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Abstract

The objective of the paper is development and implementation of the multi-objective optimization model for supporting local government (LG) long-term financial management and planning. The model assumptions are consistent with the EU regulations. The model, which is an extension of the model developed by Cichocki, provides maximum investment cumulated over an investment period $\{t_1, T_N\}$, and, at the same time, minimum of total costs associated with debt service after that period - over $\{T_{N+1}, T_M\}$, until all debts, used for financing investment, are fully repaid. Investments are implemented and debt is issued only over in the period $\{t_1, T_N\}$. Debt includes credits and bonds with various maturities.

The two objectives must be satisfied under several conditions, which result from practice and principles of sound financial management, as well as from fiscal rules, which are introduced in many countries in order to avoid budget deficits and excessive debt issued by LGs. The model is designed to support decisions made by LGs managers regarding long-term financial planning - can help them determine, each year, a safe level of investment (disaggregated into investments co-financed by the EU funds and the other investments) over an investment period, and a safe level of debt for financing investment. The financial situation of LGs is measured with the help of several financial indicators including: operating expenditures, operating surplus, the outstanding debt, and the total debt service, in relation to revenue, investment expenditures in relation to total expenditures, and debt structure. The model is solved many times using various initial conditions to find an admissible set of investment and debt (model outcomes).

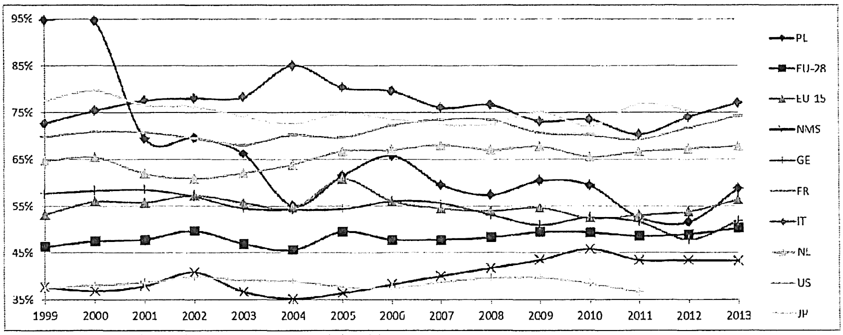
The model solutions are located on the Pareto frontier in the space of cumulative investment, and the total debt service. Less and more risky solutions are presented, which base on real Polish LG data and long term projections. Debt, investment and GFCF, which results from investment activity, are analyzed. An impact of an initial indebtedness on the admissible set of solutions and the Pareto frontier is also analyzed. The model can be implemented for analysis of alternative fiscal rules, as it is shown that the existing Polish rules are neither effective, nor efficient. Conclusions are presented.

1. Significance of the local government investment and its role in public finance

The role of local government in providing public goods and services and facilitating local societies quality of life cannot be underestimated. The functions and tasks implemented by institutions of the local government (LG) sector include many economic services – water, gas, and energy supply, sewerage and solid waste management, construction and maintenance of roads and housing - and general purpose non-economic public services: education, health and social care, culture and leisure and public security (law and justice). LG subsector is the largest public investor in the general government (public) sector.

The share of LG investment in public sector investment is high in many EU countries - in Italy (about 70%, 77% in 2013), France (74, 2% in 2013), and the Netherlands (68% in 2013). The share is also high in Japan, about 75% of the public sector investment. The share is low in Greece, Spain (about 30%), Austria, and in the U.S. (about 40%). In Poland, the share equals 58, 8% in 2013, and is higher than the EU average (about 50%). LG sector investment over 2009-2013 is lower in the new member states (NMS) - about 43% than in the old EU countries (UE15) - about 55%. The share is low in Bulgaria (below 30%), Estonia - 43% in 2013, 29% over 2010-2011, Hungary, below 40%, and Slovakia 41% in 2013 (see Fig. 1, and for details - Bitner, Cichocki, 2014).

Diagram 1. Share of LG subsector investment in the general government sector investment



Source: Eurostat data taken from Bitner, Cichocki, 2014.

In countries where local investment expenditures were low in 1999 and 2000, the investment expenditure grew fast; the average yearly growth rate in Latvia, Lithuania, Bulgaria and Romania was above 30% yearly. The investment expenditure, on average, constitutes 14%-22% of the local government total expenditure. The share is higher in EU new member states than in 15 countries of the "old EU" (the difference is about 6 - 8 percentage points - Bitner, Cichocki, 2014). Many NMS countries, also select EU15 countries will seek, over 2015-2022, to acquire European Union funds for financing local infrastructure investment. The NMS will try to maintain high investment to narrow the infrastructure gap between them and the leading EU countries. The LGs will remain the major investor in the public sector of very many countries.

Simultaneously, in Poland, the share of LGs subsector debt in GDP in 2011 equaled 4,27% (when was the highest in 15 years), and 4,25% in 2013, while the share of the public sector, including the LGs, in GDP equaled 55,6% in 2012, and 57,1% in 2013. In 2012 the share of LGs subsector deficit in GDP equaled 0,26% (0,18% in 2013), and the deficit share in GDP of the public sector, including the LGs subsector, equaled 3,9% (4,32% in 2013). The share of LGs subsector debt in GDP is low in the NMS countries (around 3%), with an exception of Latvia (above 5%), while in the Scandinavian countries, in France and Italy the share is about 8%. It is the central government sector, not the LGs sector, which mostly contribute to the excessive debt and budget deficits (see also Bitner, Cichocki, 2014).

In 2012 Polish LGs were asked in a questionnaire about the objectives of their long-term finance management and planning. Above 60% of LGs answered that they have been trying and will continue to acquire large EU funds in order to invest, and facilitate infrastructure development and enhancement of local services. 44% of LGs indicated simultaneously other objectives – decreasing indebtedness and reducing budget deficit. About 5% of LGs selected the single objective of minimizing the indebtedness to zero. Other 6% indicated that they have always been and will observe and implement the valid law regulations. Answers of about 25% of responders were not clear (see Cichocki, 2013a). It is highly probable that Polish LGs are similar to local governments in other NMS in the way of thinking about objectives of the long-term finance planning over 2015-2022.

2. Objective and methodology

2.1. Objective, subject and scope of analysis

The objective of the paper is development and implementation, with utilization of the multi-objective optimization theory, of a model for supporting LGs managers in the area of long-term financial management and planning. The model will facilitate selection, each year, of a safe level of investment, disaggregated into investments co-financed by the EU funds and other investments financed from the budget and debt, and a safe level of debt for financing investment, and its structure (medium, long term credits and bonds). These investments and debts will ensure maximum investment cumulated over an investment period, for example over 2014-2023, and, at the same time, minimum, after 2023, of total costs associated with debt service issued for investment financing during 2014-2023. The costs of debt service are minimized until debts maturities, when all debts are repaid.

Data utilized in analysis

Historical data of 2011-2013, used in the model as exogenous values, are taken from a Polish LG report. The financial projections of operating revenues and basic operating expenditure over 2014- 2023 (exogenous in the model) are taken from official long-term financial projections of the LG included in the database of the Ministry of Finance. Financial projections exogenous values over 2024-2033 are made by authors, who also use inflation and GDP data presented by the Polish National Bank, and the Ministry of Finance.

3. Model formulation

The model developed in the paper fits current Polish legal regulations and is consistent with general EU regulations. It is an extension and verification of the model presented in Cichocki, 2013a and Cichocki, 2013b and bases on general framework of local government finance and debt management (Hallerberg et all, 2007 [103], Kavanagh, 2007, Rossi & Dafflon, 2002,[164],Cichocki, Kleimo, Ley, 2001, Miranda, Picur, 2000[139],Leonard, 1996, Josef, 1994;). An earlier version of the model was described in Cichocki, 2010. The model can be used, after minor changes, for efficient long term finance, debt and investment management in many economies.

The major verifications of the optimization model described in Cichocki, 2013a, and b include: explicit and simultaneous consideration of two objectives, extension of the period of analysis and analysis of consequences of decisions regarding debt and investment until debt maturity, explicit inclusion of two types of credit and bonds (medium and long-term), and analysis of fixed assets which are the result of investment over consecutive years. In previous works the objective of the model was to maximize the total funds (from budget surplus and from debt) for financing investment. Implicitly, debt was maximized each period, determining an upper limit for safe and legally justified level of debt and lumped investment.

In the presented model we look for a maximum of accumulated investment – summed up over a given period $\{t_1, T_N\}$, which can be financed from the LG budget, the EU funds, and from debt. It is assumed that the EU funds can be used for financing investment only when the LG will provide its own share from either its own budget, or from debt - when the LGs budget surpluses allow, and debt can be issued. This is the EU regulation, which also determines, for each individual project, the maximum share of co-financing with the EU funds at 85% of a total project value. Investment projects implemented over $\{t_1, T_N\}$ can be also financed jointly from debt and from the LG budget. Capital improvement plans often use debt financing (Bitner et all, 2013, Kavanagh, 2007, Hallerberg et all, 2007, Cichocki, Leithe, 1999, Joseph, 1994).

We also assume in the model that the debt issued in consecutive years, until T_N, for financing investment projects, either together with EU funds, or with budget funds is structured in such a way that the costs of the total debt service until the debt with the longest maturity is repaid - is minimum. The total debt service includes repayment of debt principals (also of the initial debt issued prior to t_1), guarantees given by LG, and interest on the outstanding debt until all debts are repaid. Thus, the model solution enables determination of *debt structure* - bond covenants and loan terms - including medium and long-term credits and bonds, which maximize investment and minimize debt service. Repayment structure of debt results from the model assumptions regarding medium and long-term credits maturity and bond redemption. In addition, the debts are safe and legally justified, as they conform to regulations and restrictions, included in the model, regarding liquidity, maximum level of operating expenditure and fiscal rules imposed by law. Thus, the model enables assessment of debt capacity of a LG and determination of a safe level and structure of debt each year.

Investment contribute to the value of fixed assets - *gross* fixed capital formation (GFCF), which can facilitate better services, but at the same time they contribute to the value of depreciation of fixed assets, and also generate increases in operating costs associated with the maintenance of the existing assets and of new capital purchases. In the model, the GFCF at time t is defined as the value of GFCF in previous year (at t-1), plus new investment at t, minus the depreciation of GFCF at time t, and minus sales of assets (property) at t - part of fixed assets at t (see D2).

3. 1. Definition of model variables

Model variables are defined for all $t \in \{t_1, T_N\}$ and are utilized for calculations at all time instants $t = (t_1,.., T_N, T_{N+1}, T_M)$, where t_1 denotes an initial period of analysis, usually it is a budget year, T_N is the last period of investment activity and debt issuance, and T_M is the last period of debt repayment - denotes the end year of analysis. M>N are integers - number of years, or quarters. Selection of T_M results from *debt structure* - bond covenants and loan repayment terms. The model starting point is a year t_0 – end of the year proceeding the budget year (values of variables at the end of t_0 equal variables values at the beginning of year t_1). Model variables include:

- Inv^{UE}_t – investment expenditure co-financed by the EU funds
- Inv^b_t – investment expenditure financed by the LG budget
- $C1^{EU}_t$ – medium term credits financing investment co-financed by the EU funds
- $C1^b_t$ – medium term credits financing investment, jointly with the LG budget
- $C2^{EU}_t$ – long-term credits financing investment co-financed by the EU funds
- $C2^b_t$ – long-term credits financing investment, jointly with the LG budget
- B^{EU}_t – medium term bonds financing investment co-financed by the EU funds
- B^b_t – medium term bonds financing investment, jointly with the LG budget.

The newly issued debt at time t is the summation of all credits and bonds as shown in (O2A).

3. 2. Formulation of the multi-objective optimization model

The model is formulated in a way, which provides maximum level of accumulated investment over a period $\{t1, T_N\}$, where $t_1 = t_0 +1$, and at the same time minimum of total costs associated with debt service over $\{T_{N+1}, T_M\}$. Investments are implemented and debt is issued only over $\{t_1, T_N\}$. All debts are fully repaid in the analyzed period - until T_M. Time instant T_M is selected to include repayment of a debt with the longest maturity. Investment and debt are safe - they assume liquidity and balance of operational account, at all time instants, and consistency with fiscal rules of a given country. Mathematical formulation is as follows.

Given some initial values of the model variables and parameters at the initial time period t_0[1], analysis time interval (t_1, TN, T_{N+1},..T_M), and exogenous projections of select variables over $\{t_1,T_M\}$, find, for every time instant $t \in \{t_1,T_N\}$, such values of investment expenditure, co-financed by the EU funds and from the LG budget, Inv^{UE}_t, Inv^b_t, and new debt $ND_t = \{C1^{EU}_t, C1^b_t, C2^{EU}_t, C2^b_t, B^{EU}_t, B^b_t\}$ - medium term credits C1, long-term credits C2 and medium term bonds B[2], used for financing investment co-financed by the EU funds, and investment financed from the LG budget - which **maximize cumulative investment** over $\{t_1, T_N\}$, **and minimize** over $t \in \{t_{N+1},T_M\}$ the total **cost of servicing debt** issued for financing investment.

$$Maximum \{ \sum_{t=t1}^{TN} (Inv^{EU}_t + Inv^b_t)\}, \qquad \textbf{(C1)}$$

[1] The initial indebtedness Z_{t0} is very important. For some excessive Z_{t0} values the model solution may not exist.
[2] Long-term bonds can be included in the model.

$Inv^{EU}{}_{t}, Inv^{b}{}_{t}, C1^{EU}{}_{t}, C1^{b}{}_{t}, C2^{EU}{}_{t}, C2^{b}{}_{t}, B^{EU}{}_{t}, B^{b}{}_{t},$

$Inv^{EU}{}_{t} \geq 0, Inv^{B}{}_{t} \geq 0, C1^{EU}{}_{t} \geq 0, C1^{B}{}_{t} \geq 0, C2^{EU}{}_{t} \geq 0, C2^{B}{}_{t} \geq 0, B^{EU}{}_{t} \geq 0, B^{B}{}_{t} \geq 0;$

and simultaneously

$$Minimum \left\{ \sum_{t=TN+1}^{TM} [RD_t + Int(\sum_{j=1}^{tN} ND_j + D_{0j})]_t \right\}, \qquad \textbf{(C2)}$$

where $ND_j = C1^{EU}{}_{j} + C1^{b}{}_{j} + C2^{EU}{}_{j} + C2^{b}{}_{j} + B^{EU}{}_{j} + B^{b}{}_{j},$ **(C2A)**
and D_{0j} is the level of initial debt at time instant j.

The above two objectives must be satisfied under several conditions (constraints), which result from practice and principles of sound financial management as well as from fiscal regulations, which are introduced in many countries in order to avoid excessive debt, issued by LGs, and budget deficits. In the model we introduce several such constraints. The first constraint results from legal regulations in Poland[3]. It ensures operating expenditures in a given year, which do not exceed operating revenues plus surpluses on the current account from the previous year. The condition is a modification of the golden rule of finances, which states that the operating (current) budget must be balanced (Cichocki, 2013, Kavanagh, 2007, Rossi, Dafflon 2002).

$$OpRev_t - OpExp_t + Nrb'_{t-1} \geq 0, Nrb_{t-1} \geq 0, \qquad \text{(con. 1)}$$

where $OpRev_t$ denote operating revenues, and $OpExp'_t$ – total operating expenditures at year t. The operating revenues are exogenous in the model. They are specified in separate projections for all sources of revenue (see Appendix 1).

The total operating expenditures consist of two parts. Basic operating expenditures, $BOpExp_t$, which cover LGs statutory task and services, including labor costs with social insurance, and debt and investment dependent expenditures, which include costs of spending for service of the existing and planned debt, and maintenance costs of fixed assets and facilities, which usually grow with new debt and investment.

$$OpExp_t = BOpExp_t + Int_t (\sum_{j=1}^{t} ND_j, D_{t0}) + \Phi_t\, GFCF_{t-1}, \qquad \text{(D1)}$$

where $GFCF_{t-1}$ denotes *gross* fixed capital formation (*gross* fixed assets), and Φ_t is the share ratio of the maintenance costs in GFCF. Investment contribute to the value of fixed assets, but

[3] Public Finance Law (Lpf), 2009, with amendments, art. 242.

at the same time they generate increases in operating costs associated with the purchase of a new asset, and maintenance of the existing assets and of new capital purchases. $Int_t(\ )$ is the interest cost of the existing part of initial debt D_{t0}, and of cumulative new debt at period t. D_{t0} is the initial debt resulting from commitments made prior to time t_1, with given repayment schedule over $t_1,..,T_M$. In Appendix 1 we describe projections methods of operating revenues and basic operating expenditures (see also Cichocki, 2013, Kavanagh, 2007).

Gross fixed capital formation is defined by (D2). Investment contributes to the growth of the existing assets. Depreciation of fixed assets and sale of assets decrease the value of the GFCF.

$$GFCF_t = GFCF_{t-1} + Inv_t - d_t\,(GFCF_{t-1} + Inv_t) - SalGFCF_t\,, \qquad \text{(D2)}$$

where d_t is a depreciation rate of fixed assets at t, and $SalGFCF_t$ denotes sales of property at t.

The second constraint results from practical principles of sound financial management. Each period t, all cash receipts minus cash disbursements must be positive or nonnegative - liquidity must be assured. In the model we want to ensure financial balance (budget liquidity) at each year of the investment period. Thus, the revenue and expenditure forecasts and debt levels in consecutive years $t_1,..,T_N$ must ensure the LG balanced financial position each year. The condition Con. 2 ensures also continuity of investment financing.

$$OpS_t + \delta D_t + CapRev_t + OthRev_t' + Nrb_{t-1} - Inv_t \geq 0. \qquad \text{(con. 2)}$$

NO_t denotes the *operating surplus* - operating revenues minus operating expenditures at time t. It is the operating surplus *net* value, as the operating expenditures include costs of spending for service of the existing and planned debt, and the fixed assets maintenance costs. Investment (expenditures on capital purchases) contribute to the value of fixed assets, but at the same time they generate increases in operating costs associated with the purchase of a new asset, and maintenance of a new facility. Net operating surplus is gross operating surplus minus costs of debt service and fixed assets maintenance costs.

$$OpS_t = OpRev_t - OpExp_t \qquad \text{(D3)}$$

Resources defined as operating surplus can be used to fund capital expenditures. The larger the level of *OpS,* the more funds available for financing investment.

Revenue from loan proceeds, from capital shares owned by the LG and revenue from previous year budget surpluses are considered non-revenue, and may serve to finance budget deficit. Likewise, the expenditure does not include amounts allocated for the repayment of loan principal - it makes up proceeds (also non-expenditure).

The value of δD_t determines an increase in debt during a year t; it can also be called *net* debt at t, which is defined as

$$\delta D_t = D_t - D_{t-1}, \text{ also as} \quad \text{(D4a)}$$

$$\delta D_t = ND_t - RD_t - offD_t, \quad \text{(D4b)}$$

where D_t is the debt at the end of year t, D_{t-1} - the debt at the end of year t-1, RD_t is the total repayment of debt at t, including repayment of the "old" debt, issued prior to t_1, and of the new debt issued in the analyzed period, starting t_1. Debt repayment RD_t includes repayment of credits $C1^{EU}$, $C1^b$, $C2^{EU}$, $C2^b$, and repurchase of bonds B^{EU}, B^b, as well as credits and bonds of the old debt. The value of $offD_t$ denotes exogenously given debt write-offs at time t (cancellations in account books – separately for each debt category), which decreases the debt outstanding.

Thus, the debt at $t = t_1, t_2, \ldots, t_N = T_N$, equals

$$D_t = D_{t-1} + ND_t - RD_t - offD_t, \quad \text{(D5)}$$

It is assumed in further discussion, for convenience, that the debt write offs are included in the debt repayment RD_t. The values of OpS_t and δD_t can assume both, negative and positive values, while ND_t ($C1_t$, $C2_t$, B_t), and Inv_t. (model variables) can assume only positive values.

The debt outstanding at the end of t, can be also calculated as

$$D_t = D_{t0} + \sum_{k=0}^{t-t1} \delta D_{t-k}. \quad \text{(D5a)}$$

Other revenues, $OthRev_t$, are other budget net inflows considered non-revenue, which may serve to finance budget deficit. They include various inflow items, for example from privatization, minus other budget non-revenue outflows, not associated with debt.

Capital revenues $CapRev_t$, together with the operating surplus, *net* debt proceeds and other non-revenues, as well as non-revenues from previous year are used to finance investment. Capital revenues consist of three major parts: funds for financing investment from the EU, and, exogenously determined projections, over $\{t_1, T_M\}$, of sales of property and special capital grants. The EU funds actually used for financing investment are calculated in the model. However, they depend on parameters assumed in the model regarding the level of the LGs own share (for example 20%, or 30%), required by the EU for all projects co-financed by EU funds (see Appendix 2). One might consider a couple of such projections (minimum two scenarios).

The current account financial balance from the previous year, Nrb'_{t-1}, includes revenues and expenditures balance (NB_{t-1}), and debt balances (Nrb_{t-1}) – debt at the beginning of the year t-1, plus debt receipts minus repayment of debt principals at time t-1. It is assumed, for simplicity, that exogenously given debt write-offs are included in debt repayments. The Nrb'_{t-1} must be nonnegative.

The current account balance Nrb'_t is defined as follows:

$$Nrb'_t = Nrb_{t-1} + ND_t - RD_t + NB_{t-1}, \qquad \text{(D6)}$$

where NB_{t-1} denotes budget surplus, when $NB_{t-1} > 0$, or budget deficit, when $NB_{t-1} < 0$.

$$NB_t = Rev_t - Exp_t, \qquad \text{(D7)}$$

where Rev_t is the total revenue, and Exp_t, - the total expenditure of the budget at time t. Actually, the current account balance (D5) includes two separate balances defined in Polish law: the budget revenue and expenditure account balance, and the debt account balance. In Appendix 2 formation of debt outstanding, calculation of interest costs and repayment policy of the "old" and new debt are described.

The total revenue of the budget, at time t, is the sum of the operating revenue, and of the capital revenue

$$Rev_t = OpRev_t + CapRev_t, \qquad \text{(D8)}$$

and the total expenditure of the budget at time t, is the sum of the total operating expenditures and the capital expenditures

$$Exp_t = OpExp_t + CapExp_t, \qquad \text{(D9a)}$$

and the capital expenditures is a sum of investment expenditures, and of other capital expenditures. We assume in further description of the model, that the other capital expenditures can be neglected, and

$$Exp_t = OpExp_t + Inv_t. \qquad \text{(D9a)}$$

Total revenue is a summation of operating revenues, which include several categories of tax revenues, intergovernmental transfers and capital revenues

$$Rev_t = BOpRev_t + IncOpRev_t + EURev_t + SalGFCF_t + Capgrants_t, \qquad \text{(D10)}$$

where $BOpRev_t$ are basic operating revenues, Inc $OpRev_t$ denotes incidental revenues - one time revenues, for example earmarked grants, $EURev_t$ are funds received from the EU for financing investment projects, and $Capgrants_t$ are capital grants, all at time t. Discussion regarding projection of the above revenue factors over $\{t_1, T_M\}$. can be found in Appendix 1.

The current account has to be balanced - has to be nonnegative for all $t = t_1,..,T_N$. This is the third constraints resulting from practice of financial management.

$$Nrb'_t \geq 0 \qquad \text{(con. 3)}$$

It will decrease when deficit occurs, $NB_{t-1} < 0$, then, the budget deficit must be financed by additional debt.

The fourth constraint of the model, again results from legal regulations and can be of different form. It belongs to a class of fiscal rules, usually imposed by the regulator, and the government, to restrain LGs from excessive debt issues. In Poland, currently, the constraint is imposed on the total debt service at time t, including total debt repayment, payable guarantees, and total interest on the outstanding debt at t, related to the total revenue at period t. These costs must cannot exceed an affordable level (a limit). The limit is calculated for a particular municipality, and depends on the past LGs performance: the average value, over three years preceding the year t, of the operating surplus enlarged by the revenue from sales of property, in relation to total revenue (see Lpf, art. 243).[4]

$$[(RD_t + Int_t)/Rev_t] \leq \sum_{i=1}^{3} 1/3\ [(OpRev_{t-i} - OpExp_{t-i} + SalGFCF_{t-i}) / Rev_{t-i}]. \qquad \text{(con. 4)}$$

The costs of debt service are calculated for each credit and each bond issue separately. The interest costs are computed on the cumulative debt – the old debt and the new debt – credits and bonds outstanding at time t. For example credit outstanding at time *t*-1 plus new credit taken at time *t,* minus credit repayment at t. Similarly, interest costs are calculated on cumulative bond issues outstanding at time t (see debt definition D4 and Appendix 2). When in any year of the period $\{t_1, T_N\}$ either operating revenues will be lower than projected, or

[4] In order to calculate the limit for cost of debt service at time t_1, the right hand side expression of Con. 4, one has to use data for t_1-1, t_1-2, and t_1-3, where $t_1-1 = t_0$, and t_1 is the budget year. These data have to be known.

basic operating expenditure higher than projected, then, the upper limit regarding the costs of debt service will be lower, and less debt can be issued in future years. This means that less EU funds can be acquired, and less investment projects implemented.

The value of the indicator on the left had side of the condition 4 and of the limit for the total debt service is different for each period, and is calculated separately for every municipality. The value of the indicator and of the limit can be obtained from the presented model.

In Poland the conditions (con. 1) and (con. 4) must be satisfied, the first over (t_1, T_N), the second, over the whole period (t_1, T_M), otherwise, the city council cannot approve the budget for the year t_1, and the obligatory financial plan for the three following years t_1+1, t_1+2, t_1+3.

The total costs of debt service (defined in art. 243of the pfl) are defined as the sum of debt repayments - repayment of credits, bond repurchase, and interest Int_t on the total debt outstanding, as well as the guarantees extended by the LGs. We assume that the guarantees are included in the repayment principal.

$$DS_t = RD_t + Int_t\,(D_t,\ D_{t-1},\ i_t), \qquad \text{(D11)}$$

where i_t is a vector of interest rates for credits and bonds at time t. For example, the debt service in the first period of analysis, t_1, equals

$$DS_{t1} = RD_{t1} + Int_{t1}\,(D_{t1},\ D_{t0},\ i_{t1}). \qquad \text{(D12a)}$$

The debt service consists of the cost of servicing the old debt resulting from commitments made prior to time t_0 and of the new debt issued at t. For the first period t_1 we can write

$$DS_{t1} = DS_{t0} + NDS_{t1}, \qquad \text{(D12b)}$$

where the DS_{t0} is the initial debt service at t_1. The vector of the interest rates: i = (ic_{1t}, ic_{2t}, ib_t) for all t ∈ {t_1, TM}, and is charged for all credits (medium and long-term) and for bonds.

When we assume that the debts' repayments and bonds repurchase, takes place at the same time as the new debt issue, and that it happens in the middle of each year, on the 1st of July, then, the interest at time t can be calculated, separately for each debt category, as

$$Int\,(\sum_{j=1}^{t} ND_j)_t = \tfrac{1}{2}\, ic_{2t}\,(ND_{t-1} + ND_t), \qquad \text{(D13)}$$

where the interest (D13) is calculated based on new debt at the end of the year t-1 (the beginning of t), the end of the year t, and the interest rate ic_{2t} on the long-term credits paid at t. The formula can serve for calculation of interest on the medium term credits, the long-term credits and the bonds. The interest includes cost of interest charged for the new debt issued at t.

The overall debt repayment includes the repayments of the old debt at t, $(RD_{t0})_t$, and the total new debt repayments – new medium term (four years) credits, $RC1_t$, long- term (ten years) credits, $RC2_t$, and five year bonds, RB_t, all issued starting time t_1. The bond repurchase takes place once in five years.[5]

$$RD_t = (RD_{t0})_t + RND_t = (RD_{t0})_t + RC1_t + RC2_t + RB_t . \qquad \text{(D14)}$$

The repayment of debt issued at time t, starts the next year, at t+1. The repayment schedule of the medium and long-term credit, as well as the bond repurchase schedule is given. Equal nominal credit repayments are assumed. The repayment schedule of the old debt results from contracts concluded prior to time t_1 and bond prospectus.

3.3. Model assumptions

The model is solved under several assumptions. Firstly, it is assumed that the operating revenues, and the basic operating expenditures are exogenous in the model. Their projections are prepared as described in Appendix 1, and they also use arbitrary projections of inflation, and GDP growth rate. Interest rates for short, and long-term credits, and also for bonds over the period $t_1,..,T_N,..,T_M$, are assumed to be known. Exogenous are also select capital revenues: from sales of property, and from capital grants. Secondly, the share of LGs budget in investment projects co-financed from the EU funds equals 20%. It is fixed, but can easily be changed in various model scenarios. It is also assumed that the LGs share is financed from debt. There exist also in the model a reference trajectory – an exogenously assumed level of funds, each year $t_1,..,T_N$, which is an upper level for the EU funds used in the model. There are some Pareto optimal solutions, which satisfy all model constraints, but do not use all available EU funds.

The debt maturity schedule - repayment structure for each debt category (credits and bonds is known). The time and level of issue for various credits and bonds result from the model solution. It is assumed, for the simplicity of calculation, that the debts' repayments and bonds repurchase, precedes the issue of new debt, but takes place instantaneously, in the middle of each year, on the 1st of July. Such simplification was for example assumed in the US law (see 2009-2010 Wisconsin States Annotations, 6703, p. 2).

[5] In the model any time for bond maturity can be assumed. We assume that five year bonds are issued in order to better analyze the interrelation between four and ten year credits, and the repurchase of bonds.

4. Optimal solutions of the multi-criteria model

The model described in section 3. is designed to support decision made by LGs managers regarding long-term financial planning. Specifically, to help them determine a safe level of investment expenditure over a planning investment period, $\{t_1, T_N\}$, and a safe level of debt for financing investment, which will be not excessive in the investment period - until T_N, and later, until T_M, when the last debt principal is repaid, and all bonds repurchased. The optimal solutions of the model allow to select time and level of medium and long-term credits, and bonds, as well as the level of investment each year, which will maximize cumulative investment over $\{t_1, T_N\}$, and minimize over $\{t_{N+1},T_M\}$ the cost of debt service after completion of the investment process.

4.1. Multi-criteria optimization method – stylized facts

In the model there are two criteria: the cumulative investment, maximized over the time period $\{t_1, T_N\}$, and the total cost of servicing debt issued for financing investment, which is minimized over $t \in \{t_{N+1},T_M\}$. The criteria, denoted by y_1, and y_2 respectively are conflicting. Increasing investment, as a rule, facilitates using debt for financing, and contributes to the increase of debt service. Thus, we deal with the two criteria (multicriteria) optimization problem, in which we look for decision variables satisfying the formulated constraints (c1-c4), and, at the same time, jointly maximize the criterion y_1 and minimize the criterion y_2.

The problem is considered in two spaces: the first space of decision variables, and the second space of criteria. The vector of decision variable $x= Inv^{EU}{}_t, Inv^b{}_t, C1^{EU}{}_t, C1^b{}_t, C2^{EU}{}_t, C2^b{}_t, B^{EU}{}_t, B^b{}_t$ is described in sections 4.2. and 5. The model constraints define a set X_0 of admissible values of the decision variables in the first space. In the second two-dimensional space there exists a set of attainable values of the criteria, which are called outcomes, or pay-offs. The set of outcomes is denoted by Y_0. Consistently with the theory of multicriteria optimization (Keeney and Raiffa 1976, Wierzbicki 1986), we look for decision variables which are the nondominated (Pareto optimal) points in the set Y_0. In the case of our model, a point (y_1, y_2) is nondominated in the set Y_0 if there is no other point in this set improving jointly the both criteria.

The domination relation is introduced in the space $\mathbf{R}^2$ of criteria (y_1, y_2). We say that a vector $y=(y_1, y_2)$ **dominates** a vector $v=(v_1, v_2)$, where $y, v \in \mathbf{R}^2$, if $y_1 \geq v_1$ and $y_2 \leq v_2$ and $y \neq v$. A vector $y=(y_1, y_2)$ **strictly dominates** a vector $v=(v_1, v_2)$, where $v \in \mathbf{R}^2$, if $y_1 > v_1$ and $y_2 < v_2$. The domination relation defines partial ordering in the criteria space, which is not a linear ordering. Therefore, in this case traditional optimality concepts defined for one criterion optimization problems are not valid.

A vector y is **Pareto optimal (nondominated)** in the set Y_0, if $y \in Y_0$ and there is no $v \in Y_0$ dominating the vector y. In the model, y is a two element vector, where y_1 defines investment cumulated over $\{t_1, T_N\}$, and y_2 – total debt service cumulated over $\{t_{N+1}, T_M\}$. We say that a vector y is **weakly Pareto optimal (weakly nondominated)** in the set Y_0, if $y \in Y_0$ and there is no $v \in Y_0$ strictly dominating the vector y.

The admissible set Y_0 is not given explicitly. Particular points of the set can be found by computer simulations. In general there exists a set of the Pareto optimal points in Y_0 and the corresponding decision variables in X_0. This set should be derived by computation, and then analysed.

The analysis of possible Pareto optimal outcomes is made by applying the reference point approach developed by Wierzbicki (Wierzbicki, 1986), (Wierzbicki at al., 2000) with the use of the order approximation achievement functions. According to this approach, reference points in the criteria space are assumed by a system analyst and then the computer-based system generates respective outcomes which are Pareto optimal in the set of attainable outcomes. Assuming a number of the reference points, a representation of the Pareto frontier can be obtained.

Outcomes characterizing the Pareto frontier are derived by:

$$\max_{x \in X_0}[s(y(x), y^*)] \qquad \text{(M-A)}$$

where: x – is a vector of decision variables,

X_0 - is a set of admissible decisions defined by the model relations,

$y(x) = (y_1(x), y_2(x))$ – is a vector of the criteria, which depends on the vector of decision variables x through the model relations. The criteria include y_1 (the investment cumulated over $\{t_1, T_N\}$), and y_2, the total debt service cumulated over $\{t_{N+1}, T_M\}$ - after the investment process is completed.

$y^*=(y_1^*, y_2^*)$ – is a reference point (called also an aspiration point) assumed in the space $\mathbf{R}^2$ of the criteria y_1 and y_2.

$s(y,y^*)$ – is an order approximating achievement function.

The function

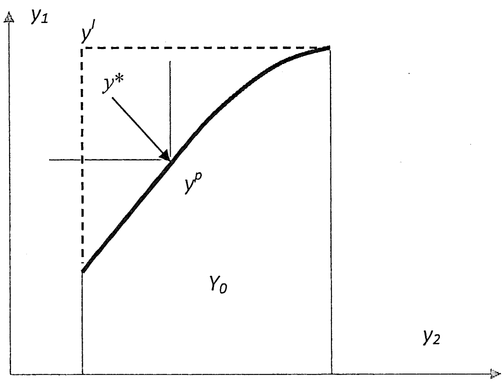
$s(y, y^*) = 1- \{(1/2)\{ [1-s_1(y_1, y_1^*)]^p + [1-s_2(y_2, y_2^*)]^p \} \}^{1/p}$, is an example of an achievement function suitable in the case of the model objectives, where $y^* \in \mathbf{R}^2$ is a reference point,

$s_1(y_1, y_1^*) = (y_1 - y_1^*)/(y_1^{up} - y_1^*)$, $s_2(y_2, y_2^*) = (y_2^* - y_2)/(y_2^* - y_2^{lo})$,

p is a given integer number $p>2$, $y^d = (y_1^{up}, y_2^{lo})$ is a given point dominating the ideal point y^I. The ideal point y^I is defined by the maximal attainable value of y_1 and the minimal attainable value of y_2.

The reference point method is illustrated in Fig. 1. A hypothetical set of attainable outcomes Y_0 is presented in the space of the criteria y_1, y_2. The set of outcomes is defined by the model relations but is not known explicitly. A system analyst implementing the model, assumes a reference point y^* in the criteria space, and solves the optimization problem (M-A). The reference point can be inside or outside the set. The corresponding Pareto optimal point y^P, including all decision variables, and other variables of the model is derived as the solution of the problem (M-A).

Fig. 1. Set of attainable payoffs Y_0, the ideal point y^I, a Pareto optimal point y^p and the domination cone shifted to the point y^p.



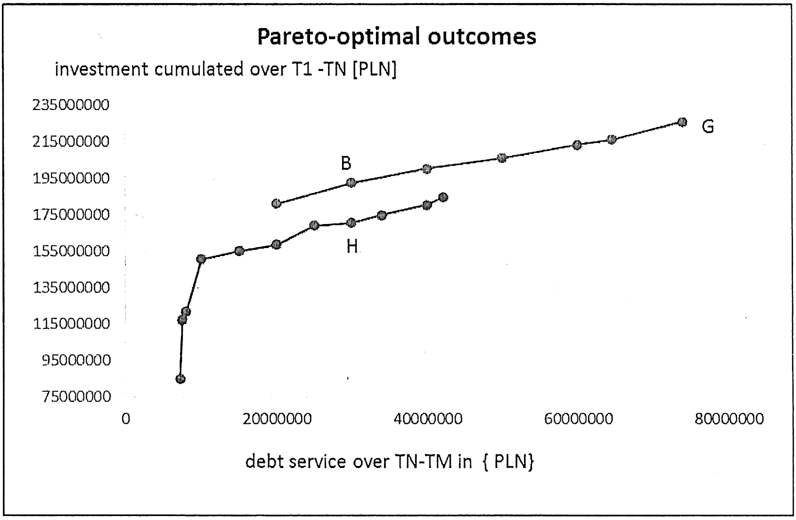
The domination cone is a set of points satisfying the conditions: $y_1 \geq 0$ and $y_2 \leq 0$.

The system analyst assumes yet another reference point, solves again the problem (M-A) and obtains a next Pareto optimal point. Then again, he selects next reference point and solves the problem (M-A). In such an interactive way a representation of the Pareto frontier of the unknown set Y_0 is obtained.

4.2. The model optimal solutions

In Figure 2, we present select model solutions, located on the Pareto frontier for different initial indebtedness D_{t0} (one of initial model conditions). For the initial D_{t0} = 15 million PLN, the Pareto optimal points are located above the Pareto frontier obtained for higher initial indebtedness D_{t0} = 35,4 million PLN. When the initial debt is lower, one can invest more, issue more new debt, and generate larger debt service costs after the investment period $\{t_1, T_N\}$ - the payoffs are much larger, and the admissible set is located higher, then in the case of higher initial indebtedness.

Fig. 2. Pareto optimal frontiers of the model solutions for different initial conditions.



In section 5.1. we present the model Pareto solutions of points B (y_1=194,8m; y_2=30m) and G (y_1=225,7; y_2=73,8m), both located on Pareto frontiers, with the initial indebtedness D_{t0} = 15 million. In section 5.2. solutions of point B, and point H (y_1=170,8m; y_2=30m), located on the Pareto frontier with the higher initial debt equal to 35,4 million PLN, are compared.

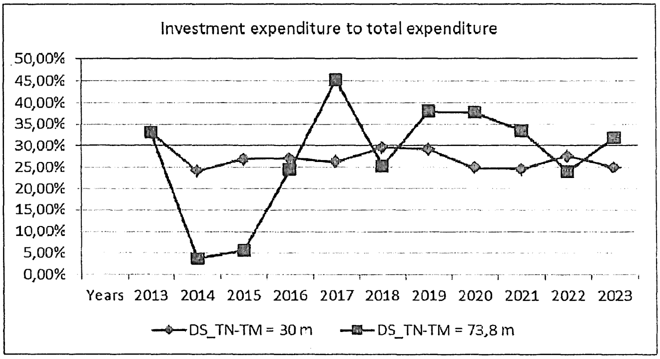
5. Presentation of computational results

The Pareto solutions of the model represented by point G are much more risky, then the solutions represented by point B. The cumulated investment in G are higher by about 30 million PLN than in B. Therefore, in G, the debt cumulated over $\{T_N+1, T_M\}$ also is much higher. However, since the Pareto solution G is located on the edge of the admissible set – on the border of Pareto frontier, a slight change in exogenous projections, for example regarding future operating revenues, or sales of property, might shift the solution outside the admissible set. Then, most probably, the constraint (con. 4) will not be satisfied. Large investment will be started, but there will be no possibility to continue the investments because, for example, the condition 4 will be violated. In case of point B, with lower investment and lower debt, a decision maker will have more flexibility. He will be able to increase investment and debt, and will remain in the admissible set, on the Pareto frontier – one can move in the direction of point G. In figures below, point B is denoted: DS_T_N-T_M = 30 m, and point G: DS_T_N-T_M = 73,8 m.

5.1. Important optimal solutions of the model

In figures 3-14 two different model solutions are compared, both with the initial debt D_0 of 15 m PLN, They are located on the Pareto frontier presented in Fig. 2. We present one solution (point B) – variables' projections generated by the model, with the cumulated, over (T_N+1, T_M), costs of debt service of 30 m PLN, and when the sum of investments over (t_1, T_N) equals 194,7 m PLN. During (T_N+1, T_M), we do not issue debt, neither invest.

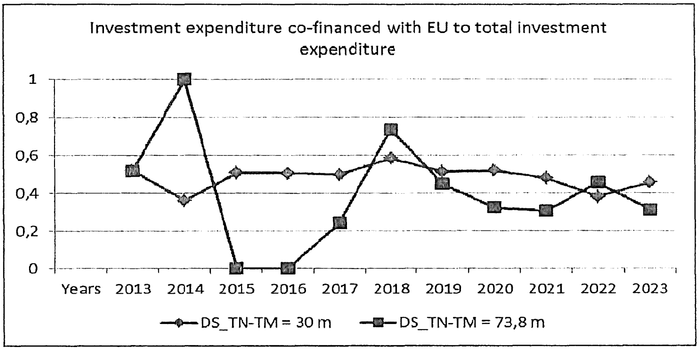
Fig. 3. Investment expenditures in relation to total expenditures over t_0-T_N



The second solution (point G) is also located on the Pareto frontier, with the cumulated debt service of 73,8 m PLN, and investment cumulated over (t_1,T_N), which equal 225,7 m PLN. These are optimal solutions, which satisfy all model constraints and maximize the investment expenditures cumulated over t_1–T_N, and at the same time minimize the total debt service costs summed up over (T_N+1,T_M).

The investment expenditures over the period t_1 = 2014 to t_N = 2023, and the share of these expenditures in total LGs expenditures are more stable for the point B solution, than for the solution G, when the cumulated investment are higher by about 16% (30 million PLN). Also, the share of investment expenditures co-financed with the EU funds in the total investment expenditures are much more stable. The optimal solution suggests that in 2015 and 2016 no UE funds are used for financing investment (Fig. 4.), and such a solution will assure maximum total cumulated investment, larger, then for solution B. The investments, specially in 2015 are very low. Such a solution may not be accepted in many practical cases, as some investment projects will have to be continued in 2015 and 2016, and the funds provided by the model optimal solution, may not be sufficient for that purpose.

Fig. 4. A share of investment expenditures co-financed with the EU funds in total investment



The total debt service, including repayment of debt, and the total debt service in relation to total revenue over (t_1,T_N) grows fast for both analyzed solutions. The rate of growth is very fast in 2015, and in 2020, 2021 and 2023 for the solution G, and in 2015 and 2023, for the solution B. The solution G, with higher debt service cumulated over (T_N+1,T_M) - after 2023, and with higher investment, generates the debt service in the period 2016-2020, which is lower than for solution B, with lower investment and lower debt service after 2023. It results from lower debt, see Fig. 11. However, the fiscal rule of the Polish law on public finance (lpf) requires that the total debt service in relation to total revenue is below the statutory limit determined by (con. 4), when the debt service associated with debt financing investment projects which are co-financed by the EU funds, is subtracted. The total debt service with such exclusions, in relation to total revenue is presented in Fig. 6, together with the limits defined by the lpf (con. 4).

Fig. 5. Total debt service in relation to total revenue

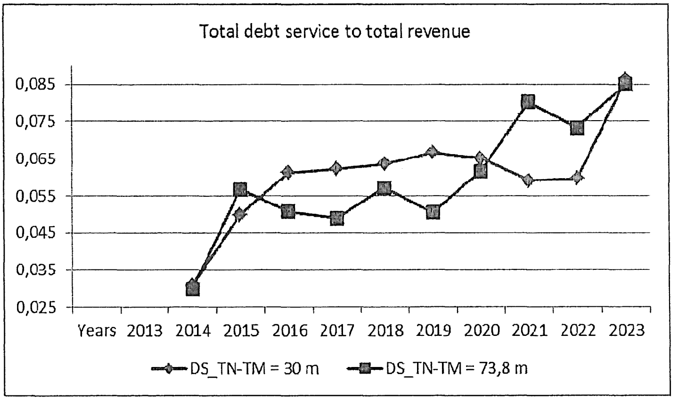
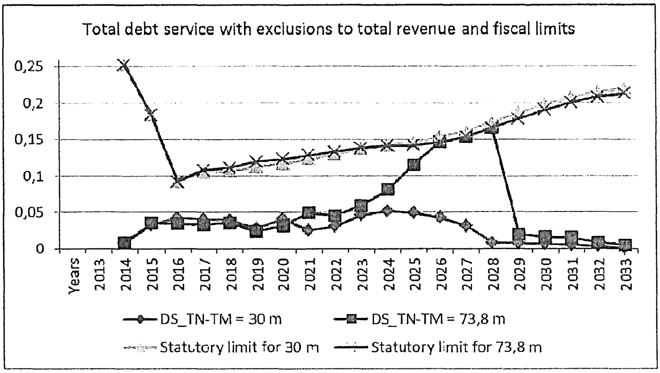


Fig. 6. Total debt service with exclusions of debt for EU investment, in relation to total revenue



The limit for the total debt service with the exclusions is slightly higher than for the debt service without exclusions, as the operating expenditures are lower, and larger is the value of the right hand side of the relation (con. 4). The actual total debt service with exclusions to revenue is below the statutory limits for both solutions B, and G. In case of the solutions G, with higher cumulated investment, the debt service grows in the period 2024-2028, equals the limit during the period of 2026-2028, and decreases sharply in 2029. The total debt service with exclusions to revenue, when we exclude expenditures associated with interest on debt which finances investment projects co-financed with the EU funds, is very close for both solutions over 2014-2020, and 2019-2033.

In Figs. 7 and 8 the operating expenditures in relation to total revenue are presented. The total operating expenditures are higher for the point G solution, than for the point B solution over the entire period of analysis. However, the operating expenditures with the exclusions, in relation to total revenue are close for the both solutions, with exception of three years: 2016-2018, when the debt service is lower for the solution G, and the maintenance costs of fixed assets are still low for the solution G, with higher cumulated investment over 2014-2023.

Fig. 7. Total operating expenditures to total revenue

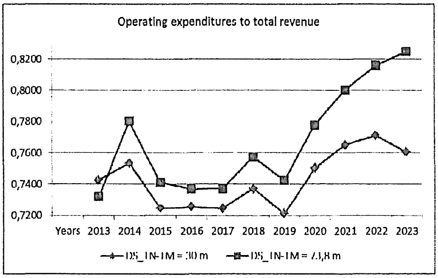


Fig. 8. Total operating expenditures with the exclusions, in relation to total revenue

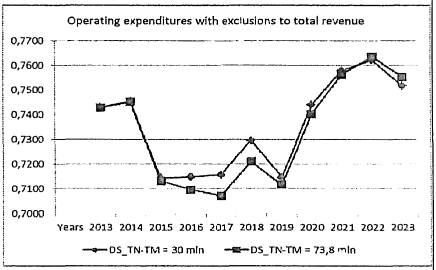


Fig. 9. Operating surplus in relation to total revenue

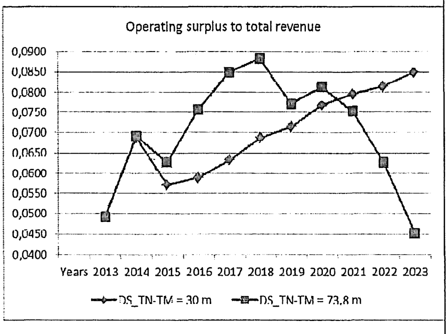
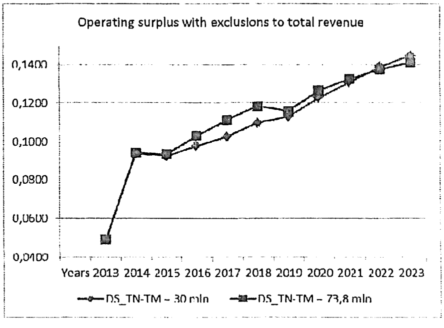


Fig. 10. Operating surplus with the exclusions, to total revenue



The *gross* operating surplus in relation to total revenue (also the operating surplus itself) is higher for the solution G, with higher cumulated investment over 2014-2023, than for the solution B. However, starting 20121 it falls sharply because of the increasing operating expenditures (Fig. 7). The operating surplus in relation to total revenue is very close for both solutions - G (with higher investment over 2014-2023) and B (with lower investment expenditures and lower debt service over 2024-2033), when we take into account the exclusions associated with the debt used for financing the EU projects.

The total outstanding debt, starting 2020 is larger, and grows fast for the scenario G, with the higher debt service over 2024-2033 (73,8 million PLN), than for the scenario B. New

Fig. 11. The debt outstanding

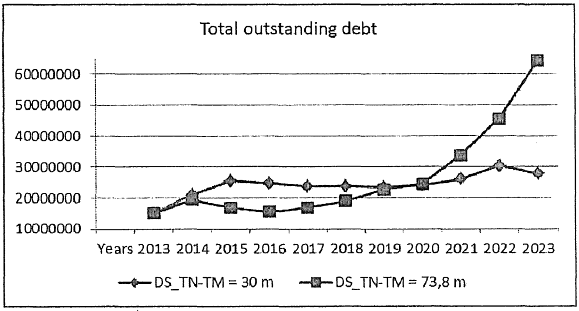
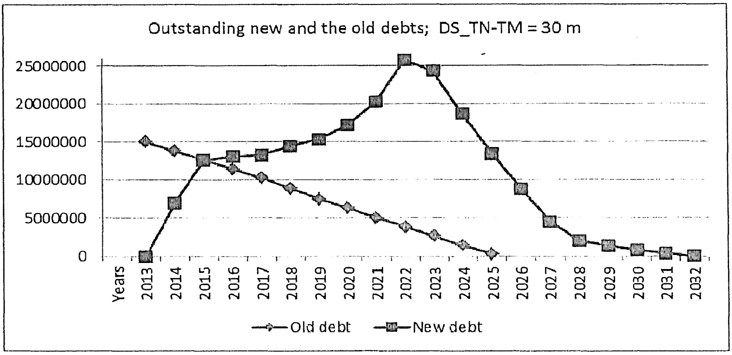


Fig. 12. The outstanding new and old debts; the cumulated debt service = 30 m PLN



debt outstanding and the old debt, issued prior to t_1 (2014) are presented in Fig. 12 for the B scenario, with lower costs of debt service over 2024-2033 (30,0 million PLN), and lower investment over 2014-2013. The old debt is repaid in 2025, and the new debt - in 2032.

The debt structure of new debt varies for the presented scenarios – the solutions B, and G. In the scenario B, bonds with relatively high face value are issued in 2018-2022. Long term credits are larger than the medium term credits, and in 2015, and 2022 the difference is substantial. In the scenario G, starting 2020, bonds are issued with very high face value. The indebtedness resulting from bonds issues in 2023 is nearly twice as high as the joint indebtedness resulting from issuance of medium and long-term credits. Starting 2017, the long term credits are larger than the medium term credits.

Fig. 13. The new debt structure; the cumulated debt service = 30 m PLN

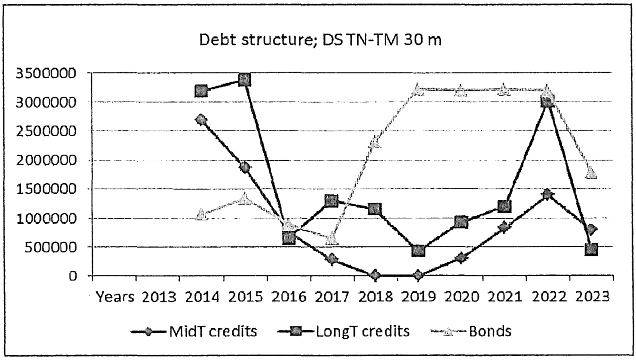
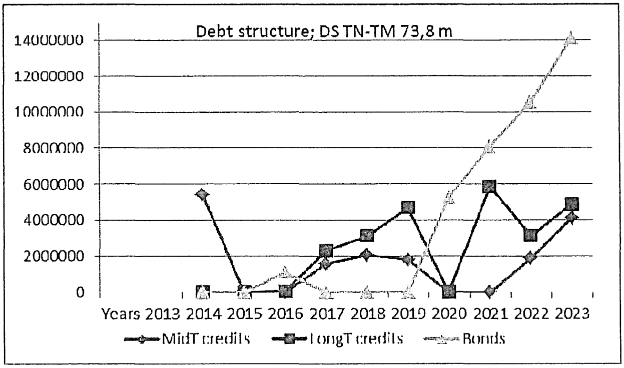


Fig. 14. The new debt structure; the cumulated debt service = 73,8 m PLN



5.2. Comparison of solutions for various LGs initial financial position

In figures 15- 20 we present select optimal variables obtained for two solutions of the two criteria optimization problem (C1) and (C2) with four types of constraints (con.1) – (con.

4). The two solution are represented by point B and the by point H - see Fig. 2. The solutions are located on different Pareto frontiers, as they are optimal solutions of the model with different initial conditions. The point B is the solution of the model with the initial debt D_{t0} of 15 million PLN, while the solution H, results from the model solution in which the initial debt D_{t0} equals 35, 4 million PLN. In order to make the results more comparable, the solution points B, and the H, have the same costs of debt service cumulated, over (T_N+1, TM), which equals 30 million PLN. The sum of investments over (t_1, T_N) equals 194, 7 million PLN for the solution B, and 170, 8 million for the solution H.

Fig. 15. Investment expenditures in relation to total expenditures for various initial debt

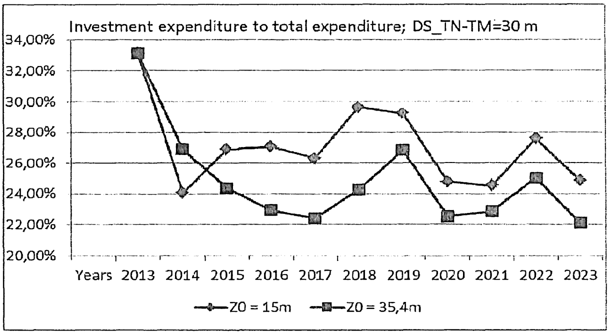
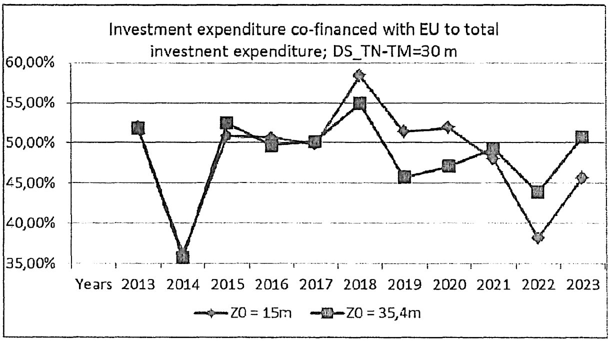


Fig. 16. A share of investment expenditures co-financed with the EU funds in total investment



The solution H, with the higher initial debt D_{t0}, allows for lower investment. More money is needed for servicing the debt. The share of investment expenditures co-financed with the EU funds in the total investment expenditures is decreasing starting 2019 for the solution B, with lower initial debt, because the costs of fixed assets maintenance (Fig. 20.), and the interest (Fig. 18.), grow. For the solution H, these cost decrease in 2021, and the EU investment share grows in 2021. In 2023, the last year of the investment process and issuance of new debt, the EU investment share rises - as a result of anticipated minimization of the total costs of debt service over the period 2024 - 2032, when all debts are repaid.

The total outstanding debt is larger for the solution H, with the higher initial debt.

Fig. 17. Total outstanding debt for various initial debts

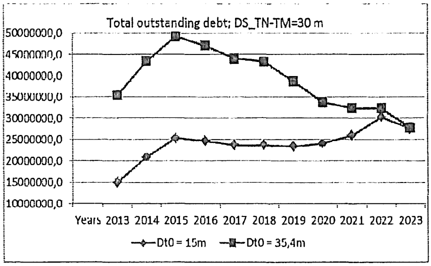
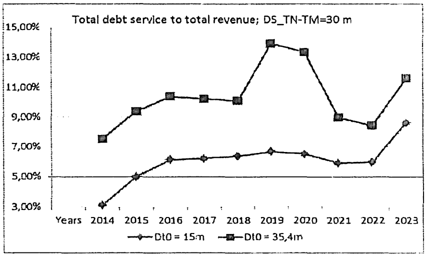
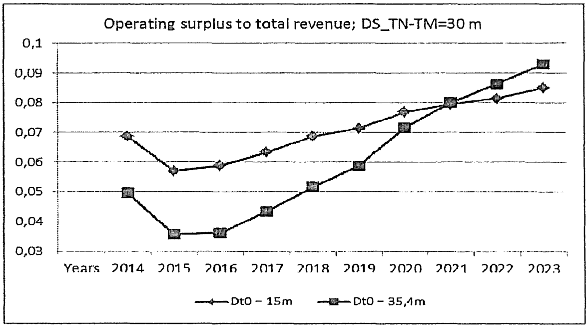


Fig. 18. Total debt service to revenue; various initial debts



However, the H solution debt decreases starting 2016, after a short rise over 2014, 2015, while the debt of the B solution rises constantly until 2022. The debts in 2023 are equal for the both solutions because the debt service over 2024-2033 is the same for the B, and the H solutions. The debt service is also higher for the solution H.

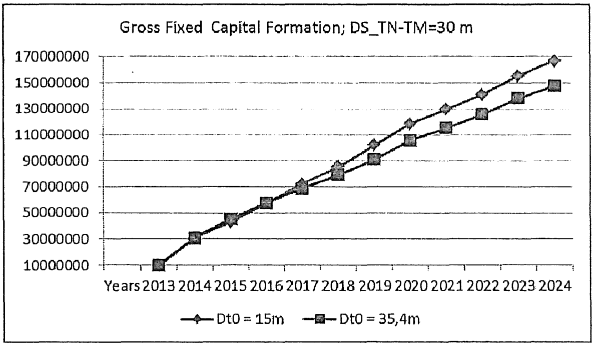
Fig. 19. Operating surplus in relation to total revenue for various initial debt



The indicator of the operating surplus in relation to the total revenue for the solution H is below the indicator for the solution B, because of the high interest, and the costs of fixed assets maintenance. Starting 2022 the fixed assets maintenance costs become so high, that the operating surplus indicator is higher for the solution H than the solution B.

The gross fixed capital formation (GFCF) – fixed assets for the solution H - is slightly higher during 2014-2016, and then is beneath the GFCF resulting from the solution B, with the lower initial debt. Higher debt service of the solution H results in a decrease of the operating surplus, and yields lower investment, and thus – lower GFCF.

Fig. 20. Gross Fixed Capital Formation for various initial debt



6. Summary and conclusions

Summary of results

The multi-objective optimization model was developed and implemented, with the use of a Polish LG data, for supporting LG managers decisions regarding long-term financial management and planning. The model is an extension of the model developed by Cichocki, 2013. It ensures a maximum investment cumulated over an investment period $\{t_1, T_N\}$, and, simultaneously, the minimum of total costs associated with debt service after the investment period - over $\{T_{N+1}, T_M\}$, until all debts, used for financing investment, are fully repaid. Investments are implemented and debt, which includes credits and bonds with various maturities, is issued only over in the period $\{t_1, T_N\}$.

The two objectives are achieved, under several conditions (constraints), which must be satisfied. They result from practical principles of sound financial management, and also include fiscal rules, which are formulated in law or constitution in many countries in order to avoid budget deficits and excessive debt issued by LGs. Liquidity and balance of operational and current accounts, at all-time instants (years), are ensured. Consistency with fiscal rules is assured, but any other rule of a given country can be included. The fiscal rule of the Polish law on public finance requires that the total debt service in relation to total revenue is below the statutory limit (con. 4). However, the limit and the total debt service are calculated in such a way that debt service associated with the debt financing investment projects which are co-financed by the EU funds, is excluded.The more EU funds a given LG acquires for investment financing, the more debt it can issue. The model is solved under several assumptions, which can easily be changed.

The model solutions include for each year, over an investment period, a safe level of investment co-financed by the EU funds and the other investments financed from the LG budget and from debt, and a safe level of debt for financing investment - two types of credit and bonds (medium and long-term), separately for the debt used together with the EU funds and the debt used with the LG budget. Several financial indicators are calculated including: operating expenditures, operating surplus, the outstanding debt, and the total debt service, in relation to total revenue, investment expenditures in relation to total expenditures, and debt structure - to show the financial position of a LG. Accumulation of the *gross* fixed capital formation is also analyzed. Investment contributes to the growth of the existing GFCF, which is depreciated, and, together with the sale of assets decreases the value of the GFCF. Growing GFCF generates increases in operating costs associated with the maintenance of the assets, which, in turn contribute to the rise of operating expenditure and the fall of operating surplus.

The formulated, two criteria model with constraints belongs to classical problems of the multi-criteria optimization theory. Consistently with the theory we look for decision variables (the model solutions), which are the non-dominated solutions – are Pareto optimal. There exist no other solutions that will jointly improve the both criteria of the model. A set of such solutions (Pareto points) determines the Pareto frontier. Solving the model several times, and allowing for alternative values of the criteria, one can obtain model solutions, which will belong to the Pareto frontier – will determine the Pareto frontier of the feasible set of all solutions which satisfy the model constraints.

The two criteria model solutions are located on the Pareto frontier in the two dimensional space of cumulative investment over $\{t_1, T_N\}$, and the debt service after completion of the investment and debt issuance, until all debts mature. Solving the model repeatedly, one obtains solutions, which are Pareto optimal – determine Pareto frontier, but various level of risk is attributed to consecutive solutions which belong to the Pareto frontier. The solutions with larger cumulated investment and higher debt service over $\{T_{N+1}, T_M\}$ are more risky. Two Pareto solutions of the model are discussed in thorough. One solution, with larger investment and higher debt service, is located on the edge of the Pareto frontier and of the admissible set. We have shown that the solution is very risky. A slight change in exogenous projections, regarding future operating revenues, or sales of property, might shift the solution outside the admissible set; one of the constraints will be violated. In such a case, the decision maker will have no alternative. Investment, which had been started, will not be continued because, for example, the condition 4 will be violated, and LG will not be able to issue debt and have the budget approved. The second solution, with lower investment and lower debt service over $\{T_{N+1}, T_M\}$, allows the decision maker for more flexibility. He will be able to increase investment and debt, and will remain in the admissible set, on the Pareto frontier.

We have also analyzed the impact of the initial indebtedness on the Pareto frontier and the model solutions. Lower indebtedness implies the Pareto optimal solutions located above the Pareto frontier obtained for higher initial indebtedness. When the initial debt is lower, one can invest more, issue more new debt, and generate larger debt service costs after the investment period $\{t_1, T_N\}$ - the model payoffs are much larger, and the admissible set is located higher, then in the case of higher initial indebtedness (see Figure 2).

Possible implementation of the model

The implementation of the model can be two fold.

The model can be implemented to support decisions made by LGs managers regarding long-term financial planning, can help determine, each year, over an investment period, a safe level of investment and EU funds - for investments co-financed by the EU funds and separately, investments financed by the LG budget, and a safe level various categories of medium and long term debt used for financing investment. Thus, the local decisions maker (treasurer, finance director) can analyze consequences of decisions regarding debt and investment until debt maturity, and analyze formation of fixed assets which are the result of investment over consecutive years, assuming various levels of risk.

The model can also be implemented for analysis of alternative fiscal rules formulated in law or constitution. We have shown with the help of the model, that the existing Polish rules are neither effective, nor efficient (see also Cichocki, 2013a). Many LGs can satisfy the rules, and the debt will grow two times in the period 2014-2023. The model can help, the central government and legal authorities, analyze explicit constraints which are designed to keep the LGs accounts in balance. It might help analyze an impact of fiscal rules on LG deficit, debt and on investment. The model can also be used for implicit constraint analysis, when some rules will be relaxed, for example the one imposed on debt and debt service. This would lead to research regarding applicability, and sufficiency of the golden rule of finances, and tax autonomy of LGs (see Dafflon, 2002).

The model assumptions are consistent with the EU regulations and the Polish law on public finance. The model, after appropriate verifications, can be implemented in many European countries. In Poland, the number of LGs, which over 2015- 2023, will not be able to use the EU funds and will have to limit investment activity and reduce debt issuance will have to be determined.

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Appendix 1.

Exogenous variables

In order to solve the model the following exogenous variables are needed: -

- D_{t0} – initial indebtedness (old debt), resulting from debt contracts concluded until t_0;
- $R(D_{t0})_t$ – repayment schedule of the old debt until its maturity, $t=t_1, .., t_m$;
- $Int\ (D_{t0})_t$ - interest paid on the old debt, $t=t_1, .., t_m$;

For some variables, in order to calculate the limit for debt service in the constraint (4) of the model, values of these variables are needed at the initial year and in two years preceding the initial year (three years preceding the budget year).

- Rev_t – the total revenue at the end of period t_0, and for $t = t_0 - 1$ and $t= t_0 - 2$;

OpS_t – the operating surplus at the end of t_0, and for $t_0 - 1$ and $t_0 - 2$; -
$SalGFCF_t$ – sales of fixed assets at the end of t_0, and for $t_0 - 1$ and $t_0 - 2$;
- Inv_{EUt0} - investment expenditures in period t_0, and for $t_0 - 1$ and $t_0 - 2$;
- Inv_{EUt0} / Inv_{t0} –the share of investment co-financed with the EU funds in the total investment expenditures; ,

In the model we also assume that values of several variables are exogenously projected over $t \in \{t_0, T_M\}$

$SalGFCF_t$ - sales of fixed assets for all $t \in \{t_0, T_M\}$

OpS_t – the operating surplus for all $t \in \{t_0, T_M\}$

$BOpRev_t$ – the basic operating revenues for all $t \in \{t_0, T_M\}$

$BOpExp_t$ - the basic operating expenditures for all $t \in \{t_0, T_M\}$

GDP_{t0} - the *gross* domestic product growth rate at t_0, and projections of GDP_t, and $t \in \{t_0, T_N\}$;

inf_t - inflation at t_0, inf_{t0}, and independent projections of GDP and inflation. for all $t \in \{t_0, T_M\}$.

The basic operating revenues $BOpRev_t$ at the end of period t, are projected as follows:

$$BOpRev_t = BRev_{t-1} [1 + (GDP_t \times inf_t)\, \gamma_t]$$

where $\gamma < 1$, denotes a local rate of growth indicator, which is smaller than the growth rate of the economy, calculated based on the GDP growth and inflation. Basic operating revenues are verified by the GDP growth and inflation growth, and are corrected by local rate of growth.

The basic operating expenditures are projected similarly to the basic operating revenues:

$BOpExp_t = BOpExp_{t-1} [1 + (GDP_t \times inf_t)\, \gamma_{1t}]$, where usually, $\gamma 1 < \gamma$, $BOpExp_{t0}$ - given.

<u>Model parameters</u>

Exogenous in the model are also forecasts of:

- the interest rates: $i_t = (ic_{1t}, ic_{2t} - i_{Bt})$ – is a vector of interest rates for credits and bonds at time $t \in \{t_0, T_M\}$. where ic_{1t}, ..,ic_{2t}, and ib_{1t}, - are interest rates charged for all credits and bonds (medium term credits, long term credits and bonds – for all debts), to be issued over $\{t_0, T_N\}$.