23.

AN INSTANTANEOUS DEMONSTRATION OF PASCAL'S THEOREM BY THE METHOD OF INDETERMINATE COORDINATES.

[*Philosophical Magazine*, XXXVII. (1850), p. 212.]

THE new analytical geometry consists essentially of two parts—the one determinate, the other indeterminate.

The determinate analysis comprehends that class of questions in which it is necessary to assume *independent* linear coordinates, or else to take cognizance of the equations by which they are connected if they are not independent. The indeterminate analysis assumes at will any number of coordinates, and leaves the relations which connect them more or less indefinite, and reasons chiefly through the medium of the general properties of algebraic forms, and their correspondencies with the objects of geometrical speculation. Pascal's theorem of the mystic hexagon, and the annexed demonstration of its fundamental property, belong to this branch of the subject, and afford an instructive and striking example of the application of the pure method of indeterminate coordinates.

Let x, y, z, t, u, v be the sides of a hexagon inscribed in the conic U. Let the hexagon be divided by a new line ϕ in any manner into two quadrilaterals, say $xyz\phi$, $tuv\phi$.

Then $$ay\phi + bxz = U = \alpha u\phi + \beta tv;$$

therefore $$(ay - \alpha u)\phi = \beta tv - bxz;$$

therefore $ay - \alpha u$ and ϕ are the diagonals of the quadrilateral $txvz$.

By construction, ϕ is the diagonal joining x, v (that is, the intersection of x and v) with z, t; and thus we see that $ay - \alpha u$ is the line joining t, x with v, z; but this line passes through y, u. Therefore x, t; y, u; z, v lie in one and the same right line. Q.E.D.