CAYLEY'S CUBIC RESOLVENT AND THE REDUCING CUBIC.

By *Irving Stringham, Ph.D.*

A HOMOGRAPHIC relation exists between the roots of Cayley's cubic resolvent $\theta^3 - M(\theta - 1) = 0$, if I may so name it (see Cayley's *Elliptic Functions*, p. 319), and those of the reducing cubic $4\zeta^3 - I\zeta - J = 0$ of the quartic equation

$$a + 4bv + 6cv^2 + 4dv^3 + ev^4 = 0.$$

We should be able to deduce this relation from the general homographic equation

$$A\zeta\theta + B\zeta + C\theta + D = 0$$

by determining its coefficients under the condition that the one cubic is hereby transformed into the other. But without attacking the problem from this general point of view, we may easily verify that the proper substitution is of the form

$$\zeta = h\frac{\theta}{3 - 2\theta} \text{ or } \theta = 3\frac{\zeta}{h + 2\zeta}.$$

Let the first of these, as the simpler, be applied to the reducing cubic. If the factor $1/(3 - 2\theta)^3$ be removed, the transformation gives, as the equation in θ,

$$4h^3\theta^3 - Ih\theta(3 - 2\theta)^2 - J(3 - 2\theta)^3 = 0,$$

which, when it is arranged according to the powers of θ, and the term containing θ^2 is omitted, becomes

$$(4h^3 - 4Ih + 8J)\theta^3 + (-9Ih + 54J)\theta - 27J = 0,$$

and the condition that the coefficient of θ^2 shall vanish is

$$Ih - 3J = 0.$$

The other coefficients then are

$$4h^3 - 4Ih + 8J = 4J\left(27\frac{J^2}{I^3} - 1\right) = -4\frac{3\Delta}{I^3}$$

$$-9Ih + 54J = 27J,$$

and the final form of the cubic in θ is

$$-\frac{4J\Delta}{I^3}\theta^3+27J\theta-27J=0,$$

or

$$\theta^3-M(\theta-1)=0,$$

where

$$\frac{1}{M}=\frac{4}{27}\left(1-\frac{27J^2}{I^3}\right).$$

This is Cayley's cubic. The homographic relation between ζ and θ therefore is

$$\zeta=\frac{3J}{I}\cdot\frac{\theta}{3-2\theta}.$$

University of California,
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A MAP OF THE COMPLEX Z-FUNCTION: A CONDENSER PROBLEM.

By *J. H. Michell, M.A.*, Fellow of Trinity College, Cambridge.

The object of this note is to consider the correspondence of two planes z, w, whose points are connected by the relation

$$z=Z(w+iK'),$$

where Z is the function of Jacobi so denoted. It appears that the relation gives the solution for an electrical condenser formed of two equal plane strips of infinite length and finite breadth, placed with parallel planes and edges so that a normal section consists of two opposite sides of a rectangle. Writing $z\equiv x+iy$ and $w\equiv\phi+i\psi$, x, y are the rectangular coordinates of the point at which the potential is ψ and the flow ϕ, the plates being at potential $-K'$, K'.

Consider the rectangle in the w plane bounded by $\phi=\pm K$, $\psi=\pm K'$.

The Z-function is single valued, and within this rectangle dz/dw only becomes infinite once, viz. at $w=0$.

Let us find the path of z when w goes around the rectangle.