another way the third letter in the triad is the same for both, for we have *ain*, *fln* and *afm*, *ilm*. Further, the three letters *m*, *n*, *o* form a triad of the system.

We may now get another solution of the problem as follows: In each column after the first one of the three *m*, *n*, *o* is taken with two of the four *a*, *f*, *i*, *l*. Interchange the other two of the three. The new arrangement is—

abc . adg . aej . afm . ahk . ain . alo,

def . bho . bdn . bgl . bjm . bfk . bei,

ghi . cij . cfh . ceo . cdl . cgm . ckn,

jkl . ekm . gko . dik . egn . djo . dhm,

mno . fln . ilm . hjn . fio . ehl . fgj.

Here the former umbral notation will not apply.

The possibility or otherwise of using this umbral notation shews an essential difference between the solutions. It may be that this classification is of importance in considering Sylvester's further problem of making thirteen such arrangements including all possible triads. It is suggested by the system of half-periods of a quadruply-periodic function.

ON THE CURVE OF INTERSECTION OF TWO QUADRICS.

By *W. Burnside.*

It is well known that the coordinates of a point on the curve of intersection of two quadrics are expressible rationally in terms of elliptic functions of an arbitrary parameter. It is proposed here to answer the question:—When the quadrics are arbitrarily given, what is the elliptic differential involved; or in other words, what is the absolute invariant of the elliptic functions?

Suppose the equations to the two quadrics reduced to the standard form

$$x^2 + y^2 + z^2 + w^2 = 0,$$

$$\alpha x^2 + \beta y^2 + \gamma z^2 + \delta w^2 = 0,$$

so that α, β, γ, δ are the roots of the quartic (Salmon's *Solid Geometry*, Chap. IX.),

$$\lambda^4\Delta + \lambda^3\Theta + \lambda^2\Phi + \lambda\Theta' + \Delta' = 0 \text{............(i).}$$

Then the ratios $x:y:z:w$ of any point referred to the common self-conjugate tetrahedron are expressible rationally in terms of the ratio of the original coordinates and of $\alpha, \beta, \gamma, \delta$.

Now the equations

$$(\beta-\gamma)(t-e_1)+(\gamma-\alpha)(t-e_2)+(\alpha-\beta)(t-e_3)+A=0,$$

$$\alpha(\beta-\gamma)(t-e_1)+\beta(\gamma-\alpha)(t-e_2)+\gamma(\alpha-\beta)(t-e_3)+\delta A=0$$

are consistent if

$$(\delta-\alpha)(\beta-\gamma)e_1+(\delta-\beta)(\gamma-\alpha)e_2+(\delta-\gamma)(\alpha-\beta)e_3=0;$$

and if at the same time

$$e_1+e_2+e_3=0,$$

so that

$$\frac{e_1}{(\delta-\beta)(\gamma-\alpha)-(\delta-\gamma)(\alpha-\beta)}=\frac{e_2}{(\delta-\gamma)(\alpha-\beta)-(\delta-\alpha)(\beta-\gamma)}$$

$$=\frac{e_3}{(\delta-\alpha)(\beta-\gamma)-(\delta-\beta)(\gamma-\alpha)},$$

then e_1, e_2, and e_3 are the roots of the equation

$$4h^3e^3-g_2he-g_3=0,$$

where g_2, g_3 are the quadrivariant and cubinvariant of the quartic (i) and h is arbitrary.

If $h=1$, it is easily verified that each of the above fractions is $\frac{1}{12}$, and then

$$A=(\beta-\gamma)e_1+(\gamma-\alpha)e_2+(\alpha-\beta)e_3$$
$$=\tfrac{1}{4}(\beta-\gamma)(\gamma-\alpha)(\alpha-\beta).$$

Hence, we may take

$$x:y:z:w$$

$$::\sqrt{(\beta-\gamma)}\sqrt{(t-e_1)}:\sqrt{(\gamma-\alpha)}\sqrt{(t-e_2)}:\sqrt{(\alpha-\beta)}\sqrt{(t-e_3)}$$
$$:\tfrac{1}{2}\sqrt{\{(\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)},$$

where e_1, e_2, and e_3 are the roots of

$$4e^3-g_2e-g_3=0.$$

If now we put

$$t=p(2u, g_3, g_3),$$

the three square roots involved can be expressed rationally in terms of pu and $p'u$ by the relation

$$p(2u)-e_\lambda=\left[\frac{(p'u-e_\lambda)^2+e_\mu e_\nu}{p'u}\right]^2;$$

so that finally

$$x : y : z : w,$$

$$:: \sqrt{(\beta-\gamma)}\,\{(pu-e_1)^2+e_2e_3\} : \sqrt{(\gamma-\alpha)}\,\{(pu-e_2)^2+e_3e_1\}$$
$$: \sqrt{(\alpha-\beta)}\,\{(pu-e_3)^2+e_1e_2\} : \tfrac{1}{2}p'u\,\sqrt{\{(\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)\}}.$$

The invariants g_2, g_3 of the elliptic functions used are the two invariants of the quartic (i), so that the absolute invariant of the elliptic functions is the absolute invariant of the quartic, and the elliptic differential involved is

$$\frac{d\lambda}{\sqrt{(\lambda^4\Delta-\lambda^3\Theta+\lambda^2\Phi+\lambda\Theta'+\Delta')}}.$$

ON THE FACT THAT SEMINVARIANTS OF A BINARY QUANTIC ARE INVARIANTS OF THAT QUANTIC AND ITS DERIVATIVES.

By Professor *E. B. Elliott.*

1. The object of the following is to exhibit without graphical or symbolical reasoning a fact which Mr. Kempe has brought into notice in a paper on the application of graphs to ordinary binary quantics (*Proc. Lond. Math. Soc.*, 1893, pp. 105, &c.).

He has shewn that $S(a_0, a_1, a_2, \ldots, a_n)$, any seminvariant of the binary quantic

$$\alpha_n \equiv (a_0, a_1, a_2, \ldots, a_n)(x, y)^n,$$

is an invariant of that quantic and its successive derivatives with regard to x,

$$\alpha_{n-1} \equiv (a_0, a_1, a_2, \ldots, a_{n-1})(x, y)^{n-1},$$

....,

$$\alpha_2 \equiv (a_0, a_1, a_2)(x, y)^2,$$

$$\alpha_1 \equiv a_0x + a_1y,$$

$$\alpha_0 \equiv a_0,$$

or some of them. He obtains, in fact, that

$$a_0^{\,W} S(a_0, a_1, a_2, \ldots, a_n),$$