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Fiscal-Monetary Game Analyzed with Use of a Dynamic Macroeconomic Model

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Abstract. The paper deals with the fiscal-monetary game. In the game the fiscal and the monetary authorities take decisions on the choice of the optimum strategy from the point of view of realization of their respective economic objectives. A macroeconomic model has been constructed and used to represent the interrelations between, on the one hand, the instruments of fiscal policy and of the monetary policy, and, on the other hand – the economic effects resulting from their application. The best response strategies of the authorities and the Nash equilibrium state are analyzed. The simulation results obtained indicate that in a general case the Nash equilibrium is not Pareto optimal. It means that the policies should be coordinated and that respective negotiations leading to a Pareto-optimal consensus are needed.

Keywords: macroeconomic modeling, fiscal-monetary game, Nash equilibrium, bargaining problem, MCDM, negotiations.

1 Introduction

The present paper concerns the problem of choice of fiscal and monetary policies in the context of mutual decision conditioning between the fiscal authority (the government) and the monetary authority (the central bank). Each policy is characterized by the definite degrees of restrictiveness and expansiveness. A noncooperative game is formulated in which the fiscal and monetary authorities play roles of players. Each authority tries to obtain his respective economic objective: a desired value of the GDP dynamics in the case of the fiscal authority, and a desired value of the inflation in the case of the monetary authority. Instruments of the policies, considered as strategies in the game include: the budget deficit in relation to the GDP value, in the case of the fiscal authority, and the real interest rate, in the case of the monetary authority.

A macroeconomic model is proposed and used in the game to express influences of the policies' instruments on the game outcomes. The model describes the business cycles mechanism and allows to analyze the state of the economy in time. Using the model the GDP growth and inflation can be derived as dependent on the instruments of the policies.

The fiscal-monetary game is analyzed under assumption of the independence of the fiscal and the monetary authorities. In many situations the Nash equilibrium in the game is not Pareto optimal. Therefore the multicriteria optimization is applied to derive Pareto optimal strategies with respect to the objectives of the authorities. Such strategies can be applied and resulting outcomes obtained under respective coordination of the policies.

The paper is constituted as follows. The macroeconomic model is presented in Section 2 including descriptions of the product and the money markets as well as interrelations between them. Section 3 includes formulation of the fiscal monetary game and analysis of the game properties. The analysis is illustrated by selected results of simulations. Conclusions and references finish (close) the paper.

The references include papers dealing with: analysis of monetary and fiscal policy interactions, independence and coordination of the policies, policy games [1, 2, 3, 14, 20, 21], macroeconomic modeling [7, 16, 6, 4, 5], methods of decision support in multicriteria bargaining [8, 9], the reference point method of multicriteria optimization [17, 18, 19]. This paper continues the discussion presented by Nordhaus [14] related to independence versus coordination of the fiscal and monetary policies.

2 The macroeconomic model

A macroeconomic dynamic model is used in this research. It describes a mechanism of the business cycles and allows for analysis of economic situation in time. On the other hand it enables analysis of the monetary and fiscal policies, namely instruments of the policies: the real interest rate, the deficit of the government budget in relations to GDP, as well as their influence on the economy, especially on the GDP growth and on inflation. The model and initial results of simulations have been published by Woroniecka-Leciejewicz [22].

The model includes two modules describing: a product market and a money market. The economy is assumed to be in the initial equilibrium state on the product market as well as on the money market. The product market is described on the basis of the multiplier model [7] and multiplier- accelerator model of business cycles [16, 6]. Undertaken assumptions and model relations are briefly presented below.

2.1 The product market

The production in the period t is calculated on the demand-side as a sum of the consumption, investments and government expenditures:

$$Y(t) = C(t) + I(t) + G(t), \tag{1}$$

where: Y is the real production, C - the real consumption, I – the real investments, G - government expenditures (spending on goods and services), t – time period.

It is measured as the real GDP value. The foreign exchange is not included in the model.

The real consumption C in the period t:

$$C(t) = \beta(r)(CA + c(1 - t_n)Y(t-1)), \tag{2}$$

where: CA denotes the autonomous consumption, c – a marginal propensity to consume, t_n – the net tax rate, r – the real interest rate, r^* - the neutral interest rate, α, λ are parameters, $\beta(r)$ is a coefficient expressing influence of the real interest rate on the consumption:

$$\beta(r) = 1 - \lambda\alpha(r - r^*), \; 0 < \lambda < 1, \; \alpha > 0. \tag{3}$$

The real investments I in the period t:

$$I(t) = a(r)(IA + k\Delta Y(t-1)), \tag{4}$$

where:

$$a(r) = 1 - \alpha(r - r^*), \; \alpha > 0, \tag{5}$$

IA denotes the autonomous investments, k – a capital-output ratio (usually, k is assumed to be in $(0,1)$ interval), $\Delta Y(t)$ - the increase of the production in the period t, $\Delta Y(t) = Y(t) - Y(t-1)$, (the increase of values for other model variables is denoted in an analogic way), $a(r)$ – a coefficient expressing influence of the interest rate on the investments.

The net tax revenue of the government budget (real) T_n in the period t:

$$T_n(t) = t_n Y(t), \tag{6}$$

where $t_n(t)$ is a net tax rate.

The government budget balance (real) BS in the period t:

$$BS(t) = T_n(t) - G(t), \tag{7}$$

where the net taxes value $T_n(t)$ equals revenues from the taxes minus the transfer payments, G are the expenditures including the government spending on goods and services without the transfer payments. In an analogical way the fiscal revenues take only into account the net tax revenues like in equation (6).

2.2 The money market

The equilibrium on the money market means that the money supply M is in a balance with the transactional demand for the money. The demand is determined by the level of prices and by the real production but depends also on velocity of money circulation.

$$M(t) = \frac{P(t)\,Y(t)}{v(t)}, \tag{8}$$

where: M denotes the money supply, P – an index of prices, v – velocity of money circulation.

The inflation p in the period t:

$$p(t) = \frac{\Delta P(t)}{P(t-1)} = \frac{\Delta M(t)}{M(t-1)} - \frac{\Delta Y(t)}{Y(t-1)} + \frac{\Delta v(t)}{v(t-1)}. \tag{9}$$

The inflation has been described in the model according to the monetarist theory by Milton Friedman [4, 5]. It is determined by an excessive growth rate of the money supply and by an increase of velocity of money.

It is assumed that the rate of change of velocity of money is in proportion to the rate in which the production is changed:

$$\frac{\Delta v(t)}{v(t-1)} = \rho \frac{\Delta Y(t)}{Y(t-1)}, \tag{10}$$

where ρ is a parameter.

The growth rate of the money supply in the period t:

$$\frac{\Delta M(t)}{M(t-1)} = m_e(t) + m_r(t) + m_b(t), \tag{11}$$

where: M is the money supply, m_e – the expected money supply growth (on the basis of expected production growth and inflation), $m_e(t)$ – the money growth being the result of the monetary policy, $m_b(t)$ – the money growth being the result of the fiscal policy.

The growth rate of the money supply as dependent on the policy of the interest rate in the period t has been described by a convex decreasing nonlinear function of the form:

$$m_r = \frac{\Delta M_r(t)}{M(t-1)} = \mu_0 (r - \mu_1)^{\mu_2} + \mu_3, \quad \mu_2 < 0, \tag{12}$$

where: μ_0, μ_1, μ_2, μ_3 are parameters, μ_2 - the elasticity of the money growth with respect to the interest rate, $\mu_2 < 0$.

The growth rate of the money supply caused by the excess budget deficit in the period t has been described by a convex increasing nonlinear function:

$$m_b = \frac{\Delta M_b(t)}{M(t-1)} = \chi_0 (b - b^*)^{\chi_1}, \qquad \chi_1 > 0, \tag{13}$$

where: b^* - a level of the budget deficit in relations to GDP which does not require to increase the money supply, χ_0, χ_1 - parameters, χ_1 - elasticity of the money growth with respect to the excess of the budget deficit, $\chi_1 > 0$.

It is assumed that in the initial state the money market as well as the product market are in the equilibrium state. Cyclic changes in the economy are introduced by impulses in the form of changes of the private investments and changes of the monetary and the fiscal policies, i.e. by changes of the interest rate value and the budget expenditures. The cyclic changes tend in time to a new equilibrium state. Besides the cyclic changes the long-term trade expressing the exogenous technical progress is taken into account in the model. Effects of the applied monetary and fiscal policies are measured by the inflation in the new equilibrium state and by the average annual GDP growth rate in the considered interval of time.

The model has been tested using hypothetic data. Some number of simulations have been done. The model parameters, initial values of the model variables, as well as initial simulation results are presented in [10].

3 The fiscal-monetary game

3.1 Formulation

Relations between the fiscal authority and the monetary authority can be described by a noncooperative game. The game is defined in the strategic form as follows:

(i) There are two players i=1, 2: the fiscal authority (the government) and the monetary authority (the central bank).

(ii) For each player a set Ω^i of pure strategies is defined. The strategies of the fiscal authority are those of the budgetary policy – from the extremely restrictive to the extremely expansive. The measure, denoted by b, of the degree of restrictiveness/ expansiveness of the fiscal policy is constituted here by the level of budget deficit in relation to GDP. The strategies of the monetary authority range from the extremely restrictive one to the extremely expansive. The degree of restrictiveness/expansiveness is equivalent simply to the value of the real interest rate and denoted by r. Let Ω denote the Cartesian product of the sets of the strategies $\Omega = \Omega^1 \times \Omega^2$.

(iii) For each player i=1, 2, a function $h^i\colon \Omega \to \boldsymbol{R}$ is given defining outcome of the player i for given strategies undertaken by the both players. The outcome of the fiscal authority is measured by the GDP growth rate, denoted by y, where $y=h^1(b, r)$. In the case of the monetary authority it is the inflation value, denoted by p, where $p=h^2(b, r)$. The functions h^i, i=1, 2, are defined by the model relations.

(iv) For each player i=1, 2, a preference relation is given in the set of the attainable outcomes. It is assumed here that each authority tries to achieve a given goal: the fiscal authority – a desired value of GDP growth, the monetary authority – a desired value of inflation.

Table 1. The fiscal-monetary game – the table of outcomes.

		Central bank – the monetary policy ← restrictive			expansive →
		Monetary strategy M_1 (interest rate r_1)	Monetary strategy M_2 (interest rate r_2)	…	Monetary strategy M_n (interest rate r_n)
Government – fiscal policy ← expansive restrictive →	Fiscal strategy F_1 (budgetary deficit b_1)	p_{11} / y_{11}	p_{12} / y_{12}	…	p_{1n} / y_{1n}
	Fiscal strategy F_2 (budgetary deficit b_2)	p_{21} / y_{21}	p_{22} / y_{22}	…	p_{2n} / y_{2n}
	…			…	
	Fiscal strategy F_m (budgetary deficit b_m)	p_{m1} / y_{m1}	p_{m2} / y_{m2}	…	p_{mn} / y_{mn}

Outcomes of the game in the discrete form are presented in Table 1 as in [20]. The strategies of the fiscal authority are shown in the first row and the strategies of the monetary authority - in the first column. The outcomes: GDB growth rate and inflation are denoted by y_{ij} and p_{ij} respectively for assumed F_j (budget deficit b_j) and M_i (interest rate r_i) strategies of the authorities.

3.2 Analysis of the game

Strategies, outcomes and payoffs of the game ware analyzed using the macroeconomic model presented in Section 2. Simulation experiments have been made for different variants of data including values of the model parameters and initial values of the model variables. Selected results are presented and discussed below.

Figures 1 and 2 present outcomes of the authorities, as dependent on assumed strategies.

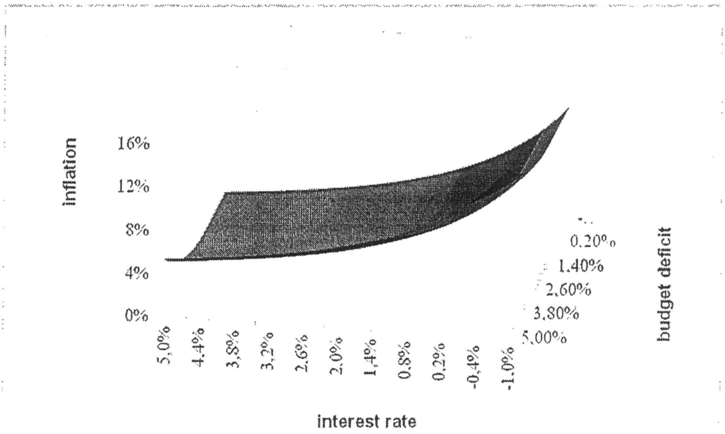

Fig. 1. Outcomes of the monetary authority

Inflation (Fig. 1) can be obtained on a low level when a restrictive monetary policy and a restrictive fiscal policy are applied. Expansive monetary and fiscal policies lead to enormous nonlinear increase of inflation. On the other hand restrictive monetary and restrictive fiscal policies lead to decrease of the economic growth (Fig. 2).

Let us assume given goals of the fiscal and the monetary authorities. Let the fiscal authority try to achieve the GDP growth rate on the level y^g, and let the monetary authority assume the inflation goal on the level p^g. Let Ω denotes the set of admissible pairs (b, r) of strategies. The respective best response strategies can be obtained as solutions of the optimization problems: Min $|h^1(b, r)- y^g|$ with respect to $b\in\Omega^1$ solved for all $r\in\Omega^2$, in the case of the fiscal authority and Min $|h^2(b, r) - p^g|$ with respect to

$r \in \Omega^2$, solved for all $b \in \Omega^1$, in the case of the monetary authority, as solutions of the problem.

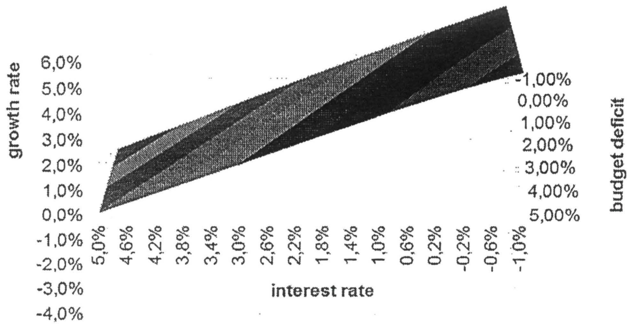

Fig. 2. Outcomes of the fiscal authority

The best response strategies (marked in the figure by triangles) of the fiscal authority when the GDP growth $y^g = 2.2\%$ can be achieved as well as the best response strategies (marked by rhombs) of the monetary authority when the inflation $p^g = 2.5\%$ can be obtained, are presented in Fig. 3. Let us see that the lines presenting the strategies have not any joint point. The Nash equilibrium exists for the combination of the most restrictive policy of the monetary authority and the most expansive policy of the fiscal authority.

Let us consider possible coordination of the policies. We assume that each player tries to minimize a distance to his goal, i.e. $d^y = |h^1(b, r) - y^g|$ and $d^p = |h^2(b, r) - p^g|$. In this case d^y and d^p can be treated as the criteria that should be minimized jointly. Let d^Ω denote the set of attainable values of the pairs (d^y, d^p) for $(b, r) \in \Omega$. We say that the pair $(\mathbf{d}^y, \mathbf{d}^p)$ is Pareto optimal in the set d^Ω if there does not exists any pair $(d^y, d^p) \in d^\Omega$, $(d^y, d^p) \neq (\mathbf{d}^y, \mathbf{d}^p)$, such that $d^y \leq \mathbf{d}^y$ and $d^p \leq \mathbf{d}^p$. A representation of the strategies leading to the Pareto optimal outcomes is presented in Fig. 3 using circle marks. The Nash equilibrium is far from the possible Pareto optimal outcomes of the authorities.

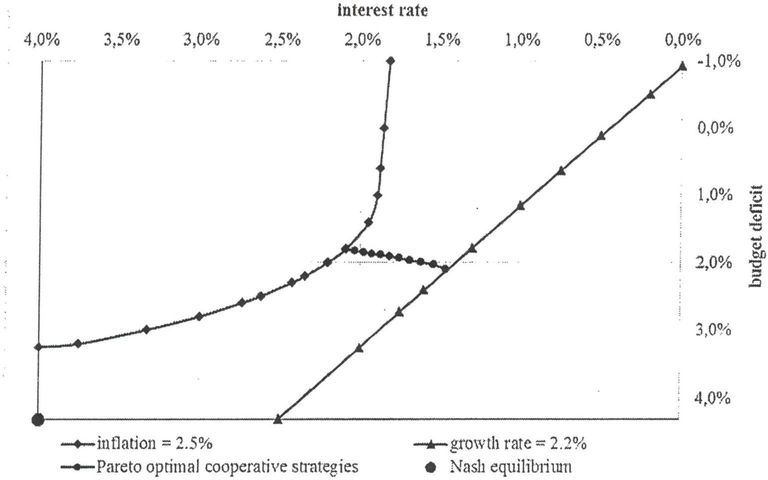

Fig. 3. The best response strategies of the authorities and possible cooperative – coordinated strategies.

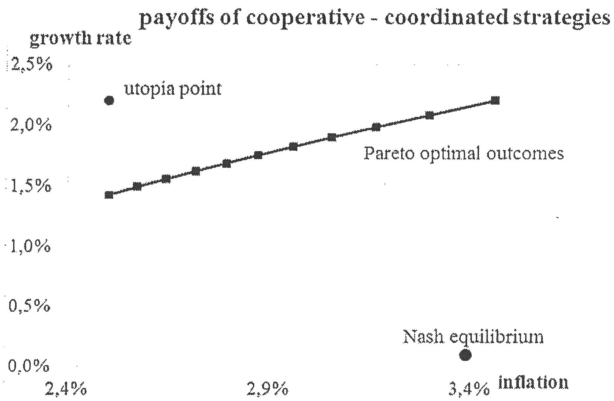

Fig. 4. Payoffs of cooperative – coordinated strategies.

The Pareto optimal strategies have been derived solving the following multicriteria optimization problem: **VMin** ($|h^1(b, r) - y^g|$, $|h^2(b, r) - p^g|$) with respect to $(b, r)\in\Omega$, where **VMin** means that the criteria are minimized jointly. The representation of the Pareto optimal strategies and the respective Pareto optimal outcomes have been obtained applying the reference point method [17,19].

Fig. 4. presents the set of the Pareto optimal outcomes which can be obtained in the case of coordination of the policies. The point defined by the outcomes corresponding to the Nash equilibrium is also presented as well as the utopia point. The utopia point represents the outcomes at which the both goals of the authorities are achieved. The utopia point is not attainable in the considered case.

Let us see that looking for cooperative strategies we deal with the bargaining problem formulated by Nash [11] and studied by many scientists. Some ideas of computer based decision support methods in bargaining problems can be found in [8, 9].

4 Conclusions

The paper presents selected results obtained within the research dealing with analysis of interactions of the fiscal and monetary policies with application of the game theory and the multicriteria optimization tools. Within the research the dynamic macroeconomic model has been constructed, the fiscal monetary game has been formulated and analyzed. The model describes a mechanism of the business cycles and allows for analysis of economic situation in time. On the other hand it enables analysis of the monetary and fiscal policies, namely instruments of the policies: the real interest rate, the deficit of the government budget in relations to GDP, as well as their influence on the economy, especially on GDP growth and on inflation.

The model relations have been implemented in the form of the computational algorithm. The algorithm is a part of a computer-based system used for simulations and analysis of the game in an interactive way. The simulations were done for some number of variants including different model parameters and initial values of model variables describing different states of economy.

The computational results presented show a typical case when the Nash equilibrium in the game is not Pareto optimal. In this case questions arise: how to support the players, when they are playing a noncooperative game, in looking for a Pareto optimal consensus, how to rich the consensus taking into account specific, in generally conflicting objectives and preferences of the players. In this case a bargaining problem can be formulated and analyzed with use of the multicriteria optimization tools. Formulation and analysis of the bargaining problem can be basis for designing negotiations leading to a Pareto optimal consensus.

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