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APPLICATION OF SOFT COMPUTING IN A DISCRETE OPTIMIZATION PROBLEM

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In the paper two methods of solving a discrete optimization problem are discussed. The problem itself is related to air quality protection on a regional scale. The approach refers to optimal allocation of financial means for emission reduction in a given set of power and heating plants.

The implementation considered is sulfur-oriented. The problem is formally stated as cost-constrained minimization of environmental damage function by the optimal choice of desulfurization technologies, within the predefined set of the controlled plants. The receptor-oriented objective function utilizes air pollution forecast preprocessed by a regional scale dispersion model.

An integer-type optimization problem is solved by two methods. The first method utilizes a heuristic algorithm designed for solving this specific problem, which directly finds discrete solution. Another approach is based on the classical gradient optimization algorithm and gives continuous, technologically not applicable solution. Then the continuous solution is transformed to the discrete form by enumeration of some discrete cases.

Both algorithms has been implemented and tested on the real data for selected region. The case study relates to the set of major power plants in Silesia Region (Poland) and the basic desulfurization technologies, which are to be allocated. The test calculations allows us to evaluate accuracy of the heuristic method as well as applicability of both approaches for supporting decisions concerning optimal strategies of emission abatement on a regional scale.

1. Introduction

The paper addresses the problem of regional strategy of environmental quality protection and computer methods, which allow to implement respective decision support tools. The main task deals with the regional-scale strategy for air quality control, mainly due to sulfur oxide pollution. The objective is to allocate emission reduction technologies to emission sources in such a way that certain environmental quality index is minimized under the constraint on total cost of the operation.

Regional-scale abatement strategy depends on the criteria upon which the environmental damage is evaluated. The straightforward approach is based on the emission reduction in all the plants by the fixed percentage or proportionally to the current emission intensity. This solution, however, is not the most efficient from environmental and economic point of view. Other strategies can also be formulated [1,2,7], where the final environmental impact is considered as the main criterion. Moreover, the problem of cost-effectiveness of emission control can be taken into account. This motivates formulation of the problem in terms of optimization techniques.

The task of defining optimal allocation of emission abatement technologies is formulated as integer optimization problem. Exactly one of technologies available must be assigned to each emission source. To solve this problem, two algorithms have been implemented. One of them is a purely heuristic method that directly finds a discrete solution. Another approach utilizes one of continuous, gradient optimization algorithms, respectively adapted to the specific problem. In the paper two methods are presented and compared on the real-data case study.

Test computations were performed on the set of major power plants located in Upper Silesia Region, where optimal strategy of reduction of SO_2 concentration is considered. Calculations have been performed for real emission values and meteorological scenarios. Results characterize efficiency and accuracy of the discussed methods.

2. Statement of the problem

Assume that there are N controlled SO_2 emission sources in a region Ω . Moreover, we have M technologies for emission reduction in our disposal. Effectiveness and the unit cost (both for investment and operational costs) characterize each of them. Our goal is to allocate emission reduction technologies to all the sources in such a way, that the value of certain environmental damage index (the objective function) will be minimized subject to constraints on investment and operational costs, in given period T.

Let us denote:

$\Omega = L_x \times L_y$ -- rectangle area under consideration,

N -- number of controlled sources,

M -- number of available desulfurization technologies,

C -- constraint on total (investment and operational), year averaged costs,

$\vec{u} = [u_1, u_2, \ldots, u_N]$ -- emission vector of controlled sources,

$\vec{e} = [e_1, e_2, \ldots, e_M]$ -- effectiveness vector of desulfurization technologies,

$F = \{f_{ij}\},\ 1 \le i \le N,\ 1 \le j \le M$ -- matrix of abatement cost per unit emission,

$X = \{x_{ij}\},\ 1 \le i \le N,\ 1 \le j \le M$ -- "0-1" matrix of technology assignment to the sources (decision variable matrix).

Definition of the environmental criterion, which is to be minimized, depends on the objectives of the control strategy considered. We define here a global environmental cost function of the following form:

$$J(c) = \frac{1}{2} \int_{\Omega} w(x,y)[\max(0, c(x,y) - c_{ad})]^2 d\Omega, \tag{1}$$

where

$w(x,y)$ -- area sensitivity (weight) function,

c_{ad} -- admissible level of SO_2 concentration.

The concentration forecast used in (1) is calculated as

$$c(x,y) = c_0(x,y) + \sum_{i=1}^{N} A_i(x,y) \cdot u_i, \qquad (x,y) \in \Omega, \tag{2}$$

where

$c_0(x,y)$ -- background concentration (impact of uncontrolled sources),

$A_i(x,y)$ -- transfer matrix (relation emission → concentration) of the i-th source.

The unit transfer matrix $A_i(x,y)$ -- represents here the contribution of the i-th source, referred to the unit emission intensity. All the matrices $A_i(x,y)$; ($i = 1,\dots,N$), for the controlled sources are preprocessed by the respective forecasting model. In a similar way, the background pollution field $c_0(x,y)$ is computed for uncontrolled, background emissions, including the inflow from the neighboring regions. The current emission intensity of the i-th source depends on the initial emission value -- u_i^0 and efficiency of the abatement technology applied, according to the formula

$$u_i = u_i^0 \sum_{j=1}^{N} (1 - e_j) \cdot x_{ij}, \quad \sum_{j=1}^{N} x_{ij} = 1, \quad x_{ij} \in \{0,1\}, \quad 1 \le i \le N, \tag{3}$$

where

u_i -- current emission intensity of the i-th source,

u_i^0 -- initial emission intensity of the i-th source.

Cost of emission abatement in each source consists of two components: investment cost and operational cost. Both investment and operational costs depend on the specific abatement technology and on the parameters of the energy installation where this technology is to be applied. Here a simplified approach is utilized, where the investment cost of the j-th abatement technology applied in the i-th emission source is calculated as annual cost, averaged over the entire amortization period. Thus, the total emission abatement cost per year, considered as a sum of desulfurization costs in the respective plants, is used to formulate the financial constraint

$$\sum_{i=1}^{N} F_i = \sum_{i=1}^{N} u_i^0 \sum_{j=1}^{M} f_{ij} x_{ij} = \sum_{i=1}^{N} u_i^0 \sum_{j=1}^{M} (f_{ij}^1 + f_{ij}^2) x_{ij} \le C, \tag{4}$$

where F_i is the annual total cost of the abatement strategy assigned to the i-th source, while the cost coefficients (cost per unit emission) related to the j-th technology applied to the i-th source are as follows:

f_{ij} -- averaged annual total cost,

f_{ij}^1 -- averaged annual investment cost,

f_{ij}^2 -- averaged annual operational cost.

Now we can formulate the following problem of allocation of emission reduction technologies to emission sources

ALLOCATION PROBLEM: *Determine the set of emission reduction technologies*

$$X^* = \{x_{ij}^* \in \{0,1\}: \sum_{j=1}^{M} x_{ij}^* = 1, \quad 1 \le i \le N, \quad 1 \le j \le M\},$$

such that the environmental cost function (1) s minimized

$$J(c(X^*)) \Rightarrow \min,$$

subject to the total cost constraint

$$\sum_{i=1}^{N} F_i \le C\ .$$

3. Implementation of the optimization algorithms

In this section three algorithms of solving the problem stated above are presented. The first one is a heuristic algorithm (*heuristic method*) directly solving the discrete programming task formulated in Section 1.

Another approach discussed here (*continuous method*) is based on formulation the main task as the respective continuous problem, where the decision variables are real numbers $x_{ij} \in \langle 0,1 \rangle$, $(i = 1,\dots,N;\ j = 1,\dots,M)$. That means, combination of several technologies allocated to a source is an accepted solution. Despite technologically unrealistic, such a continuous solution can be used as a reference base for evaluation of accuracy of the heuristic, discrete programming algorithm.

On the other hand, the solution in a form of real numbers can be utilized as the starting point for searching by enumeration the neighboring discrete solution. Such an approach is used in the implementation of the third algorithm presented in the sequel and referred to as *combined method.*.

3.1. A heuristic discrete-programming algorithm

The flow diagram of the *heuristic algorithm* is shown on Figure 1. In order to compare solutions, we construct for each x_{ij} -- i.e. for each source i and each technology j -- an efficiency factor r_{ij}, defined as follows:

$$r_{ij} = \alpha[u_i^0 \cdot e_j / f_{ij}] + (1-\alpha)[\int_{\Omega} A_i(x,y) \cdot e_j / f_{ij}], \qquad \alpha \in \langle 0,1 \rangle,$$

where α is the weight factor ranging from 0 to 1, reflecting the influence of emission reduction versus concentration reduction (via transfer matrix $A_i(x,y)$).

The main loop of the algorithm consists of the following steps:

1) Set the value of α to 0.
2) Construct the list of all x_{ij}, where i is the number of the source and j is the number of technology, ordering the list in descending mode due to the value of r_{ij} for x_{ij}.
   Set the value of the temporary variable C_{tmp} to 0. C_{tmp} stores the sum of products $f_{ij} \cdot u_i^0$ for x_{ij} present in the solution $\{x_{ij}\}$.
3) Take the first x_{ij}^* on the list , i.e. the x_{ij}^* with the greatest r_{ij} .
4) Check, if x_{ij}^* can be added the solution $\{x_{ij}\}$, i.e. check, if the sum $C_{tmp} + f_{ij}^* \cdot u_i^o$ is greater than the cost constraint C. If so, proceed to step 6. Otherwise proceed to step 5.
5) Update the solution $\{x_{ij}\}$ by adding x_{ij}^* .
   Update C_{tmp} by adding $f_{ij}^* \cdot u_i^o$.
   Remove x_{ij}^* from the list, and all x_{ij} concerning the same source.
   Proceed to step 7.
6) Remove x_{ij}^* from the list.
   Proceed to step 7.
7) Check, if the list is empty. If not, go to step 3. Otherwise proceed to step 8.
8) Check, if the solution obtained in the current iteration is better than the best one known yet. If so, set the current solution as the best one. Proceed to step 9.
9) Increase α by some value (in our case 0.1).
10) Check, if the value of α is equal 1. If so, stop calculations. Otherwise, go to step 2.

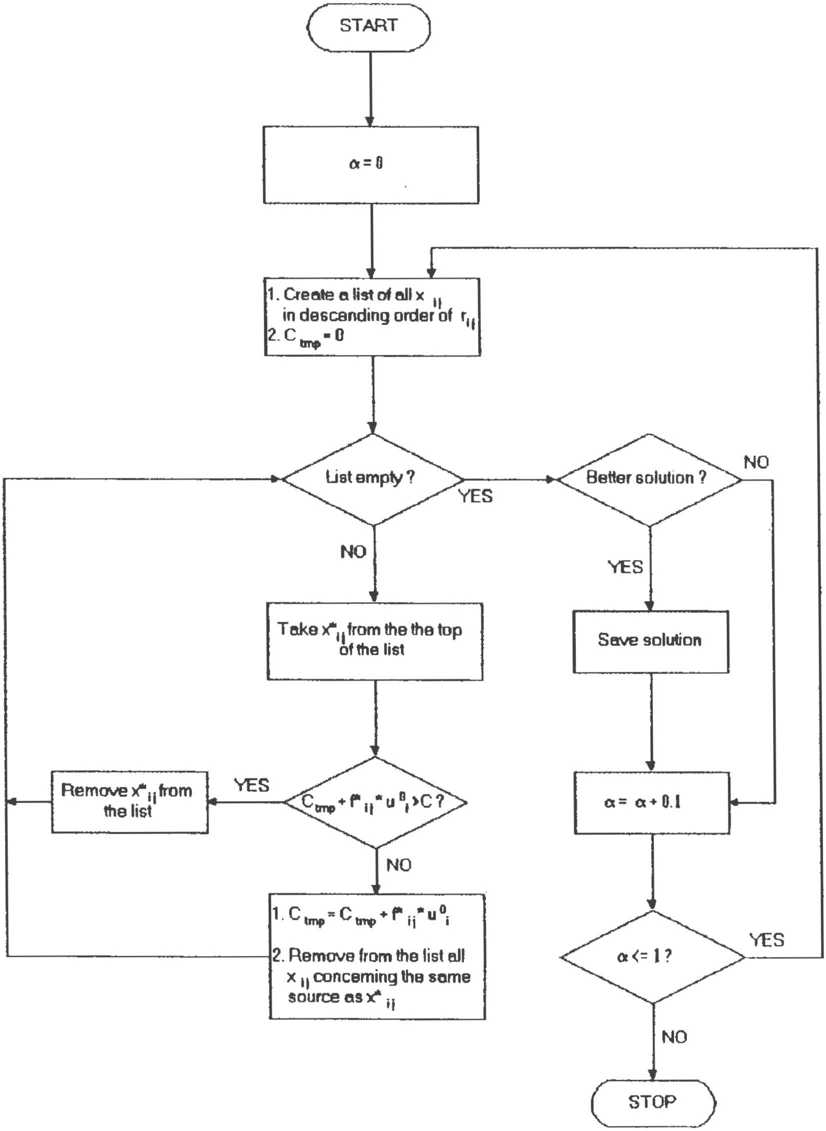

Figure 1. Flow-diagram of the heuristic algorithm

3.2. An algorithm based on the continuous optimization

The continuous optimization task is solved by a modified version of the method of linearization proposed by Pshenitchny [6]. General problem formulation consists in minimization of the objective function

$$f_0(\vec{x}) \to \min \tag{5}$$

subject to the constraints

$$f_j(\vec{x}) < 0, \quad j = 1,\ldots,m, \qquad \vec{x} \in R^n. \tag{5a}$$

The implementation of the method utilizes the following functions:

$$F(\vec{x}) \overset{df}{=} \max_{1 \le j \le m} f_j(\vec{x}), \qquad \Phi_N(\vec{x}) \overset{df}{=} f_0(\vec{x}) + N(\vec{x})\,F(\vec{x}),$$

where the function $N(\vec{x})$ depends on the dual variables of the main optimization problem and is defined in details in [3]. Moreover the set of the active constraints with the tolerance $\delta > 0$ is defined as follows:

$$M_\delta(\vec{x}) \overset{df}{=} \left\{ j\colon\ j \ge 1 \ \text{ and } \ f_j(\vec{x}) \ge F(\vec{x}) - \delta \right\}$$

The consecutive steps of the algorithm are as follows:

1) Set the initial point $\vec{x}_0$ and the parameters $\delta, \varepsilon, \theta > 0;\ \ 0 < \eta < 1$. Put $i = 0$.
2) Calculate $F(\vec{x}_i)$ and $M_\delta(\vec{x}_i)$.
3) Solve the quadratic programming problem to find the descent direction $\vec{p}$

$$\frac{1}{2}\|\vec{p}\|^2 + \nabla f_0(\vec{x}_i)\cdot\vec{p} \to \min \tag{6}$$

subject to the constraints

$$f_j(\vec{x})\cdot\vec{p} + f_j(x_i) \le 0, \qquad j \in M_\delta(\vec{x}_i) \qquad \vec{p} \in R^n. \tag{6a}$$

4) If $\|p\|^2 \le \varepsilon$ -- *END* of the algorithm; $\vec{x}_i$ is the minimum point.
5) If the solution of (6)--(6a) does not exist, put $\delta = \delta / 2$ and go step 2).
6) Calculate $N(\vec{x}_i)$ (details can be found in [3]).
7) Find the minimum number $k = 0,1,\ldots$ such tat

$$\Phi_N\left(\vec{x}_i + \left(\frac{1}{2}\right)^k \vec{p}\right) \le \Phi_N(x_i) - \left(\frac{1}{2}\right)^k \eta\,\|p\|^2 \tag{7}$$

8) Substitute $\vec{x}_{i+1} = \vec{x}_i + \left(\frac{1}{2}\right)^k \vec{p}$ and go to step 2).

To formulate the *continuous method* for the *ALLOCATION PROBLEM* stated in Section 2, we assume the continuity of the decision variables x_{ij}, $(i = 1,\dots,N;\ j = 1,\dots,M)$. That means, the solution is a set of real values $x_{ij} \in \langle 0,1 \rangle$. The task generates therefore certain artificial, continuous solution, which can be used as a reference base for the heuristic algorithm discussed above.

The objective function to be minimized is applied in the form (1) along with the constraint (2), which relates the environmental impact of the source (SO_2 concentration) with the actual emission intensity.

Some additional constraint was imposed on the original problem to get the final values of the decision variables close to the integer solution. To this end, the constraints (3) where modified as follows:

$$u_i = u_i^0 \sum_{j=1}^{N} (1-e_j)\cdot u_i, \quad \sum_{j=1}^{M} x_{ij} = 1, \quad x_{ij} \in \langle 0,1 \rangle, \quad 1 \le i \le N; \quad 1 < j < M, \tag{8}$$
$$x_{ij} = y^2{}_{ij}, \qquad 1 \le i \le N; \quad 1 < j < M.$$

We apply the algorithm (5)--(7) to the continuous optimization problem (1), (2), (4) and (8) in terms of the auxiliary decision variable $y_{ij} \in \langle 0,1 \rangle$. Due to (8), gradient of the objective function (1) has the following form

$$\frac{\partial J}{\partial y_{ij}} = \frac{\partial J}{\partial d} \cdot \frac{\partial c}{\partial u_i} \cdot \frac{\partial u_i}{\partial y_{ij}} = 2u_i^0 (1-e_j) \int_\Omega w(x,y)(\max(0, c(x,y) - c_{ad}) A_i(x,y) d\Omega \tag{9}$$

$$(i=1,\dots,N;\quad j=1,\dots,M)$$

where

$$\frac{\partial J}{\partial d} = \int_\Omega w(x,y)(\max(0, c(x,y) - c_{ad}) d\Omega, \tag{9a}$$

$$\frac{\partial c}{\partial u_i} = A_i(x,y), \qquad (i = 1,\dots,N), \tag{9b}$$

$$\frac{\partial u_i}{\partial y_{ij}} = 2u_i^0 \cdot (1-e_j), \qquad (i = 1,\dots,N;\ j = 1,\dots,M). \tag{9c}$$

Solution of this problem will be directly applied for evaluating accuracy of the previous, heuristic algorithm. Moreover, it will be utilized for searching the closed integer-form solution by enumeration method (*combined method*). Results of the test computations and comparison of two methods are presented in the next section.

4. Case study analysis

Three optimization algorithms discussed in Section 3 were applied in the real-data case for selection of desulfurization technologies in the major power plants of the industrial Upper Silesia Region. The region is characterized by high concentration of heavy industry and the energy sector installations.

The domain considered is a rectangle area 110 km x 76km. In this area 20 major power plants were selected and considered as the controlled sources. Moreover, certain number of medium and small industrial sources constitutes the background emission field.

In the example presented, 8 desulfurization technologies are taken into account (5 basic technologies and 3 combined). The technologies and the respective emission reduction efficiencies are as follows:

1. "do nothing" technology $(e_1 = 0)$,
2. low-sulfur fuel $(e_2 \cong 30)$,
3. dry desulfurization method $(e_3 \cong 35)$,
4. low-sulfur fuel + dry desulfurization method $(e_4 \cong 545)$,
5. half-dry desulfurization method $(e_5 \cong 75)$,
6. low-sulfur fuel + half-dry desulfurization method $(e_6 \cong 825)$,
7. MOWAP method $(e_7 \cong 85)$,
8. low-sulfur fuel + MOWAP method $(e_8 \cong 895)$.

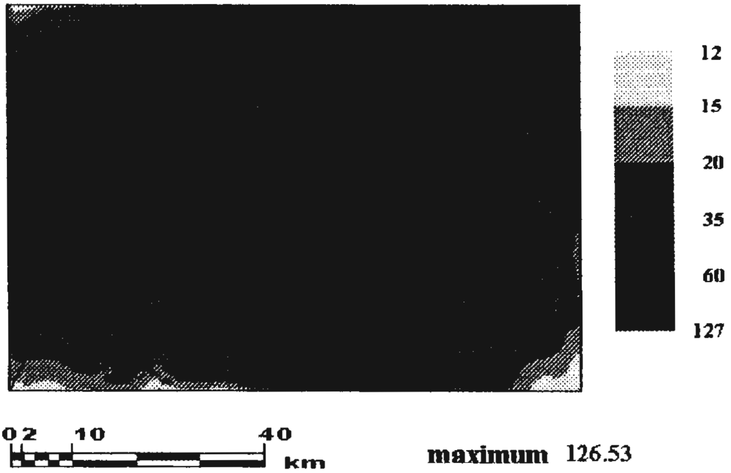

Figure 2. Initial SO$_2$ concentration in the region [$\mu g / m^3$].

The annual unit concentration maps for the controlled sources (the transfer matrices $A_i(x,y)$, $i = 1,\dots,N$) are preprocessed off-line by the regional-scale forecasting model REGFOR3 defined in [4,5]. This is a dynamical, single-layer model based on the meteorological input data for the period of simulation.

The same technique is used for generating the background concentration field for intermediate, pointwise and area sources. Computations were performed for one representative year, where a sequence of meteorological data with 12-hrs time resolution was applied. Calculations were repeated for several levels of the total cost constraint. In Tables 1--4 some selected results are presented for cost constraints 150 mill $/yr. and 250 mill $/yr., respectively.

Table 1. Solutions obtained by continuous optimization and cost constraint 150 mill $/yr.

sou rce		solution by optimization method environmental cost reduction = **0.1175** abatement cost = **150.00** mill $/yr.								
No	initial emiss.	abatement technology								final emiss.
		1	2	3	4	5	6	7	8	
1	303.20	0.00	0.00	0.00	0.00	0.00	0.00	**1.00**	0.00	45.48
2	225.30	**1.00**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	225.30
3	104.00	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	0.00	47.32
4	91.80	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	0.00	41.77
5	90.10	0.00	0.00	0.00	0.00	0.00	**1.00**	0.00	0.00	40.99
6	78.00	**0.55**	**0.45**	0.00	0.00	0.00	0.00	0.00	0.00	54.60
7	65.00	0.00	**1.00**	0.00	0.00	0.00	0.00	0.00	0.00	45.51
8	52.00	**1.00**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	52.00
9	52.00	0.00	**1.00**	0.00	0.00	0.00	0.00	0.00	0.00	36.40
10	45.10	0.00	**1.00**	0.00	0.00	0.00	0.00	0.00	0.00	31.57
11	34.70	0.00	0.00	0.00	0.00	0.00	0.00	**1.00**	0.00	5.21
12	33.80	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**1.00**	3.55
13	29.90	0.00	0.00	0.00	0.00	0.00	**0.05**	**0.94**	**0.01**	4.51
14	25.10	0.00	0.00	0.00	0.00	0.00	0.00	**0.16**	**0.84**	2.82
15	26.00	0.00	0.00	0.00	**0.91**	0.00	**0.09**	0.00	0.00	11.15
16	18.70	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	0.00	8.51
17	16.90	**0.51**	**0.49**	0.00	0.00	0.00	0.00	0.00	0.00	14.40
18	15.10	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	0.00	6.87
19	12.30	**0.43**	**0.26**	**0.31**	0.00	0.00	0.00	0.00	0.00	10.00
20	11.60	0.00	**0.01**	0.00	**0.99**	0.00	0.00	0.00	0.00	5.30

Figure 2 presents the map of the initial SO_2 concentration in the region as well as location of the main emission sources. The initial emission intensities of 20 selected sources are presented in Table 1. This table also presents the solution obtained by continuous optimization method for the cost constraint 150 mill $/yr. Relative share of the abatement technologies selected by the algorithm for specific sources are shown in the consecutive rows of Table 1. Columns refer to 8 abatement technologies, according to their increasing efficiencies. It can be seen that, due to the specific form of the algorithm and the additional constraint imposed, most of the solutions is of integer form.

Table 2. Integer-type solutions obtained by two approaches and cost constraint 150 mill $/yr.

source		solution by heuristic method env. cost reduction = **0.125** abatement cost = **149.731** mill $/yr.									solution by optimization method env. cost reduction = **0.118** abatement cost = **149.768** mill $/yr.								
	initial emiss.	abatement technology								final emiss.	abatement technology								final emiss.
No		1	2	3	4	5	6	7	8		1	2	3	1	5	6	7	8	
1	303.20	0	0	0	0	0	0	1	0	45.48	0	0	0	0	0	0	1	0	45.48
2	225.30	1	0	0	0	0	0	0	0	225.30	1	0	0	0	0	0	0	0	225.30
3	104.00	0	0	0	1	0	0	0	0	47.32	0	0	0	1	0	0	0	0	47.32
4	91.80	0	0	0	0	1	0	0	0	22.95	0	0	0	1	0	0	0	0	41.77
5	90.10	0	0	0	0	0	0	0	1	9.46	0	0	0	0	0	1	0	0	15.77
6	78.00	1	0	0	0	0	0	0	0	78.00	0	1	0	0	0	0	0	0	54.60
7	65.00	1	0	0	0	0	0	0	0	65.00	0	1	0	0	0	0	0	0	45.50
8	52.00	1	0	0	0	0	0	0	0	52.00	1	0	0	0	0	0	0	0	52.00
9	52.00	0	0	0	1	0	0	0	0	23.66	0	1	0	0	0	0	0	0	36.40
10	45.10	1	0	0	0	0	0	0	0	45.10	0	1	0	0	0	0	0	0	31.57
11	34.70	1	0	0	0	0	0	0	0	34.70	0	0	0	0	0	0	1	0	5.21
12	33.80	0	0	0	0	0	0	0	1	3.55	0	0	0	0	0	0	0	1	3.55
13	29.90	0	0	0	0	0	1	0	0	5.23	0	0	0	0	0	0	1	0	4.49
14	25.10	0	0	0	0	0	0	1	0	3.77	0	0	0	0	0	0	1	0	3.77
15	26.00	0	0	0	1	0	0	0	0	11.83	0	0	0	1	0	0	0	0	11.83
16	18.70	0	0	0	1	0	0	0	0	8.51	0	0	0	1	0	0	0	0	8.51
17	16.90	1	0	0	0	0	0	0	0	16.90	0	1	0	0	0	0	0	0	11.83
18	15.10	0	0	0	1	0	0	0	0	6.87	0	0	0	1	0	0	0	0	6.87
19	12.30	1	0	0	0	0	0	0	0	12.30	1	0	0	0	0	0	0	0	12.30
20	11.60	0	1	0	0	0	0	0	0	8.12	0	0	0	1	0	0	0	0	5.28

On the other hand, solutions found for sources no. 6, 13--15, 17,19 are fuzzy and suggests technologically unrealistic combination of several technologies. The last column shows the emissions of the sources, related to selected reduction strategies.

Table 2 shows the results for the same cost limit, but generated by heuristic and combined methods, respectively. Solution get by the heuristic method is shown in the first part of Table 2. In this case, general strategy suggested is similar to that obtained by continuous algorithm, but significant differences appear in some specific sources. The rate of reduction of the environmental cost function is much worse for heuristic method comparing to those of the other two algorithms.

Table 3. Solutions obtained by continuous optimization and cost constraint
250 mill $/yr.

sou rce		solution by optimization method environmental cost reduction = **0.0420** abatement cost = **250.00 $**/yr.								
	initial emiss.	abatement technology								final emiss.
No		1	2	3	4	5	6	7	8	
1	303.20	0.00	0.00	0.00	0.00	0.00	0.00	**1.00**	0.00	45.48
2	225.30	0.00	0.00	**1.00**	0.00	0.00	0.00	0.00	0.00	146.45
3	104.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**1.00**	10.92
4	91.80	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**1.00**	9.64
5	90.10	0.00	0.00	0.00	0.00	0.00	**0.12**	0.00	**0.88**	10.23
6	78.00	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	0.00	35.53
7	65.00	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	0.00	29.58
8	52.00	0.00	0.00	**1.00**	**0.00**	0.00	0.00	0.00	0.00	33.80
9	52.00	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	0.00	23.66
10	45.10	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	0.00	20.52
11	34.70	0.00	0.00	0.00	0.00	0.00	0.00	**1.00**	0.00	5.21
12	33.80	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**1.00**	3.55
13	29.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**1.00**	3.14
14	25.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**1.00**	2.64
15	26.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**1.00**	2.73
16	18.70	0.00	0.00	0.00	**0.67**	**0.33**	0.00	0.00	0.00	7.25
17	16.90	0.00	**0.99**	0.00	**0.01**	0.00	0.00	0.00	0.00	11.77
18	15.10	0.00	0.00	0.00	0.00	0.00	**0.70**	0.00	**0.30**	2.33
19	12.30	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	0.00	5.60
20	11.60	0.00	0.00	0.00	**0.99**	0.00	**0.01**	0.00	0.00	5.24

As explained in the previous section, the combined algorithm utilizes the optimal solution of the continuous method as the starting point. Then all the fuzzy solutions x_{ij}, with the value neither equal 0 nor 1, are enumerated. In the enumeration process the values of those x_{ij} are set to 0 or 1. The result usually gives a slightly worse rate of cost function reduction comparing to that of continuous method but is much better than heuristic solution.

Tables 3 and 4 show analogous solutions of allocation problem obtained for 250 mill \$/yr. Figures 3 and 4 present respective maps of SO_2 concentration for the optimal emission reduction strategy (combined method) and the financial constraints 150 mill \$/yr and 250 mill \$/yr., respectively.

Table 4. Integer-type solutions obtained by two approaches and cost constraint 250 mill \$/yr.

source		solution by heuristic method env. cost reduction = **0.0456** abatement cost = **249.881** mill \$/yr.									solution by optimization method env. cost reduction = **0.0423** abatement cost = **249.181** mill \$/yr.								
	initial emiss.	abatement technology								final emiss.	abatement technology								final emiss.
No		1	2	3	4	5	6	7	8		1	2	3	1	5	6	7	8	
1	303.20	0	0	0	0	0	0	0	1	31.84	0	0	0	0	0	0	1	0	45.48
2	225.30	0	0	0	1	0	0	0	0	102.51	0	0	1	0	0	0	0	0	146.45
3	104.00	0	0	0	0	0	0	0	1	10.92	0	0	0	0	0	0	0	1	10.92
4	91.80	0	0	0	0	0	0	0	1	9.64	0	0	0	0	0	0	0	1	9.64
5	90.10	0	0	0	0	0	0	0	1	9.46	0	0	0	0	0	0	0	1	9.46
6	78.00	0	1	0	0	0	0	0	0	54.60	0	0	0	1	0	0	0	0	35.49
7	65.00	0	0	1	0	0	0	0	0	42.25	0	0	0	1	0	0	0	0	29.58
8	52.00	1	0	0	0	0	0	0	0	52.00	0	0	1	0	0	0	0	0	33.80
9	52.00	0	0	0	1	0	0	0	0	23.66	0	0	0	1	0	0	0	0	23.66
10	45.10	0	0	1	0	0	0	0	0	29.32	0	0	0	1	0	0	0	0	20.52
11	34.70	0	1	0	0	0	0	0	0	24.29	0	0	0	0	0	0	1	0	5.21
12	33.80	0	0	0	0	0	0	0	1	3.55	0	0	0	0	0	0	0	1	3.55
13	29.90	0	0	0	0	0	0	0	1	3.14	0	0	0	0	0	0	0	1	3.14
14	25.10	0	0	0	0	0	0	0	1	2.64	0	0	0	0	0	0	0	1	2.65
15	26.00	0	0	0	0	0	0	0	1	2.73	0	0	0	0	0	0	0	1	2.73
16	18.70	0	0	0	0	0	1	0	0	3.27	0	0	0	1	0	0	0	0	8.51
17	16.90	1	0	0	0	0	0	0	0	6.90	0	1	0	0	0	0	0	0	11.83
18	15.10	0	0	0	0	1	0	0	0	3.78	0	0	0	0	0	1	0	0	2.64
19	12.30	0	0	1	0	0	0	0	0	7.99	0	0	0	1	0	0	0	0	5.60
20	11.60	0	0	0	1	0	0	0	0	5.28	0	0	0	1	0	0	0	0	5.28

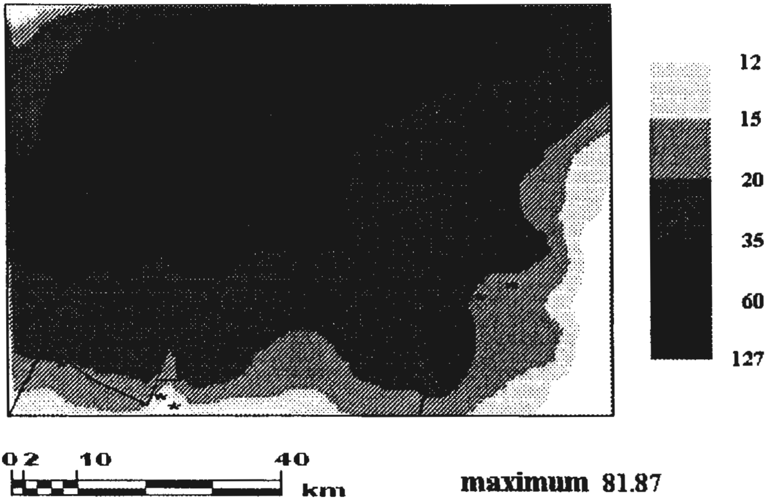

Figure 3. SO_2 concentration [$\mu g / m^3$] map for abatement technology constraint 150 mill \$/yr.

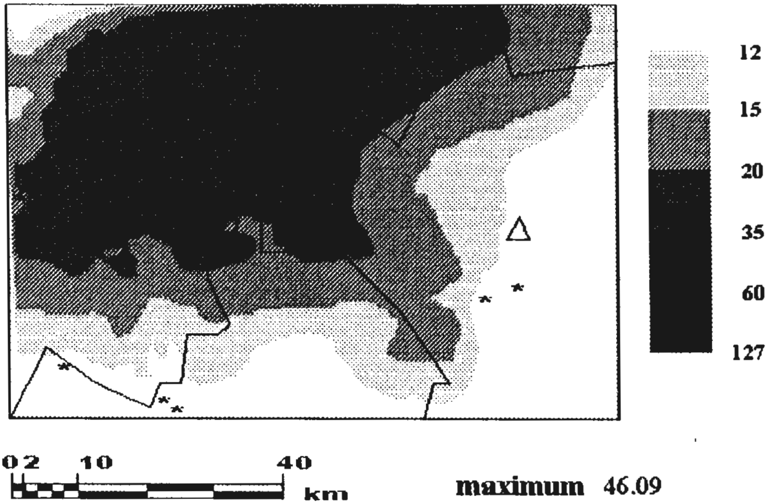

Figure 4. SO_2 concentration [$\mu g / m^3$] map for abatement technology constraint 250 mill \$/yr.

5. Concluding remarks

Results presented in the previous section for two levels of cost limit show that general allocation strategies are similar for three algorithms discussed. Reduction of the objective function is the best for continuous method and definitely the worse for heuristic one. On the other hand, the absolute values of that index obtained by three methods only slightly differ from each other. This is the result of very flat shape of criterion function (1) in the neighborhood of the optimum point.

The methods discussed here were also tested on 50 different, randomly generated data sets, with the same cost constraint. The comparison of the achieved results is shown on Figure 5.

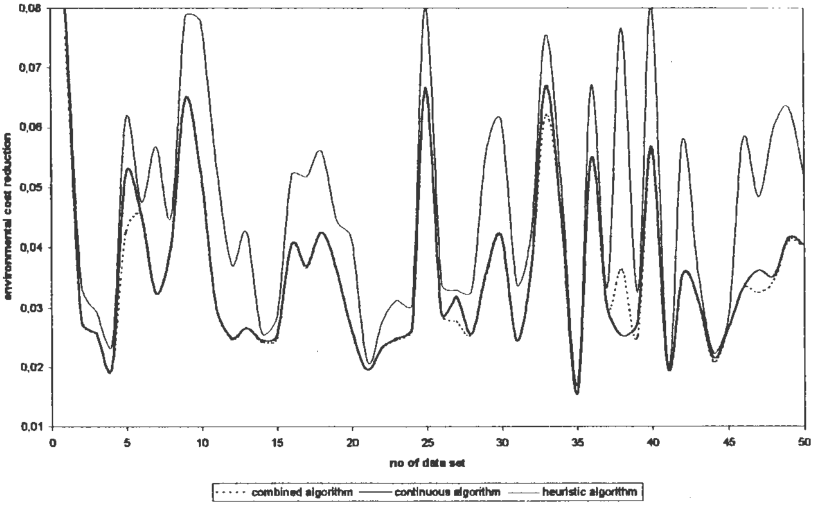

Figure 5. Comparison of the efficiency of different algorithms

As one can observe, the heuristic method gives the worst results (reduction of the environmental objective function), although it is the fastest method. The difference in solution quality calculated by the continuous method and the combined method varies from case to case. It depends on the stop criterion of the continuous algorithm, which is the compromise between computing time and the final accuracy of the solution. In cases, where the continuous solution is computed with high accuracy condition, the solution obtained by the combined method must be no better than the continuous one. On the contrary, where the continuous algorithm stops early, the better solution can be obtained by enumerating some possible solution, what is done by the combined method.

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