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Rules for control of emissions of pollutants and marketable permits under high observation uncertainty. The greenhouse gases case

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Abstract

Implementation of the Kyoto Protocol raises the question of how to verify changes in net greenhouse gas (GHG) emissions on the country scale. Estimates show that the uncertainties of the reported net emissions can be great, depending on how they are accounted and which GHG emissions and/or removals are considered. We study the problem of testing fulfilment of the Kyoto obligations taking into account big uncertainty of the observed emissions. Our proposition for doing it is based on introduction of undershooting emission limitation or reduction commitments and on specifying the risk of not doing so. This leads to conditions that can be easily checked both deterministically and stochastically. In a follow-up step, we then address the issue of emission trading in the presence of uncertainty. Trading rules are influenced as the costs of bargained emission units change due to the uncertainty of emission observation.

1. Introduction

The Kyoto Protocol contains the first legally binding commitments to limit or reduce the emissions of six greenhouse gases (GHGs) or groups of gases (CO_2, CH_4, N_2O, HFCs, PFCs, and SF_6). For so-called Annex I Parties, the targets agreed upon under the Protocol by the first commitment period (2008 to 2012) add up to a decrease in greenhouse gas emissions of 5.2% below 1990 levels in terms of CO_2 equivalents (CO_2-eq)[1]. Among other mechanisms, the Protocol endorses emission trading (Article 17) ([3]; see also [10]).

The Kyoto Protocol also mentions uncertainty. However, it does not put uncertainty (and, thus, verification) at the centre of its efforts to slow global warming ([14,15]). So far, the number of countries that have quantified and made their uncertainty assessments available is limited to Austria, Netherlands, Norway, Poland, Russia and the United Kingdom ([18, 11, 1, 17, 5, 13, 7, 2]). Their uncertainty estimates are summarized in Table 1.

These findings signal difficulties associated with calculation of the net emission changes on the country level. The uncertainty estimates published by these countries so far show that they may dwarf committed limitations or reductions. Therefore, hitting a "Kyoto target" provides little information if uncertainties are great, as it is also probable that the countries' emissions lie above or below their respective targets. The situation is even more difficult because also the targets are not exactly known due to the countries' uncertain emissions in the base year.

The idea developed in this paper starts with the observation that, to be credible, a country's uncertain emission estimate should undershoot the reduction target in the commitment period, in proportion to the amount of uncertainty that is assigned to its net emissions. Our proposition starts with setting up the concept of undershooting and specifying the risk that the country's real (unknown) emissions may actually be above the original target value.

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[1] For some countries the base year is different from 1990.

Table 1. First-order comparison by country: Quantified total uncertainties of net emissions for 1990, which are compared with the countries' emission limitation/reduction commitments.

Country	Reduction Commitment[a)] [%]	Total Uncertainty [%]	GHGs Considered	LUCF[b)] In- or Excluded	Reference
AT	8 (13)	~ 12 ~ 9.8	CO_2, CH_4, N_2O	included excluded	[18] Tab. 3
		~ 15 ~ 7.5		included excluded	[12] Tab. 14
NL	8 (6)	~ 5	all[e)]	included	[1] p. xxi
NO	-1	~ 21	all[e)]	excluded	[17] Tab. 4
PL	6	~ 6	CO_2, CH_4, N_2O	included	[5] Tab. 3
RU	0	~ 17[c)] (energy sector)	CO_2	included	[14] p. 158
UK	8 (12.5)	~ 42[d)] ~ 19	all[e)]	included excluded	[8] Tab. 1

[a)] A positive number refers to a committed reduction in the emissions and a negative number to a limitation in their increase. A number in parentheses refers to a national GHG emission target agreed upon under the EU burden sharing.

[b)] LUCF stands for *Land Use Change and Forestry*.

[c)] The uncertainties reported by Nilsson *et al.* [13] in their Russian case study are presently scrutinized as new knowledge is unfolding, which may justify their reduction.

[d)] This uncertainty is derived by applying the law of uncertainty (error) propagation. [7] reports a total relative uncertainty of 19% for all emissions by sources and 38% for all removals by sinks.

[e)] All gases as mentioned in Annex A to the Kyoto Protocol [3].

Based on this work, we then address the issue of emission trading in the presence of uncertainty. Purchasing excess reductions must take into account the inaccuracy of the sellers' emissions. As a consequence, excess emission reductions of a country exhibiting great uncertainty should be cheaper than those of a country exhibiting small uncertainty. Our idea starts with appropriate correction for the buyers' uncertainty and ends with the formulae for correction of the traded amounts of emission.

The idea of undershooting committed emission reduction targets has already been addressed in [16] and [6]. However, in this paper, undershooting emerges as a result of specifying the risk of not satisfying emission limitation or reduction commitments. Apart of the interval uncertainty, we consider the stochastic uncertainty in the reported emissions, and grasp changes in net emissions. In our study we do not consider a market optimisation model (e.g., based on bilateral trade) as in [6]. However, we demonstrate the necessity of considering uncertainty in the trading process. In our proposition trading rules are influenced as the costs of bargained emission units change due to the uncertainty of emission observation.

2. Testing Committed Changes in Emissions

2.1. Notations and problem presentation

We denote true emissions expressed in CO_2 equivalents (CO_2eq), which are a function of time, by $x(t)$. The emissions in the base year t_0 are denoted by $x(t_0) = x_0$. The years 2008–2012, during which GHG net emission accounts will have to be tested, are denoted by T_i, where for brevity i = 8–12. Here we study the problem of testing with respect to any individual year during this period. However, our methodology is also applicable if net emissions are aggregated over the commitment period and represented, e.g., by their arithmetic mean. This only requires prior agreement of how the emissions and their corresponding uncertainties are compounded.

We consider true emissions to be unknown, that is, they can only be estimated. Hats, i.e., $\hat{x}(t)$ mark the estimated emissions that are reported by Annex I Parties. We note that the reported emissions can be utilized for more sophisticated estimates of $x(t)$, the procedure of which we present elsewhere. Here we presuppose that this step has already been carried out.

The estimated emissions are contaminated by errors due to reporting. Inaccuracies inherent in the emission estimates can be treated differently, e.g., with the help of uncertainty intervals, or in a stochastic or fuzzy manner. The choice of handling these inaccuracies is important, as it determines the rules of calculus that have to be applied.

Let us denote the fraction of a committed emission reduction by δ. Thus, in the ideal case of perfect knowledge, emissions in the year T_i should not be greater than $(1-\delta)x_0$. The problem arising is that we can not directly compare $x(T_i)$ and $(1-\delta)x_0$ but can only calculate the difference

$$\hat{x}(T_i) - \hat{x}_0(1-\delta), \tag{1}$$

where the two observed emission values $\hat{x}(T_i)$ and $\hat{x}_0$ are inaccurately known.

In addition, let us introduce the risk α ($0 \le \alpha \le 0.5$) that the emission reduction in the year T_i is not fulfilled. The lower bound ($\alpha = 0$) corresponds to the case where the reduction target is undershot by one half of the respective two-sided uncertainty interval or distribution. The upper value ($\alpha = 0.5$) corresponds to the case of completely ignoring uncertainty. We assume that the value of α is set beforehand and that it is valid for all Parties.

2.2. Interval uncertainty

In the first case that we consider, the difference between the true (but unknown) emissions x_0 and our best estimate $\hat{x}_0$ at T_0 as well as the corresponding difference between $x(T_i)$ and $\hat{x}(T_i)$ at T_i are known to be smaller than some upper bounds. Let us assume that these uncertainty bounds are given by:

$$|x_0 - \hat{x}_0| \le \Delta_0, \quad |x(T_i) - \hat{x}(T_i)| \le \Delta_i. \tag{2a,b}$$

By using interval calculus or the triangle inequality, the combined interval uncertainty of $\{(\hat{x}_0(1-\delta) - \hat{x}(T_i)) - (x_0(1-\delta) - x(T_i))\} = \{(1-\delta)(\hat{x}_0 - x_0) + (x(T_i) - \hat{x}(T_i))\}$ at time T_i is Δ_{0i}, where $\Delta_{0i} = (1-\delta)\Delta_0 + \Delta_i$, and the difference between the actual emissions and its corresponding target at T_i can be bounded by:

$$x(T_i) - x_0(1-\delta) \in [D\hat{x} - \Delta_{0i}, D\hat{x} + \Delta_{0i}], \tag{3}$$

where $D\hat{x} = \hat{x}(T_i) - \hat{x}_0(1-\delta)$. To be sure that $x(T_i) - x_0(1-\delta) \le 0$ we have to require $D\hat{x} + \Delta_{0i} \le 0$. By accepting the risk α that $x(T_i)$ is actually greater than $x_0(1-\delta)$, the following condition holds (see Fig. 1):

$$D\hat{x} + \Delta_{0i} \leq 2\alpha\Delta_{0i}$$

Substituting for $D\hat{x}$ and rearranging yields:

$$\hat{x}(T_i) \leq \hat{x}_0(1-\delta) - (1-2\alpha)\Delta_{0i} \qquad (4)$$

which necessitates $D\hat{x} \leq 0$. As $(1\text{-}\delta)\Delta_0$ is typically close to Δ_i, $\Delta_{0i} \approx 2\Delta_i$. Thus, if α is small, inequality (4) requires that the best estimate of the target, $\hat{x}_0(1-\delta)$, is undershot by almost $2\Delta_i$. If a greater risk α is accepted, the required undershooting is considerably reduced.

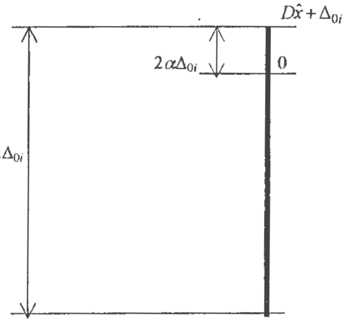

Figure 1: Determination of the condition for fulfilment of the risk α.

2.3. Stochastic uncertainty

In the second case, we assume that the true (but unknown) emissions x_0 and $x(T_i)$ can be grasped with the help of the uncertainty distributions that underlie $\hat{x}_0$ and $\hat{x}(T_i)$, with the mean values $E(\hat{x}_0) = x_0$ and $E(\hat{x}(T_i)) = x(T_i)$, and with $\sigma_{\hat{x}}(0)$ and $\sigma_{\hat{x}}(T_i)$ as their standard deviations. The derivation below is valid for arbitrary distributions with finite variances provided $(x_0(1-\delta) - x(T_i))$ is the median of the distribution $(\hat{x}_0(1-\delta) - \hat{x}(T_i))$, as it is, for example, the case for any symmetric distributions of $\hat{x}_0$ and $\hat{x}(T_i)$.

As before, we require that the probability of not satisfying emission limitation or reduction commitments is α ($0 \leq \alpha \leq 0.5$). This can be written as

$$P\left(\frac{(\hat{x}_0(1-\delta) - \hat{x}(T_i)) - (x_0(1-\delta) - x(T_i))}{\sigma_{\hat{x}}} \geq q_{1-\alpha}\right) = \alpha\ , \qquad (5)$$

where $\sigma_{\hat{x}}$ is the standard deviation of the distribution of the variable $(\hat{x}_0(1-\delta) - \hat{x}(T_i))$ and $q_{1-\alpha}$ is the $(1\text{-}\alpha)$th quantile of the distribution of the corresponding standardized variable. We note that due to standardization $q_{0.5} = 0$.

According to the rules for calculating the variance of the linear combination of two random variables, we find

$$\sigma_{\hat{x}}^2 = (1-\delta)^2\sigma_{\hat{x}}^2(0) - 2(1-\delta)\rho_{0i}\sigma_{\hat{x}}(0)\sigma_{\hat{x}}(T_i) + \sigma_{\hat{x}}^2(T_i)\ , \qquad (6)$$

where $\sigma_{\hat{x}}^2(0)$ is the variance of $\hat{x}_0$, $\sigma_{\hat{x}}^2(T_i)$ the variance of $\hat{x}(T_i)$, and ρ_{0i} the correlation of $\hat{x}_0$ and $\hat{x}(T_i)$.

Equation (5) provides the condition that needs to be satisfied with respect to the risk α:

$$\hat{x}(T_i) \le \hat{x}_0(1-\delta) - x_0(1-\delta) + x(T_i) - q_{1-\alpha}\sigma_{\hat{x}} \ . \tag{7}$$

As our goal is to achieve $x(T_i) = x_0(1\text{-}\delta)$, let us tentatively assume $-x_0(1\text{-}\delta) + x(T_i) = 0$, which yields for Equation (7)

$$\hat{x}(T_i) \le \hat{x}_0(1-\delta) - q_{1-\alpha}\sigma_{\hat{x}} \ . \tag{8}$$

When our tentative assumption is not true and if $-x_0(1\text{-}\delta) + x(T_i) > 0$, then (8) implies (7). But if $-x_0(1\text{-}\delta) + x(T_i) < 0$, then the Kyoto obligation is actually fulfilled. Thus, the condition (8) with all variables known can be used instead of (7). Similarly to the interval uncertainty case it necessitates $D\hat{x} \le 0$.

The calculation of $q_{1-\alpha}$ depends on the probability distributions of $\hat{x}_0$ and $\hat{x}(T_i)$, and can be computed according to known rules. For the standardized normal distribution the quantiles are tabulated for different α's.

2.4. Reducing overshooting requirement

The value δ is the reduction fixed in the Kyoto protocol. Within the methodology proposed up to now, it is required from all countries to undershoot their reduction target. Such additional requirement is costly and may cause objections.

To overcome this difficulty, a smaller δ than fixed in the Kyoto protocol can be used. We propose to use the redefined value δ_D that is constructed as follows. Consider a hypothetical country with the emission value $\hat{x}(T_i)$ equal to the Kyoto target and with a reference (e.g. average, minimal) uncertainty Δ_s. This country is considered to be fulfilling the obligations, see Fig. 2. Also all other countries with the relative uncertainty distribution mass above the level $\hat{x}_0(1-\delta_D)$ smaller than the agreed risk α are considered to fulfil the obligation. Thus, in fact, the value δ_D replaces in our approach the original value, which we denote in this subsection as δ_0.

For the interval uncertainty the condition to be satisfied by δ_D is as follows:

$$\hat{x}_0(1-\delta_D) - \hat{x}_0(1-\delta_0) = (1-2\alpha)\Delta_s$$

where Δ_s is the reference interval uncertainty. Rearranging we get the redefined reduction as:

$$\delta_D = \delta_0 - (1-2\alpha)v_{s0}$$

where $v_{s0} = \Delta_s / \hat{x}_0$ is the relative reference uncertainty.

Discussing the stochastic uncertainty, we restrict our attention to the normal distribution. Let δ_0 be the original Kyoto reduction, σ_s the reference standard deviation of the normal distribution (i.e. the reference distribution is $N_s\left(\hat{x}_0(1-\delta_0),\sigma_s^2\right)$). Then the proposed redefined reduction emission value δ_D satisfies

$$\frac{\hat{x}_0(1-\delta_D) - \hat{x}_0(1-\delta_0)}{\sigma_s} = q_{s,1-\alpha}$$

where $q_{s,1-\alpha}$ is the $(1-\alpha)$th quantile of the standard normal distribution N_S. Then, rearranging yields

$$\delta_D = \delta_0 - \frac{q_{s,1-\alpha}\sigma_s}{\hat{x}_0} = \delta_0 - q_{s,1-\alpha}v_s \qquad (9)$$

where $v_s = \sigma_s / \hat{x}_0$ is the relative standard deviation.

Under the above regulation, a country with big uncertainty may be still considered as not fulfilling the obligations even if it actually reports undershooting of the Kyoto reduction target, see Fig. 2. But a country, which has not achieved the target value, may be considered as fulfilling the obligations, if only its uncertainty is small enough.

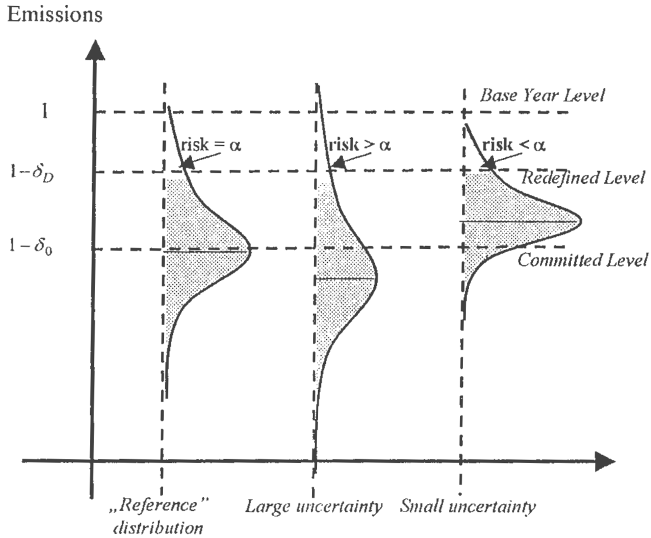

Figure 2. Definition of the redefined level (left) and situation of two countries: reporting undershooting of the Kyoto obligations but with big uncertainty that is considered not to fulfil the obligations (centre) and reporting emission over the committed level with small uncertainty that is considered to fulfil the obligations (right).

2.5. Consequences of specifying uncertainty differently

In this section we investigate three uncertainty models, which refer to the interval uncertainty, and the stochastic uncertainties with the uniform and normal distributions (see Fig. 3). The axes on the figure are normalized, so to get the real dimensions the x-axis should be multiplied and the y-axis divided by Δ.

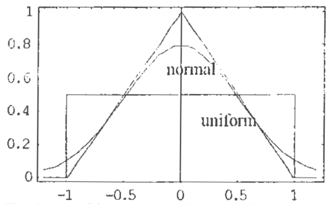

Fig.3. Considered probability functions.

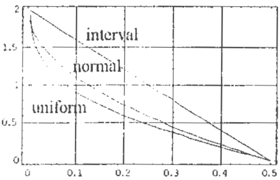

Fig.4. δ_α / ν as a function of α and in dependence of how uncertainty has been treated (deterministically or stochastically, in the latter case with a uniform and normal distribution).

To render the comparison possible, we assume that the interval of uncertainty is $[-\Delta, \Delta]$ and that the uniform distribution is defined with respect to this interval. Thus, the standard deviation of the uniform distribution is $\sigma_u = \Delta / \sqrt{3}$. For the normal distribution we take $\sigma_n = \Delta / 2$. Moreover, we assume that $\Delta_i = (1-\delta)\Delta_0 = \Delta$ in the deterministic case and $\sigma_{\hat{x}}(T_i) = \sigma_{\hat{x}}(0)(1-\delta) = \sigma$ in the stochastic case. Additionally, we assume that in the stochastic case the variables $\hat{x}_0$ and $\hat{x}(T_i)$ are uncorrelated.

Let us now consider the difference $\hat{x}_0(1-\delta) - \hat{x}(T_i)$. Its uncertainty for the interval case is 2Δ. In the stochastic case, for the uniform distributions of both $\hat{x}_0$ and $\hat{x}(T_i)$ the distribution of the difference is triangular with $\sigma_{\hat{x}} = \sqrt{\frac{2}{3}}\Delta$. For the normal distributions of the above variables the distribution of the difference is normal with $\sigma_{\hat{x}} = \Delta / \sqrt{2}$.

Table 2. New reduction commitments $\delta + \delta_\alpha$ (in %) for the countries listed in Table 1 and α = 0.1 and α = 0.3.

Country	Reduction Commit-ment	New Reduction Commitment $(\delta + \delta_\alpha)$ [%]					
		$\alpha = 0.1$			$\alpha = 0.3$		
		interval	uniform	normal	interval	uniform	normal
AT	8	27.2	21.2	18.9	17.6	13.4	12.4
		23.7	18.8	16.9	15.8	12.4	11.6
		32.0	24.5	21.7	20.0	14.8	13.6
		20.0	16.3	14.8	14.0	11.4	10.8
NL	8	16.0	13.5	12.6	12.0	10.3	9.9
NO	-1	32.6	22.1	18.1	15.8	8.5	6.8
PL	6	15.6	12.6	11.5	10.8	8.7	8.2
RU	0	27.2	18.7	15.5	13.6	7.7	6.3
UK	8	75.2	54.2	46.2	41.6	26.9	23.5
		38.4	28.9	25.3	23.2	16.6	15.0

We now introduce a common notation for the deterministic and stochastic cases, namely $\delta_\alpha = (1-2\alpha)\Delta_{0i} / \hat{x}_0$ and $\delta_\alpha = q_{1-\alpha}\sigma_{\hat{x}} / \hat{x}_0$, respectively. The new target to be reached is then equal to $\hat{x}_0(1-\delta-\delta_\alpha)$. Dividing δ_α by $\nu=\Delta/\hat{x}_0$ we obtain a dimensionless coefficient, which characterises the influence of the selected approach of treating uncertainties (deterministically or stochastically, in the latter case with a uniform, triangular or normal distribution) on α, the risk associated with the undershooting δ_α. The dependence of δ_α/ν on α is shown in Fig. 4. Taking the values of ν for different countries from Table 1 (see column *Total Uncertainty*), we can derive with the help of Fig. 4 the values of δ_α and then, in a next step, the new reduction commitments $\delta+\delta_\alpha$. These are listed in Table 2 for $\alpha=0.1$ and $\alpha=0.3$ in dependence of the various ν values specified in Table 1. The undershooting is smaller in the stochastic case, to big extend due to the different rules of summing up of the stochastic variables. It will decrease further if the variables $\hat{x}_0$ and $\hat{x}(T_i)$ are positively correlated.

3. Emission trading

3.1. Interval uncertainty

The methodology proposed for the verification of Kyoto limitation and reduction commitments will also influence emission trading. Let us consider two countries. Country 1 emits $x_1(T_i)$ units of GHG emissions with an interval uncertainty of $\Delta_{i,1}$. It needs to buy $\hat{E}$ units of additionally avoided emissions from country 2, which emits $x_2(T_i)$ units of GHG emissions with the interval uncertainty of $\Delta_{i,2}$. To begin with, let us agree on the following conditions:

$$|x_1(T_i)-\hat{x}_1(T_i)|\le\Delta_{i,1},\qquad |E-\hat{E}|\le\hat{r}\Delta_{i,2},\qquad \hat{r}=\frac{\hat{E}}{\hat{x}_2(T_i)}<1$$

Let us notice that $\hat{r}\Delta_{i,2}=\hat{E}\frac{\Delta_{i,2}}{\hat{x}_2(T_i)}$ is actually easy to calculate.

The corrected emissions of country 1 after purchasing $\hat{E}$ excess reduction emission units from country 2 will be $\hat{x}_1(T_i)\ -\hat{E}$. The uncertainty underlying the emissions of country 1 has also to be corrected accordingly. It increases (applying interval calculus or the triangle inequality) to

$$(1-\delta_1)\Delta_{0,1}+\Delta_{i,1}+\hat{r}\Delta_{i,2}\ .$$

However, this way purchasing of the reduction would always increase the uncertainty interval of the buying country. To counteract this we can subtract from $\Delta_{i,2}$ a reference uncertainty Δ. Different values can be taken for Δ. Two obvious candidates would be a commonly agreed value Δ_s (i.e. a kind of a benchmark value) or the uncertainty of the buing country $\Delta_{i,1}$. The latter introduces a nice invariance property, that is, the uncertainty of a country buying the excess reduction from itself does not change.

Thus, our proposition for calculation of the uncertainty interval, called *the effective uncertainty interval*, of the buying country is:

$$\Delta_{eff}=(1-\delta_1)\Delta_{0,1}+\Delta_{i,1}+\hat{r}(\Delta_{i,2}-\Delta)=(1-\delta_1)\Delta_{0,1}+\Delta_{i,1}+\hat{r}\Delta_{i,2}(1-\zeta_2)$$

where $\zeta_2 = \Delta / \Delta_{i,2}$ is the ratio of the reference and the seller uncertainty intervals. When admitting $\Delta = 0$ we have $\zeta_2 = 0$ and therefore the entire buying country uncertainty is taken into account.

As a consequence, inequality (4) changes from

$$\hat{x}_1(T_i) + (1-2\alpha)[(1-\delta_1)\Delta_{0,1} + \Delta_{i,1}] \leq \hat{x}_{0,1}(1-\delta_1) \tag{10}$$

before emission trading to

$$\hat{x}_1(T_i) - \hat{E} + (1-2\alpha)[(1-\delta_1)\Delta_{0,1} + \Delta_{i,1} + \hat{r}\Delta_{i,2}(1-\zeta_2)] \leq \hat{x}_{0,1}(1-\delta_1) \tag{11}$$

after emission trading. Comparing inequalities (10) and (11), we see that they differ from each other by the component, which we call the *effective excess reduction* (of the selling country):

$$E_{eff} = \hat{E} - (1-2\alpha)\hat{r}\Delta_{i,2}(1-\zeta_2) = \hat{E}[1-(1-2\alpha)\nu_2(1-\zeta_2)] \quad \text{with} \quad \nu_2 = \frac{\Delta_{i,2}}{\hat{x}_2(T_i)} , \tag{12}$$

where $\hat{E}$ is the excess reduction and ν_2 the relative uncertainty underlying the emissions of country 2. Country 1 may be willing to pay for the effective excess reduction E_{eff} rather than for the excess reduction $\hat{E}$.

We can introduce *the effective relative uncertainty* of the selling country $\widetilde{\nu}_2 = \nu_2(1-\zeta_2)$, which can be also expressed as:

$$\widetilde{\nu}_2 = \nu_2 - \nu_2\zeta_2 = \nu_2 - \frac{\Delta_{i,2}}{\hat{x}_2(T_i)}\frac{\Delta}{\Delta_{i,2}} = \nu_2 - \nu_{s,2}$$

where $\nu_{s,2} = \Delta / \hat{x}_2(T_i)$ is the relative (with respect to $\hat{x}_2(T_i)$) reference uncertainty. Thus, we can also reformulate (12) as:

$$E_{eff} = \hat{E}[1-(1-2\alpha)\widetilde{\nu}_2] = \hat{E}[1-(1-2\alpha)(\nu_2 - \nu_{s,2})] \tag{13}$$

The effective excess reduction of a country is corrected by the term depending on its effective relative uncertainty $\widetilde{\nu}_2$. If $\Delta = 0$, then $\nu_{s,2} = 0$ and $\widetilde{\nu}_2 = \nu_2$. Depending on the sign of $\widetilde{\nu}_2$ the effective excess reduction E_{eff} may be smaller or bigger than $\hat{E}$.

The formula (13) can be simplified by introducing some abstract relative reference uncertainty ν_s. This carries us to the expression:

$$E_{eff} = \hat{E}[1-(1-2\alpha)(\nu_2 - \nu_s)] \tag{14}$$

The above expressions (12)-(14) give the transformation of the original quantity $\hat{E}$ offered for sale to the effective emission E_{eff} as envisaged by the buying country with respect to its need to fulfil the Kyoto obligations. This quantity may form the basis for financial liabilities between countries.

At the same time the effective uncertainty interval of the buying country is:

$$\Delta_{eff} = (1-\delta_1)\Delta_{0,1} + \Delta_{i,1} + \hat{E}\nu_2(1-\frac{\nu_{s,2}}{\nu_2}) = (1-\delta_1)\Delta_{0,1} + \Delta_{i,1} + \hat{E}\widetilde{\nu}_2$$

or, when turning to the relative uncertainties:

$$\nu_{1,eff} = (1-\delta_1)\nu_{0,1} + \nu_{i,1} + \hat{R}\widetilde{\nu}_2 = \nu_1 + \hat{R}\widetilde{\nu}_2 \tag{15}$$

where $\nu_{1,eff} = \Delta_{eff} / \hat{x}_1(T_i)$ and $\hat{R} = \hat{E} / \hat{x}_1(T_i)$. As before, we can simplifying above admitting ν_s in place of $\nu_{s,2}$. The effective uncertainties above are the original uncertainties corrected for the effective uncertainty of the selling country multiplied by the amount bought or the relative amount bought, respectively.

Taking $\nu_{s,2} = \nu_1$ gives:

$$\nu_{1,eff} = (1 - \hat{R})\nu_1 + \hat{R}\nu_2 ,$$

which is the weighted mean of the relative uncertainties of the selling and buying countries.

Table 3. Interval uncertainty effective excess reductions, E_{eff}, in percent of the excess reduction, $\hat{E}$, for the countries listed in Table 1 offering them for sale, where: $\alpha = 0.1$ and 0.3, and $\nu_{s,2} = \nu_s = 0.1$.

Country	Uncertainty	$E_{eff} / \hat{E}$ [%]	
	[%]	α=0.1	α=0.3
AT	12	98.4	99.2
	9.8	100.2	100.1
	15	96.0	98.0
	7.5	102.0	101.0
NL	5.0	104.0	102.0
NO	21	91.2	95.6
PL	6	103.2	101.6
RU	17	94.4	97.2
UK	42	74.4	87.2
	19	92.8	96.4

Table 4. Interval uncertainty effective reductions, E_{eff}, in percent of the excess reduction, $\hat{E}$, for the countries listed in Table 1 wishing to buy them from Poland and Russia, where: $\alpha = 0.1$ and 0.3, and $\nu_{s,2} = \nu_1$.

Country	Uncertainty	$E_{eff} / \hat{E}$ [%]			
		Poland		Russia	
		0,1	0,3	0,1	0,3
AT	12	104,8	102,4	96,0	98,0
	9,8	103,0	101,5	94,2	97,1
	15	107,2	103,6	98,4	99,2
	7,5	101,2	100,6	92,4	96,2
NL	5	99,2	99,6	90,4	95,2
NO	21	112,0	106,0	103,2	101,6
PL	6	100,0	100,0	91,2	95,6
RU	17	108,8	104,4	100,0	100,0
UK	42	128,8	114,4	120,0	110,0
	19	110,4	105,2	101,6	100,8

We quantify the effective excess reduction in Table 3 for the countries listed in Table 1 with $\alpha = 0.1$ and $\alpha = 0.3$, where we select a common reference relative uncertainty ν_s of 10%. Likewise, in Table 4 we present the same functions for $\nu_{s,2} = \nu_1$. The figures in Table 3 and 4 are the factors, in percents, for multiplication of the excess emission offered for sale by the countries listed at the table to get the effective emission, which can be directly subtracted from the emission of the buying country.

3.2. Stochastic uncertainty

To fulfill its emission limitation or reduction commitment, the emissions of the country 1 must satisfy

$$\hat{x}_1(T_i) + q_{1-\alpha}\sigma_{\hat{x}_1} \le \hat{x}_{0,1}(1-\delta_1) \ , \qquad (16)$$

where $\sigma_{\hat{x}_1}$ is given by Equation (6) and we assume that $q_{1-\alpha}$ is identical for all countries. In the case that country 1 needs to purchase $\hat{E}$ excess reduction units from country 2, the variance of the difference $\hat{x}_0(1-\delta) - [\hat{x}_1(T_i) - \hat{E}]$ in consideration of $\hat{E} = \hat{r}\hat{x}_2(T_i)$ (as before) and $\sigma^2_{\hat{x}_2}(T_i)$ as the variance of $\hat{x}_2(T_i)$ is

$$\sigma^2_{\hat{x}_1} + \hat{r}^2\sigma^2_{\hat{x}_2}(T_i) \ ,$$

where we assume that the countries' emissions, i.e., $\hat{x}_1$ and $\hat{x}_2$ are uncorrelated.

Similar to the interval case, we consider however the effective variances calculated as:

$$\sigma^2_{\hat{x}_1,eff} = \sigma^2_{\hat{x}_1} + \hat{r}^2[\sigma^2_{\hat{x}_2}(T_i) - \sigma^2] = \sigma^2_{\hat{x}_1} + \hat{r}^2\sigma^2_{\hat{x}_2}(T_i)(1-\zeta^2)$$

where σ is a reference standard deviation and $\zeta = \sigma / \sigma_{\hat{x}_2}(T_i)$. Thus, instead of (16), the new condition is

$$\hat{x}_1(T_i) - \hat{E} + q_{1-\alpha}\sqrt{\sigma^2_{\hat{x}_1} + \hat{r}^2\sigma^2_{\hat{x}_2}(T_i)(1-\zeta^2)} \le \hat{x}_0(1-\delta_1) \ .$$

The effective excess reduction E_{eff} can be expressed in analogy to Section 3.1 (see Equations (13) and (14)) as

$$E_{eff} = \hat{E}(1 - q_{1-\alpha}\nu_2\sqrt{1-\zeta^2}) = \hat{E}(1 - q_{1-\alpha}\tilde{\nu}_2) \qquad \text{for } \rho_{0i} = 1$$

$$E_{eff} = \hat{E}(1 - \frac{q_{1-\alpha}\hat{R}}{2\sqrt{2(1-\rho_{0i})}}\frac{\tilde{\nu}_2}{\nu_1}\tilde{\nu}_2) = \hat{E}(1 - \frac{q_{1-\alpha}\hat{R}}{2\sqrt{2(1-\rho_{0i})}}\frac{\nu_2^2 - \nu_{s,2}^2}{\nu_1}) \ . \qquad \text{for } \rho_{0i} \ll 1,$$

where $\nu_1 = \sigma_{\hat{x}_1}(T_i)/\hat{x}_1(T_i)$, $\nu_2 = \sigma_{\hat{x}_2}(T_i)/\hat{x}_2(T_i)$, $\tilde{\nu}_2^2 = \nu_2^2(1-\zeta^2) = \nu_2^2 - \nu_{s,2}^2$ and $\nu_{s,2} = \sigma / \hat{x}_2(T_i)$ are the appropriate relative uncertainties. $\hat{R} = \hat{E}/\hat{x}_1(T_i)$ is the purchased emissions of country 1, expressed as a fraction of its emissions at T_i, and ρ_{0i} is the correlation of $\hat{x}_{1,0}$ and $\hat{x}_1(T_i)$ (see Appendix). In contrast to the case of interval uncertainty (Section 3.1), the effective excess reduction E_{eff} now depends on the uncertainty of both country 1 (ν_1) and the effective uncertainty of the country 2 ($\tilde{\nu}_2$) and, in addition, on the purchased emission fraction $\hat{R}$. This is why we call E_{eff} *the effective excess reduction of country 2 for country 1.*

The effective relative variance can be now expressed as follows:

$$\nu_{1,eff}^2 = \nu_1^2 + \hat{R}^2\nu_2^2(1-\zeta^2) = \nu_1^2 + \hat{R}^2\tilde{\nu}_2^2$$

Taking $\nu_{s,2} = \nu_1$ we get:

$$E_{eff} = \hat{E}\left[1 - \frac{q_{1-\alpha}\hat{R}}{2\sqrt{2(1-\rho_{0i})}}\left(\frac{\nu_2^2}{\nu_1} - \nu_1\right)\right]. \quad (17)$$

However, unlike for the interval uncertainty, admission for $\nu_{s,2}$ of a common constant reference uncertainty ratio ν_s may give unreasonable results for $\rho_{0i} << 1$. Then we would have:

$$E_{eff} = \hat{E}\left(1 - \frac{q_{1-\alpha}\hat{R}}{2\sqrt{2(1-\rho_{0i})}}\frac{\nu_2^2 - \nu_s^2}{\nu_1}\right).$$

Let us consider now the case $\nu_2 < \nu_s$ and assume that there are two buying countries A and B with $\nu_{1A} > \nu_{1B}$. Then from the above expression we get $E_{eff}(\nu_{1a}) < E_{eff}(\nu_{1B})$, which counteracts intuition of fairness. Further assumptions can provide some reasonable approximations. For example, assuming $\nu_s / \nu_1 \cong 1$ gives:

$$E_{eff} = \hat{E}\left[1 - \frac{q_{1-\alpha}\hat{R}}{2\sqrt{2(1-\rho_{0i})}}\left(\frac{\nu_2^2}{\nu_1} - \nu_s\right)\right] \quad (18)$$

Table 5. Stochastic uncertainty effective excess reduction, E_{eff}, in percent of the excess reduction, $\hat{E}$, sold by Poland and Russia to the countries listed in Table 1, where: $\alpha = 0.1$ and 0.3, $\nu_{s,2} = \nu_1$, $R = 0.1$, $\rho_{0i} = 0$, and the emissions of all countries are normally distributed and uncorrelated to each other.

Country	Uncertainty [%]	$E_{eff} / \hat{E}$ [%]			
		Poland		Russia	
		α=0.1	α=0.3	α=0.1	α=0.3
AT	12	100,41	100,17	99,45	99,78
	9,8	100,28	100,11	99,11	99,63
	15	100,57	100,23	99,81	99,92
	7,5	100,12	100,05	98,59	99,42
NL	5	99,90	99,96	97,61	99,02
NO	21	100,87	100,36	100,33	100,13
PL	6	100,00	100,00	98,09	99,22
RU	17	100,67	100,28	100,00	100,00
UK	42	101,86	100,76	101,59	100,65
	19	100,78	100,32	100,17	100,07

In Table 5 we present the values of $E_{eff} / \hat{E}$ for both Poland and Russia, which offer their excess reductions $\hat{E}$ for sale, assuming $\rho_{0i} << 1$ and $\nu_{s,2} = \nu_1$. We keep the purchased emission fraction constant at $\hat{R} = 0.1$. The $E_{eff} / \hat{E}$ values for Poland are greater than 100% because Poland has smaller relative uncertainty (6%) than most of other countries. On the other hand, Russia exhibits a great relative uncertainty (17%) resulting in $E_{eff} / \hat{E}$ values mostly smaller than 100%.

Likewise, in Table 6 analogous factors are given for expression (18), with ν_s = 0,1. The approximation effects can be noticed there. For example, Russia has smaller relative uncertainty then United Kingdom. However, the factor $E_{eff} / \hat{E}$ is smaller then 100% for ν_1 = 19%.

Table 6. Stochastic uncertainty effective excess reduction, E_{eff}, in percent of the excess reduction, $\hat{E}$, sold by Poland and Russia to the countries listed in Table 1, where: α = 0.1 and 0.3, $\nu_{s,2} = \nu_s = 0.1$, R = 0.1, ρ_{0i} = 0, and the emissions of all countries are normally distributed and uncorrelated to each other.

Country	Uncertainty [%]	$E_{eff} / \hat{E}$ [%]			
		Poland		Russia	
		α=0.1	α=0.3	α=0.1	α=0.3
AT	12	100,32	100,13	99,36	99,74
	9,8	100,29	100,12	99,12	99,64
	15	100,34	100,14	99,58	99,83
	7,5	100,24	100,10	98,71	99,47
NL	5	100,13	100,05	97,83	99,11
NO	21	100,38	100,15	99,83	99,93
PL	6	100,18	100,07	98,27	99,29
RU	17	100,36	100,15	99,68	99,87
UK	42	100,41	100,17	100,14	100,06
	19	100,37	100,15	99,76	99,90

4. Conclusions

The paper addresses the problem of testing fulfilment of the Kyoto obligations. The present knowledge makes it obvious to us that fulfilment of the obligations cannot be confirmed without taking into account the uncertainty of the reported values. This paper addresses this problem and proposition of a solution is given.

The main idea of our proposition concentrates in replacing the emission reduction by a combination of the reduction and uncertainty. The exact proportions are related with the risk that the real emission has not satisfied the obligations.

Two basic ways of modelling the uncertainty: the deterministic (interval uncertainty) and the stochastic are considered. The deterministic case is easier to manipulate with. The stochastic case involves much more complicated formulae. But it is more in line with the IPCC recommendation to treat the emission uncertainties (7, 8) and it also provides much smaller shifts of the original reduction target.

Acceptation of the idea of testing proposed in the paper makes it necessary: 1) to introduce an initial/obligatory undershooting (possibly with the redefined Kyoto target, as discussed in section 2.4) before countries are permitted to sell their excess emission reductions, and 2) to change the emission trading rules. When bargaining the price, the buyer should combine the reduction of emission with uncertainty of its reporting, because both of them will count in the final testing of fulfilment of the Kyoto obligations. The paper contains a proposition of solving this problem. Specifically, a construction of an effective excess reduction value, corrected for uncertainty, is proposed as the basis for bargaining the price. The deterministic case provides us with the linear formula, the stochastic one ends with nonlinear formulae, depending also on factors characterising both bargaining parties.

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Appendix. Derivation of the effective excess emission for the stochastic case

Using the formula for the sum of variances of two variables we have for the stochastic approach

$$\sigma^2_{\hat{x}_1-\hat{E}}(T_i) = \sigma^2_{\hat{x}_1}(T_i) + \hat{r}^2\sigma^2_{\hat{x}_2}(T_i)$$

To fulfil the obligations, the original emission of the country 1 should satisfy the following condition

$$\hat{x}_1(T_i) + q_{1-\alpha}\sigma_{\hat{x}_1} \le \hat{x}_0(1-\delta_1)$$

where

$$\sigma^2_{\hat{x}_1} = (1-\delta_1)^2\sigma^2_{\hat{x}_1}(0) - 2(1-\delta_1)\rho_{0i}\sigma_{\hat{x}_1}(0)\sigma_{\hat{x}_1}(T_i) + \sigma^2_{\hat{x}_1}(T_i)$$

is the correlation coefficient of $\hat{x}_0$ and $\hat{x}(T_i)$. After purchasing $\hat{E}$ excess reduction units from the country 2 the effective variance of the difference $\hat{x}_0(1-\delta_1) - [x_1(T_i) - \hat{E}]$ is

$$\sigma^2_{\hat{x}_1} + \hat{r}^2\sigma^2_{\hat{x}_2}(T_i)(1-\zeta^2)$$

Thus, the new condition will be

$$\hat{x}_1(T_i) - \hat{E} + q_{1-\alpha}\sqrt{\sigma^2_{\hat{x}_1} + \hat{r}^2\sigma^2_{\hat{x}_2}(T_i)(1-\zeta^2)} \le \hat{x}_0(1-\delta_1)$$

This can be written in the form

$$\hat{x}_1(T_i) - \hat{E} + q_{1-\alpha}\sigma_{\hat{x}_1} + q_{1-\alpha}\left(\sqrt{\sigma^2_{\hat{x}_1} + \hat{r}^2\sigma^2_{\hat{x}_2}(T_i)(1-\zeta^2)} - \sigma_{\hat{x}_1}\right) \le \hat{x}_0(1-\delta_1)$$

The component in the parenthesis, denoted as P, is transformed as follows

$$P = \sqrt{\sigma^2_{\hat{x}_1} + \hat{r}^2\sigma^2_{\hat{x}_2}(T_i)(1-\zeta^2)} - \sigma_{\hat{x}_1} =$$

$$= \hat{r}\sigma_{\hat{x}_2}(T_i)\sqrt{1-\zeta^2}\left(\sqrt{\frac{\sigma_{\hat{x}_1}^2}{\hat{r}^2\sigma_{\hat{x}_2}^2(T_i)(1-\zeta^2)}+1}-\frac{\sigma_{\hat{x}_1}}{\hat{r}\sigma_{\hat{x}_2}(T_i)\sqrt{1-\zeta^2}}\right)=$$

$$= \frac{\hat{r}\sigma_{\hat{x}_2}(T_i)\sqrt{1-\zeta^2}}{\sqrt{\frac{\sigma_{\hat{x}_1}^2}{\hat{r}^2\sigma_{\hat{x}_2}^2(T_i)(1-\zeta^2)}+1}+\frac{\sigma_{\hat{x}_1}}{\hat{r}\sigma_{\hat{x}_2}(T_i)\sqrt{1-\zeta^2}}}$$

We have

$$\hat{r}\sigma_{\hat{x}_2}(T_i) = \hat{E}\frac{\sigma_{\hat{x}_2}(T_i)}{\hat{x}_2(T_i)} = \hat{E}v_2$$

where now

$$v_2 = \frac{\sigma_{\hat{x}_2}(T_i)}{\hat{x}_2(T_i)}$$

is the relative uncertainty of the country 2 at time T_i. Although formally v_2 here is different from the relative uncertainty used in the interval uncertainty case, it will be convenient to keep the same notation for both cases. This should not cause any confusion.

Furthermore, it holds

$$\frac{\sigma_{\hat{x}_1}^2}{\hat{r}^2\sigma_{\hat{x}_2}^2(T_i)} = \frac{(1-\delta_1)^2\sigma_{\hat{x}_1}^2(0) - 2(1-\delta_1)\rho_{0i}\sigma_{\hat{x}_1}(0)\sigma_{\hat{x}_1}(T_i) + \sigma_{\hat{x}_1}^2(T_i)}{\hat{r}^2\sigma_{\hat{x}_2}^2(T_i)} =$$

$$= (1-\delta_1)^2\frac{\hat{x}_1^2(0)}{\hat{x}_1^2(T_i)}\frac{\frac{\sigma_{\hat{x}_1}^2(0)}{\hat{x}_1^2(0)}}{\frac{\sigma_{\hat{x}_2}^2(T_i)}{\hat{x}_2^2(T_i)}}\frac{\hat{x}_1^2(T_i)}{\hat{E}^2} - 2(1-\delta_1)\rho_{0i}\frac{\hat{x}_1(0)}{\hat{x}_1(T_i)}\frac{\frac{\sigma_{\hat{x}_1}(0)}{\hat{x}_1(0)}\frac{\sigma_{\hat{x}_1}(T_i)}{\hat{x}_1(T_i)}}{\frac{\sigma_{\hat{x}_2}^2(T_i)}{\hat{x}_2^2(T_i)}}\frac{\hat{x}_1^2(T_i)}{\hat{E}^2} + \frac{\frac{\sigma_{\hat{x}_1}^2(T_i)}{\hat{x}_1^2(T_i)}}{\frac{\sigma_{\hat{x}_2}^2(T_i)}{\hat{x}_2^2(T_i)}}\frac{\hat{x}_1^2(T_i)}{\hat{E}^2} =$$

$$= \frac{v_{10}^2}{v_2^2}\frac{1}{\eta^2\hat{R}^2} - 2\rho_{0i}\frac{v_{10}v_1}{v_2^2\eta\hat{R}^2} + \frac{v_1^2}{v_2^2}\frac{1}{\hat{R}^2} = \frac{1}{\hat{R}^2v_2^2}\left(\frac{v_{10}^2}{\eta^2} - 2\rho_{0i}\frac{v_{10}v_1}{\eta} + v_1^2\right) =$$

$$= \frac{1}{\hat{R}^2v_2^2}\left[\left(\frac{v_{10}}{\eta} - v_1\right)^2 + 2\frac{v_{10}v_1}{\eta}(1-\rho_{0i})\right]$$

where v_1 is the relative uncertainty of the country 1 at time T_i, v_{10} is the relative uncertainty of the country 1 at time 0. The reduction factor

$$\eta = \frac{\hat{x}_1(T_i)}{(1-\delta_1)\hat{x}_1(0)}$$

will be close to 1 and even smaller, as the country 1 must at least fulfil the obligations to sell the excess reduction. Moreover

$$\hat{R} = \frac{\hat{E}}{\hat{x}_1(T_i)}$$

is the purchased fraction of emission of the country 1. It will be close, and possibly even smaller than δ, and therefore of the order of few percent.

We can assume that ν_{10}/η does not differ significantly from ν_1. So, we drop $\nu_{10}/\eta - \nu_1$. Thus, the expression for P can be simplified to

$$P = \frac{\hat{E}\widetilde{\nu}_2}{\sqrt{\frac{2\nu_1^2}{\hat{R}^2\widetilde{\nu}_2^2}(1-\rho_{0i})+1} + \sqrt{\frac{2\nu_1^2}{\hat{R}^2\widetilde{\nu}_2^2}(1-\rho_{0i})}}$$

where $\widetilde{\nu}_2 = \nu_2\sqrt{1-\zeta^2}$.

If $\rho_{0i} = 1$, then $P = \hat{E}\widetilde{\nu}_2$. Arguing as for the interval uncertainty we get

$$E_{eff} = \hat{E}(1 - q_{1-\alpha}\widetilde{\nu}_2) \qquad \text{for } \rho_{0i} = 1$$

in close analogy to (14).

If $\rho_{0i} \neq 1$, then we have

$$P = \frac{\hat{E}\hat{R}\widetilde{\nu}_2^2}{\nu_1\sqrt{2(1-\rho_{0i})}} \frac{1}{\sqrt{1+\frac{1}{S}}+1}$$

where

$$S = \frac{2\nu_1^2}{\hat{R}^2\widetilde{\nu}_2^2}(1-\rho_{0i})$$

The value R will be usually of the order of few percents while $\widetilde{\nu}_2/\nu_1$ is not more than, say, 5-6, and often smaller. Thus, for ρ_{0i} small enough, $1/S$ is negligible with respect to 1. This will be the case for most practical cases and then we can use the approximate formula

$$E_{eff} = \hat{E}(1 - \frac{q_{1-\alpha}\hat{R}}{2\sqrt{2(1-\rho_{0i})}}\frac{\widetilde{\nu}_2}{\nu_1}\widetilde{\nu}_2) \qquad \text{for } \rho_{0i} << 1$$

In particular, for $\rho_{0i} = 0$ ($\hat{x}_0$ and $\hat{x}(T_i)$ uncorrelated)

$$E_{eff} = \hat{E}\left(1 - q_{1-\alpha}\frac{\sqrt{2}}{4}\hat{R}\frac{\widetilde{\nu}_2}{\nu_1}\widetilde{\nu}_2\right) \qquad \text{for } \rho_{0i} = 0$$

Also this expression resembles that of (14). However, now the effective excess reduction depends on the uncertainty ratios of both countries and, moreover, on the purchased fraction R. This is why we call *it the effective excess reduction of the country 2 for the country 1.*

Let us notice that if we assume that $\hat{x}_1(0)$ is known exactly, then the coefficient $\sqrt{2}/4$ will be replaced by $1/2$.