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Analysis of incentive compatible multicriteria decisions for producer and clients' problem

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ANALYSIS OF INCENTIVE COMPATIBLE MULTICRITERIA DECISIONS FOR A PRODUCER AND CLIENTS PROBLEM

Abstract

The paper deals with analysis of incentive compatible multicriteria decisions within a computer-based multiagent framework. The general design problem is discussed on an example of a market decision problem, where a producer is introducing a product and some clients are considering purchase of the product. Decisions of the producer and clients are multicriterial. Each of the clients is seeking for the product variant according to his own preferences. The producer decides which variant of the product is introduced to the market. In order to incentivize the decisions, one of his criterions takes into account an aggregated satisfaction of the clients. Solutions compatible to the preferences of the producer and to the preferences of the clients are searched for. A multiagent computer-based system has been constructed for supporting mulicriteria analysis made by clients and by the producer. Selected results of interactive sessions made with use of the system are presented and analyzed.

Keywords

Incentive compatible decision mechanisms, multiagent systems, multicriteria optimization.

1. Introduction

Analysis of incentive compatible decisions is made on an example of two stage mechanism in which a producer and his potential clients participate. In the first stage, each client makes independent multicriteria analysis of possible variant of a product and selects the variant preferred according to his individual criteria. In the presented example, each client minimizes a cost criterion and maximizes a criterion defined by a usefulness of the product. In the second stage, the producer makes also multicriteria analysis in the set of possible variants of the product but with respect to his criteria, including a profit criterion. A reputation of the product on the market has been assumed as one of important criterions of the producer. The reputation is expressed by an aggregated measure of satisfaction of clients from the variant offered by the producer. The incentive compatibility in market mechanisms was analyzed previously by Toczyłowski (2003, 2009). The ideas developed in the papers have inspired the presented research.

A special multiagent computer-based system has been designed. It enables problem formulation and supports multicriteria analysis made by clients and by a producer. The system has been implemented using Optimization Software for Operations Research Applications (AIMMS). Information about the AIMMS environment can be found in www.aimms.com and (Bisschop and Roelofs, 2009). Details referring to functionality of the system, its implementation and user instructions can be found in the eng. diploma thesis (Skorupiński, 2010). The system secures confidentiality of information of users playing parts of

clients and a producer. The producer has no access to the information introduced by clients, nor to results of their analysis made with use of the system.

We could imagine that the system is in disposal of an institution, to which the producer and the clients trust. The institution secures confidentiality of the individual information, makes market analysis of the new product among potential clients. It supports also the producer to select the variant of the product, which would be favorable with respect to his criteria but also would have a reputation on the market.

This paper includes mathematical formulation of multicriteria optimization tasks for clients and for a producer. The tasks are solved during multicriteria analysis with use of the reference point method (Wierzbicki, 1986), (Wierzbicki at al., 1993, 2000). A question arises how to define and derive clients satisfaction levels with respect to a variant of the product offered on the market. Then, how to calculate a cumulative reputation of the product variant on the market. Respective proposals are presented.

Series of interactive sessions have been made with use of the system. Different results have been obtained showing possible behaviors of clients and a producer, as well as relations among solution variants chosen by them. Several results are presented and analyzed. In the final remarks, directions of further research are discussed.

2. Mathematical description

Producer is going to offer a new variant of his product to a set L of clients. Variants of the product that can be produced are described by a

vector of decision variables $x \in D \subset R^n$, where D is a set of admissible values of the variables. The set D is not given explicitly. We assume that it is given by a set of linear constraints of the form: $A\,x^{\mathrm{T}} \le b$, where A and b are matrix and vector of coefficients respectively.

The vector x includes among others, criteria of clients, such as:

e – variable describing economic attributes of the product, like purchasing cost, operating cost,

u – variable describing usefulness of the product, quality, technological advantage, reliability.

Each client can generate, review and analyze nondominated product variants in his space of criteria, using reference point method (Wierzbicki, 1986), (Wierzbicki at al., 1993, 2000). The following optimization tasks are formulated:

$$\max_{x}\{[\phi(r,a,y)] : x \in D \subset R^n\}, r \in R^n, a \in R^n,$$

where Φ denotes scalarizing achievement function, r, a are vectors of controlling parameters. The vectors r, a play roles of reservation and aspiration points respectively. Criteria y are selected variables of the vector x.

A nondominated solution is derived for reservation and aspiration points given by a client, solving the optimization problem:

$$\max z + \varepsilon \sum_{k \in \bar{Y}} z_k ,$$

subject to constraints of the reference point method:

$$z \le z_k, \forall k \in \overline{X},$$
$$z_k \le \gamma(x_k - r_k)/(a_k - r_k), \forall k \in \overline{X},$$
$$z_k \le (x_k - r_k)/(a_k - r_k), \forall k \in \overline{X},$$
$$z_k \le \beta(x_k - a_k)/(a_k - r_k) + 1, \forall k \in \overline{X},$$

and constraints of admissible values of the variables x:

$$A\,x \le b.$$

In the formulation z, z_k, x denote variables, $\overline{X}$ is a set of criteria indexes.

Analysis is made in some number of iterations. In each iteration a client assumes reservation and aspiration points according to the reference point method. The computer-based system solves the above problem and calculates a respective variant, nondominated in the set D.

We have assumed, that the reservation point of each client is not selected arbitrarily but is defined on the base of the BATNA concept, similarly as it is assumed in the procedures supporting cooperative decisions (Kruś, 2002, 2004, 2008). The BATNA concept (Best Alternative to Negotiated Agreement) is widely applied in negotiations (Fisher and Ury, 1981), (Raiffa, 1982). It means the best alternative a negotiating party can have if negotiations will not succeed. In our case, it relates to a product, which is accessible on the market already and can be compared to the variants of the product offered by the producer. We assume that a client is interested in a variant proposed by the producer if the variant is better than that defined by BATNA. The BATNA concept is important for calculation of the client satisfaction, proposed further in the paper.

For a given in this way reservation point and different aspiration points proposed by a client the system derives respective

nondominated variants, so the client can obtain a representation of a Pareto frontier of the set D. The client is asked then to indicate the preferred variant. The multicriteria analysis is made independently by all clients acting with use of the system. The system stores information about the variants indicated by all the clients.

Criteria of the producer include a profit obtained from the product variant offered on the market and a reputation among clients accepting the offered variant. Other producer criteria can be also formulated using the system. The profit implies sales revenues minus total expanses referring to the product variant. The profit criterion is included in the optimization task as the constraint:

$$y_{profit} \leq (p_e x_e - p_u x_u) \sum_{l \in L} v_l ,$$

where v_l is a binary variable indicating who of the clients accepts the offered product variant. A simplifying assumption has been made that the revenues are in proportion to the variable x_e, and the costs are in proportion to the usefulness x_u, with coefficients p_e i p_u respectively.

The reputation is defined as an aggregated measure of satisfaction levels of clients. The satisfaction level of a client is calculated for the product variant offered by the producer when the client has already made the multicriteria analysis, has defined reservation point and has chosen the preferable aspiration point and nondominated accessible variant. The reservation points, aspiration points and variants preferred by different clients are in generally different. The satisfaction levels of clients accepting the variant proposed by the producer are aggregated, so that the reputation expresses aggregated satisfaction of the clients.

An interval scale has been assumed to measure the satisfaction level of a client. The scale has to be normalized with respect to different clients and should be manipulation free. The interval scales are constructed based on two uniquely defined points. The Celsius temperature scale defined by the temperature of ice thawing and the temperature of water boiling serves an example. We have assumed that the satisfaction level of a client is measured, based on his reservation point (with level s_d=0) and of the accessible variant preferred by him (with the level s_g=100). An arbitrary variant may have of course attached a satisfaction level lower than 0, or greater than 100. Discussions on different types of scales and applicability of the scales to measuring can be found in (Torgerson, 1958), (Coombs, Dawes, Tversky, 1970).

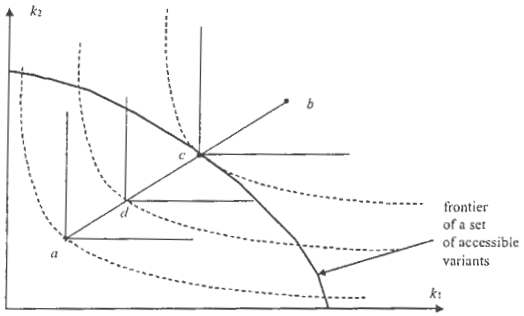

Figure 1. Indifference sets of a function measuring client's satisfaction level

We discussed during the research also other definitions of the scale and different ways of calculating the satisfaction level. It seems natural to take the aspiration point chosen by a given client as a variant with the maximum satisfaction level equal to 100. In the last case, the

client can manipulate with the distance of the aspiration point to the reference point. Increasing the distance, he could inflow on the producer decisions, increasing his importance in comparison to other clients.

Figure 1 presents an example illustrating how the client satisfaction level is derived. Two maximized criteria are considered. The satisfaction level is defined by a scalar function defined in the space of the criteria. In a general case, it is a nonlinear utility function. Indifference sets of the function are presented by dropped lines in the space of criteria k_1, k_2. In the present version of the computer-based system, we assumed a specific variant of the function defined by frontiers of the shifted positive cone presented by thin continuous lines. In further research, other form of the utility functions will be discussed including problems of its estimation and implementation in the system. The points presented in Figure 1 denote: a – reservation point, b – aspiration point indicated by a client after his multicriteria analysis, c – chosen preferred accessible variant. According to the assumed scale, all the points on the continues lines originated from point d have the satisfaction level equal to

$$s = (s_g - s_d)\cdot|a, d| / |a, c|.$$

The maximized reputation criterion is included in the optimization task by the constraint:

$$y_{reputation} \le \sum_{l \in L} s_l ,$$

where $y_{reputation}$ denotes a value of the reputation and s_l is a satisfaction level of the client $l \in L$.

The producer makes multicriteria analysis in the space o his own criteria, assuming respectively reservation and aspiration points. The

computer-based system derives and stores respective nodominated solutions. The producer can review generated solutions and can select the preferred one. The system derives the nondominated solution solving the following optimization problem for given reservation and aspiration points:

$$\max z + \varepsilon \sum_{i \in \overline{Y}} z_i$$

subject to the constraints due to the reference point method:

$$\begin{aligned}
&z \le z_i, \forall i \in \overline{Y},\\
&z_i \le \gamma(y_i - r_i)/(a_i - r_i), \forall i \in \overline{Y},\\
&z_i \le (y_i - r_i)/(a_i - r_i), \forall i \in \overline{Y},\\
&z_i \le \beta(y_i - a_i)/(a_i - r_i) + 1, \forall i \in \overline{Y},
\end{aligned}$$

to the reputation criterion

$$y_{reputation} \le \sum_{l \in L} s_l,$$

$$\begin{aligned}
&s_l \le (s_g - s_d) f_{l_k}, \forall l \in L^+, k \in \overline{X},\\
&f_{l_k} \le (x_k - \hat{r}_{l_k})/(\hat{x}_{l_k} - \hat{r}_{l_k}) + M(1 - v_l), \forall l \in L^+, k \in \overline{X},\\
&(x_k - \hat{r}_{l_k})/(\hat{x}_{l_k} - \hat{r}_{l_k}) \ge -M(1 - v_l), \forall l \in L^+, k \in \overline{X},\\
&s_l \ge \varepsilon^* - M(1 - v_l), \forall l \in L^+,\\
&v_l \le 0, \forall l \in L^-,\\
&f_{l_k} \le s_d + M v_l, \forall l \in L,
\end{aligned}$$

to the profit criterion

$$y_{profit} \le \sum_{l \in L} w_l,$$

$$\begin{aligned}
&w_l \le M v_l, \forall l \in L,\\
&w_l \le p_c x_c - p_u x_u + M(1 - v_l), \forall l \in L,\\
&p_c x_c - p_u x_u - w_l + M v_l \le M, \forall l \in L,
\end{aligned}$$

to the model constraints of admissible variants of the product:

$$A x \le b.$$

In the above relations, w_l, v_l, f_{l_i} denote additional variables, $\bar{Y}$ is a set of indexes of the producer criteria, , L^+ is a set of clients for which there exists a product variant better than that defined by the reservation point, L^- respectively the set of clients for which such a variant does not exists. M is a great positive number, $\hat{x}_{l_i}, \hat{r}_{l_i}$ denote components of the accessible solution selected by a client l, and of his reservation point respectively. Not all clients of the set L^+ can be interested in the variant offered by the producer. It has been assumed that a client is interested in the variant of the product if the level of his satisfaction is at least ε^* value greater than the level of his reservation point.

3. Analysis of some results

Computing experiments and series of sessions have been made with use of the system. In the first experiments, the system was intensively tested. Next, interactive sessions were carried on by a producer and by several clients. It was interesting to check, how preferences of clients take effects on decisions of the producer maximizing his profit but also attaching an importance to the reputation of his product. On the other hand, producer's decisions take effects on satisfaction levels of particular clients. Selected results and respective analysis is presented below.

Figure 2 presents results of multicriteria analysis made by one of the clients. Selected reservation and aspiration points as well as the respective nondominated solutions are presented in the space of client's criteria: e (minimized cost), u (maximized usefulness).

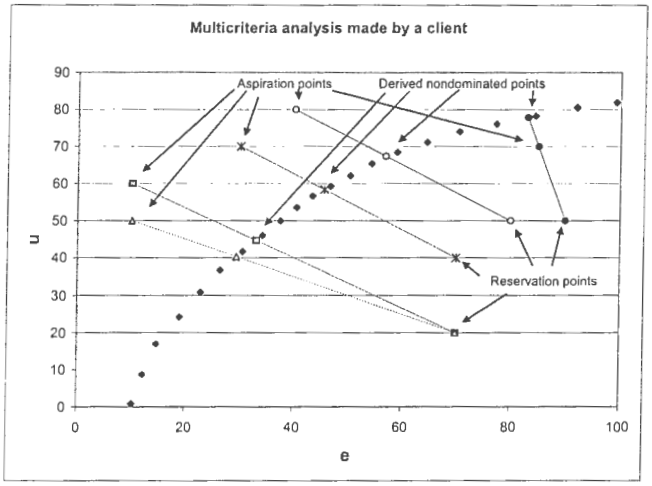

Figure 2. Illustration of multicriteria analysis made by a client.

All the nondominated points shown in the figure have been derived by the reference point method, but only several selected reservation point, aspiration points and respective nondomminated points are presented. Let us see that even if the aspiration point is assumed in the interior of a set of accessible solutions, the methods leads to the point on the Pareto frontier of the set, and the aspiration point is improved (the example on the right hand side of the figure).

Each client, assuming different reservation and aspiration points, can derive a representation of the set of Pareto optimal variants. He is asked to select the preferred variant and the respective aspiration point.

The producer can start analysis when all the clients have already selected their preferred variants. He has no access to information related to particular clients, their analysis, decisions nor preferences. The computer-based system derives values of the producer criteria: the reputation of the variant among the clients, and the profit, dependently on the product variant considered by the producer to be offered to the clients.

Multicriteria analysis conducted by a producer is made with use of the reference point method in an analogical way as in the case of clients. The producer can make a representation of the set of Pareto solutions and can select the preferred variant.

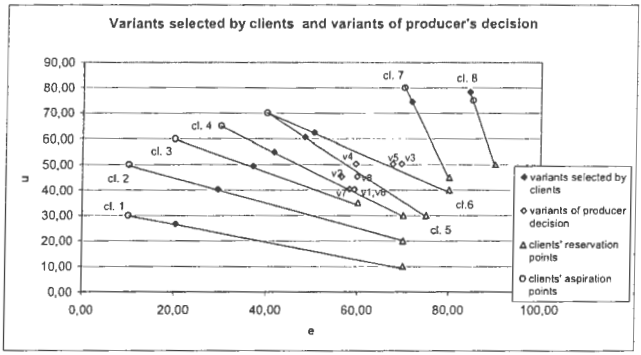

Figure 3. Results of a session with eight clients

Figures 3, 4, 5 present results of a session conducted with a producer and eight clients. Several different decisions of the producer have been assumed. The clients have different reservation points and different preferences. Figure 3 presents nondominated variants and the respective aspiration points indicated by the clients.

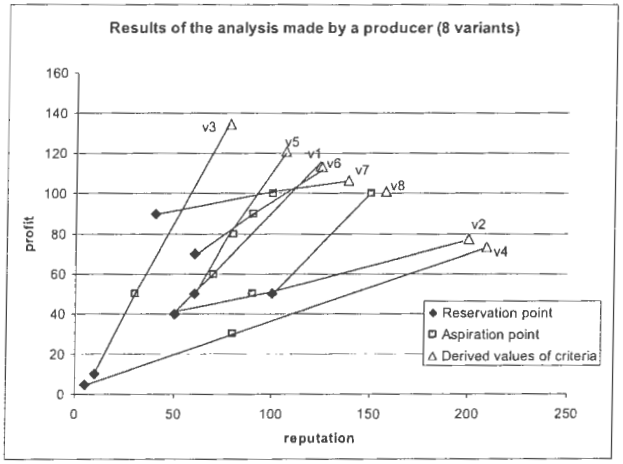

Figure 4. Different variants of producer's decision in the case of eight clients

It was interesting to check effects of producer's preferences on his choice of the variant offered to clients; which clients accept the variant; what will be their satisfaction levels and the resulting reputation criterion. Eighth variants of producer's decisions differing with respect to reservation points, aspiration points and the respective nondminated solutions are shown in Figure 4 in the space of profit and reputation criteria.

Let us see that in the case of variant 8, the nondominated solution is significantly deviated from the direction defined by the reservation and aspiration points. In Table 1 values of decision variables e and u characterizing this variant, as well as the number of clients accepting this variant are given.

Table 1

Reservation and aspiration points assumed by a producer and respective nondominated solutions (criteria and decision variables)

	Analysis made by a producer				Decision variables		Number of satisfied clients
Variant	Criterion	Reservation point	Aspiration point	Derived values of criteria	e	u	
1	reputation profit	50 40	70 60	123,97 113,98	59,22	40,22	6
2	reputation profit	50 40	90 50	199,59 77,42	56,36	45,29	7
3	reputation profit	10 10	30 50	77,85 134,54	69,5	50,28	7
4	reputation profit	5 5	80 30	208,95 73,32	59,45	50,28	8
5	reputation profit	60 50	80 80	106,55 121,24	67,60	50,28	7
6	reputation profit	60 70	90 90	125,08 113,42	59,13	40,22	6
7	reputation profit	40 90	100 100	138,74 106,45	58,13	40,39	6
8	reputation profit	100 50	150 100	157,51 101,33	59,77	45,29	7

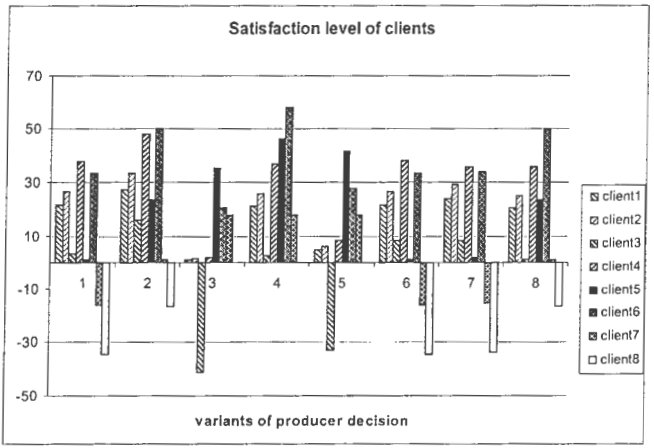

Figure 5. Clients' satisfaction levels dependent on the producer's decision

Figure 5 shows satisfaction levels of clients dependently on the variants of product offered by the producer. Negative values of the level mean that the respective variant is not accepted by the respective clients.

Variants 1, 6, 7 are not accepted by clients 7 and 8. Variant 3 and 5 are not accepted by client 3. Variant 3 gives the greatest profit to the producer in the set of variants analysed here. The greatest number of clients is interested in variant 4. This variant has the greatest reputation among clients but it gives the lowest profit to the producer.

4. Final remarks

A mathematical model describing the producer and clients problem has been proposed. It includes formulations of optimization tasks solved during multicriteria analysis conducted by the clients and by the producer. The optimization tasks have been implemented in specially designed multiagent computer-based system.

An original proposal for derivation of satisfaction levels of individual clients is presented. On this basis, the reputation can be calculated. It is one of producer's criteria. It harmonizes producer's and clients' interests. The client's satisfaction level is derived with use of the BATNA concept and with use of an assumed form of the client's utility function. In further research, different ways of the derivation will be analyzed. In particular, different forms of the utility function, interactive procedures for scaling the function with use of information obtained from clients will be discussed.

A number of interactive sessions with use of the computer-based system have been conducted. They illustrate interactive

multicriteria analyses made by clients and by the producer. Possible behaviors of the clients and the producer have been analyzed, especially how the clients' decisions can inflow on the producer's decision and how the producer's decision is seen among the clients.

In this paper a single round decision making process is proposed. It includes the stage of analyses made by the clients and the stage of analysis made by the producer. In further research, a multi round process will be considered, in which the clients and the producer could successively correct their decisions, similarly as it is in the interactive procedures supporting cooperative decisions, discussed in papers (Kruś, 2002, 2004, 2008).

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