by any plane $t-ey$ drawn through the line (t, y) in a pair of right lines; or, in other words, that one of the cones containing the intersection of the two variable conoids (V and U) will have its vertex in the *invariable line* which is the common tangent to the two fixed conics: this proves the theorem stated by me hypothetically in a foot-note in one of my papers in the last number of the *Magazine*†. The steps of the geometrical proof there hinted at are as follows.

* These relative or conjugate forms are taken from a table which I shall publish in a future number of this *Magazine*, exhibiting the conjugate characteristics in their simplest forms, correspondent to all the various species of contacts possible between lines and surfaces of the second degree. This table is as important to the geometer as the fundamental trigonometrical formulæ to the analyst, or the multiplication table to the arithmetician; and it is surprising that no one has hitherto thought of constructing such.

[† p. 149 above.]

The four consecutive points in which the two conics intersect will be consecutive points in the curve of intersection of the two variable conoids. This curve lies in each of four cones of the second degree. Every double tangent plane to it passes through the vertex of one amongst these. The plane containing four, that is, two (consecutive) pairs of consecutive points, is a double tangent plane, and will therefore pass through a vertex; but four consecutive points of a curve of the fourth order described upon a cone, and lying in one tangent plane thereto, can only be *conceived* generally as disposed in the form of an ∫, of which the belly part will point to the vertex; or, in other words, at any point where two consecutive osculating planes coincide so that the *spherical* curvature vanishes, the linear curvature will also vanish, that is, there will be a point of inflexion at which, of course, the tangent line must pass through the vertex of the cone. This is the assumption felt to be true, but stated by me hypothetically in the paper referred to, because a ready demonstration did not at the moment occur to me. The legitimacy of this inference is now vindicated by the above analytical demonstration.

The methods of general and correlative coordinates and of determinants combined possess a perfectly irresistible force (to which I can only compare that of the steam-hammer in the physical world) for bringing under the grasp of intuitive perception the most complicated and refractory forms of geometrical truth.