Raport Badawczy
Research Report

RB/37/2011

Modeling uncertainty structure of greenhouse gases inventories-revised. Analysis of national inventory reports data for EU countries

Z. Nahorski, J. Jarnicka

Instytut Badań Systemowych
Polska Akademia Nauk

Systems Research Institute
Polish Academy of Sciences

POLSKA AKADEMIA NAUK

Instytut Badań Systemowych

ul. Newelska 6

01-447 Warszawa

tel.: (+48) (22) 3810100

fax: (+48) (22) 3810105

Kierownik Zakładu zgłaszający pracę:
Prof. zw. dr hab. inż. Zbigniew Nahorski

Warszawa 2011

MODELING UNCERTAINTY STRUCTURE OF GREENHOUSE GASES INVENTORIES – REVISITED. ANALYSIS OF NATIONAL INVENTORY REPORTS DATA FOR EU COUNTRIES

ZBIGNIEW NAHORSKI, JOLANTA JARNICKA
SYSTEMS RESEARCH INSTITUTE, POLISH ACADEMY OF SCIENCES
NEWELSKA 6, 01-447 WARSAW, POLAND
EMAIL: ZBIGNIEW.NAHORSKI@IBSPAN.WAW.PL
EMAIL: JOLANTA.JARNICKA@IBSPAN.WAW.PL

ABSTRACT. The paper addresses the problem of reducing uncertainty in national greenhouse gases inventories. We analyze the annual reports, containing data on GHG emission, from a given year and revisions of past data. These reports are provided by national centres for reporting greenhouse gases inventories according to the UNFCCC. Our goal is to investigate how the uncertainty of inventory reports changes over the consecutive yearly revisions. This is done by proposing a parametric model that describes the structure of uncertainties in the inventories. A procedure for estimating parameters is described and preliminary results of fitting the model to the data from several EU countries are given.

Keywords: greenhouse gases inventories, uncertainty, modeling

1. INTRODUCTION

According to the United Nations Framework Convention on Climate Change (UNFCCC), each of the cosignatories is obliged to provide annual data on greenhouse gas inventory. Independent data on emission of greenhouse gases for various countries are also gathered by the Carbon Dioxide Information Analysis Center (CDIAC), in Oak Ridge, USA. In both cases, each report contains data from a given year and revisions of past data, whenever required. Data for previous years are revised when more precise information is obtained. This means that revisions made in different years use different knowledge, and hence uncertainties in different revisions are incomparable.

The question therefore arises, whether it is possible to compare and organize data on GHG emission, as well as model time evolution of inventory uncertainties, to get as much information as possible. The point is to estimate the uncertainty in national emission inventories, taking into account both the data revised in consecutive years[1] and possible correlations between inventories for neighbouring countries.

The article is a continuation of the considerations carried out in [9], where an attempt to analyze, how the uncertainty of inventory reports changes over the consecutive yearly revisions, was made. At the same time it is a reference to the results of [2], although the latter work presented an approach from a different point of view.

Our studies, carried out so far in [9], enabled better understanding of the nature of the data and their interpretation in a way that allowed us to introduce an appropriate model. The earlier approach in which we considered only the linear relationship between the parameters of the model adopted and the years when the data were revised, did not prove to be sufficiently good. Also introducing the elements of nonlinearity, by using the square root did not help a lot. However, the analysis conducted and our attempts to fit different parameters depending on the year of revision, gave a clue to improve the model. It was necessary to use several stages of

[1]A preliminary methodology for this has already been proposed by Khrystyna Hamal (Boychuk) in her IIASA Interim Report [2].

estimation, where the previous approach we used during the initial estimation. We also had to reach for other tools, such as nonlinear regression or the AR model. In such a – revised form, the model is presented in this report and applied to the data.

In Section 2 we explain the way of interpreting the data and introduce a parametric model, that describes the structure of uncertainties in the inventories. Section 3 describes a method for estimating parameters in this model. Section 4 contains the results of fitting the model to data from the national inventory reports for several EU countries. Summary and conclusions of the study are given in Section 5.

2. Presentation of the idea

We analyze data from the national inventory reports. Let $E^n_{y,i}$ – denote the inventory data for the country i, in the year n revised in the year y, and let Y – denote the last year, when the revision is made. For a given country i all the inventory data form a table, in which each row contains consecutive revisions of the data for a given year (see Table 1).

$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\dots$	$E^{n-1}_{y,i}$	$E^n_{y,i}$	$E^{n+1}_{y,i}$	$\dots$	$E^y_{y,i}$	0	$\dots$	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\dots$	$E^{n-1}_{Y,i}$	$E^n_{Y,i}$	$E^{n+1}_{Y,i}$	$\dots$	$E^y_{Y,i}$	$E^{y+1}_{Y,i}$	$\dots$	$E^Y_{Y,i}$

Table 1. Indexing the data.

We use the fact that, each revision data, for a given country, forms a realization of a stochastic process. These stochastic processes for a fixed country are different, but related (see Figure 1). They form a bunch of stochastic processes.

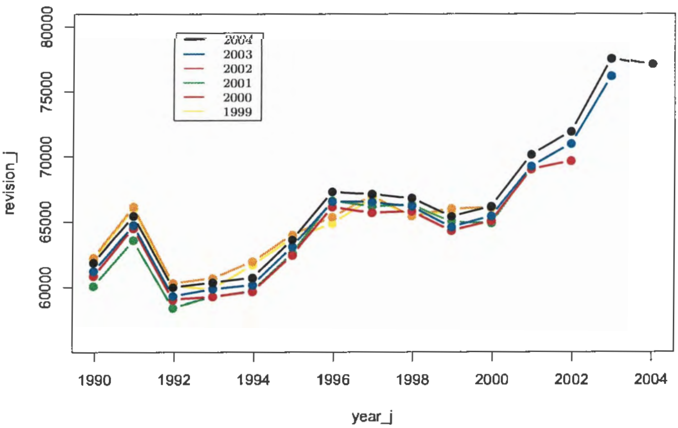

Figure 1. Revisions of the data for Austria, made in 1999-2005; data from National Inventory Reports.

For a given country i, we model any revision data to be composed of the "real" emission, which we call the "deterministic" fraction and a "stochastic" fraction, related to our lack of knowledge and imprecision of observation of the real emission. We assume that the uncertainty is related to the stochastic part of the model. For simplicity assume that we work on absolute errors, i.e. that the stochastic part is expressed in the units of weight. Following the notation from [2], we can write

$$E^n_{Y,i} = D^n_{Y,i} + S^n_{Y,i}, \qquad S^n_{Y,i} \sim \mathcal{N}(0, \sigma_{Y,i}),$$

where E – stands for the emission inventory, D – for its deterministic fraction, S – for the stochastic fraction, and n – is the year, for which the revised data were recalculated.

Now, the data revised in the year $y < Y$ are modeled as having the same deterministic fraction. Thus they follow the same type of decomposition

$$E^n_{y,i} = D^n_{Y,i} + S^n_{y,i}, \qquad \text{with} \quad S^n_{y,i} \sim \mathcal{N}\big(m^n_{y,i}, \sigma^n_{y,i}\big).$$

where the means $m^n_{y,i}$ and the standard deviations $\sigma^n_{y,i}$ are of the form

$$m^n_{y,i} = a_i(Y - y), \qquad \sigma^n_{y,i} = \sigma_{Y,i} + b_i\, f(Y - y), \quad b_i \neq 0, \tag{1}$$

and f is a given function, such that $f(Y - y) > -\frac{\sigma_{Y,i}}{b_i}$.

The parameters a_i and b_i, associated with the stochastic fraction $S^n_{y,i}$, can be estimated from the data together with $\sigma_{Y,i}$. Parameter a_i describes a shift in the accuracy of the inventory gathering, and b_i – a shift of the precision level. They both depend on the difference between the revision year y and the most recent revision year Y, due to the learning. To find the deterministic fraction $D^n_{Y,i}$, the smoothing splines can be used, as presented in [8]. This approach, when applied to the most recently revised data $E^n_{Y,i}$ will give not only the estimate of the deterministic fraction, but also an estimate of the variance $\sigma^2_{Y,i}$.

The procedure for a given country i, can be presented in the algorithmic way.

1. For the most recently revised inventory data $E^n_{Y,i}$ calculate the smoothing spline $\mathrm{Sp}_{Y,i}$ and estimate the variance $\sigma^2_{Y,i}$ of the stochastic fraction $S^n_{Y,i}$.
2. Subtract the spline data $\mathrm{Sp}_{Y,i}$ from all earlier revisions, calculating differences $v^n_{y,i} = E^n_{y,i} - \mathrm{Sp}_{Y,i}, \quad y < Y$.
3. Estimate parameters a_i and b_i, and hence also $m^n_{y,i}$, and $\sigma^n_{y,i}$.

3. Estimators of the model parameters

In this section, we present a method for estimating parameters a_i and b_i in (3), for a fixed country. Fix i and Y, and consider differences v^n_j of the inventory data $E^n_{y_j,i}$ in the year n for $n = 1, \ldots, N_j$, revised in the year y_j, $j = 1, \ldots, J$, and the smoothing spline Sp_Y built on the data from the year Y, $v^n_j = E^m_{y_j,i} - \mathrm{Sp}_{Y,i}$. For some years the difference v does not exist, due to lack of revised inventories in this year. These years are skipped from the sequence of N_j data. Assume that

$$v^n_j \sim \mathcal{N}\Big(m_j, \sigma_j\Big), \quad n = 1, \ldots, N_j, \quad j = 1, \ldots, J, \tag{2}$$

where

$$m_j = a(Y - y_j), \quad \sigma_j = \sigma_Y - b\,(Y - y_j)^{c+1}, \quad b \neq 0. \tag{3}$$

Assume also that differences (2) are independent. Our goal is to estimate the unknown parameters a, b, c, and σ_Y. Estimate for the variance σ^2_Y is obtained when creating the smoothing spline Sp_Y. Other parameters we estimate in a three-step procedure.

3.1. Estimation of sequences α_j and β_j.

In the first step we assume that σ_j is an affine function of $Y - y_j$, that is we take $c = 0$. Thus, for a fixed j, consider differences v^n_j (2), taking

$$m_j = \alpha_j(Y - y_j), \tag{4}$$

$$\sigma_j = \hat\sigma_Y + \beta_j\,(Y - y_j), \quad \beta_j \neq 0, \tag{5}$$

where α_j, β_j are parameters, and $\hat{\sigma}_Y$ is estimated when building the smoothing spline Sp_Y. The likelihood function $L(\mathbf{p})$, where $\mathbf{p} = (\alpha_j, \beta_j)$, has the form

$$L(\mathbf{p}) = \frac{1}{(\sqrt{2\pi})^{N_j}(\sigma_j)^{N_j}} \Pi_{n=1}^{N_j} \exp \frac{\left[v_j^n - m_j\right]^2}{2(\sigma_j)^2}$$

so its logarithm is given by

$$\ln L(\mathbf{p}) = -N_j \ln \sqrt{2\pi} - N_j \ln \sigma_j - \frac{1}{2(\sigma_j)^2} \sum_{n=1}^{N_j} \left[v_j^n - m_j\right]^2,$$

where N_j is the number of differences v_j^n. Taking into account equations (4)-(5), we obtain the necessary optimality conditions of the form

$$\frac{\partial \ln L(\mathbf{p})}{\partial \alpha_j} = \frac{Y - y_j}{(\sigma_Y + \beta_j (Y - y_j))^2} \sum_{n=1}^{N_j} \left[v_j^n - \alpha_j(Y - y_j)\right] = 0, \tag{6}$$

$$\frac{\partial \ln L(\mathbf{p})}{\partial \beta_j} = - \frac{N_j (Y - y_j)}{\sigma_Y + \beta_j (Y - y_j)} + \frac{(Y - y_j)}{(\sigma_Y + \beta_j (Y - y_j))^3} \sum_{n=1}^{N_j} \left[v_j^n - \alpha_j(Y - y_j)\right]^2 = 0, \tag{7}$$

From equation (6) we have the estimator of α_j

$$\widehat{\alpha}_j = \frac{1}{N_j(Y - y_j)} \sum_{n=1}^{N_j} v_j^n, \tag{8}$$

and from equation (7) we get the estimator of β_j.

$$\widehat{\beta}_j = \Big(\sqrt{\frac{1}{N_j} \sum_{n=1}^{N_j} \left(v_j^n - \bar{v}_j\right)^2} - \hat{\sigma}_Y\Big)/(Y - y_j), \tag{9}$$

where $\bar{v}_j = \frac{1}{N_j} \sum_{n=1}^{N_j} v_j^n$.

Now, take a look at the estimator of the model parameters covariance matrix. Under mild assumptions the maximum likelihood parameter estimators are asymptotically normal and unbiased, with the asymptotic covariance matrix

$$\Gamma = -E\Big[\frac{\partial^2 \ln L(\mathbf{p})}{\partial \hat{\mathbf{p}} \partial \hat{\mathbf{p}}^T}\Big]^{-1}$$

where now $\hat{\mathbf{p}}^T = [\hat{\alpha}_j \quad \hat{\beta}_j]$ is the vector of the maximum likelihood parameter estimators.

To find the estimator of the covariance matrix the Hessian matrix of the second derivatives

$$\mathbf{H} = \frac{\partial^2 \ln L(\mathbf{p})}{\partial \mathbf{p} \partial \mathbf{p}^T} = \begin{bmatrix} \frac{\partial^2 \ln L(\mathbf{p})}{\partial \alpha_j^2} & \frac{\partial^2 \ln L(\mathbf{p})}{\partial \beta_j \partial \alpha_j} \\ \frac{\partial^2 \ln L(\mathbf{p})}{\partial \alpha_j \partial \beta_j} & \frac{\partial^2 \ln L(\mathbf{p})}{\partial \beta_j^2} \end{bmatrix}$$

has to be calculated. Its entries are given by

$$\frac{\partial^2 \ln L(\mathbf{p})}{\partial \alpha_j^2} = - \frac{(Y - y_j)^2}{(\sigma_Y + \beta_j(Y - y_j))^2},$$

$$\frac{\partial^2 \ln L(\mathbf{p})}{\partial \beta_j \partial \alpha_j} = \frac{\partial^2 \ln L(\mathbf{p})}{\partial \alpha_j \partial \beta_j} = - \frac{2(Y - y_j)(Y - y_j)}{(\sigma_Y + \beta_j(Y - y_j))^{\frac{3}{2}}} \sum_{n=1}^{N_j} (v_j^n - \alpha_j(Y - y_j)),$$

$$\frac{\partial^2 \ln L(\mathbf{p})}{\partial \beta_j^2} = \frac{(Y - y_j)^2}{(\sigma_Y + \beta_j(Y - y_j))^2} \sum_{n=1}^{N_j} \left(1 - \frac{3(v_j^n - \alpha_j(Y - y_j))^2}{(\sigma_Y + \beta_j(Y - y_j))^2}\right).$$

Now inserting the maximum likelihood estimators we get

$$\frac{\partial^2 \ln L(\mathbf{p})}{\partial \widehat{\alpha}_j^2} = -\frac{(Y-y_j)^2}{\frac{1}{N_j}\sum_{n=1}^{N_j}(v_j^n - \bar{v}_j)^2},$$

$$\frac{\partial^2 \ln L(\mathbf{p})}{\partial \widehat{\beta}_j \partial \widehat{\alpha}_j} = \frac{\partial^2 \ln L(\mathbf{p})}{\partial \widehat{\alpha}_j \partial \widehat{\beta}_j} = -\frac{2(Y-y_j)^2}{(\frac{1}{N_j}\sum_{n=1}^{N_j}(v_j^n - \bar{v}_j)^2)^{\frac{3}{2}}} \sum_{n=1}^{N_j}(v_j^n - \bar{v}_j),$$

$$\frac{\partial^2 \ln L(\mathbf{p})}{\partial \widehat{\beta}_j^2} = \frac{(Y-y_j)^2}{\frac{1}{N_j}\sum_{n=1}^{N_j}(v_j^n - \bar{v}_j)^2} \sum_{n=1}^{N_j}\left(1 - \frac{3(v_j^n - \bar{v}_j)^2}{\frac{1}{N_j}\sum_{n=1}^{N_j}(v_j^n - \bar{v}_j)^2}\right) =$$

$$= N_j\,(Y-y_j)^2\left(1 - \frac{3}{\frac{1}{N_j}\sum_{n=1}^{N_j}(v_j^n - \bar{v}_j)^2}\right).$$

Observe that the element $\frac{\partial^2 \ln L(\mathbf{p})}{\partial \hat{\beta}_j^2}$ may either be positive or negative, depending on the data. If it is positive, the Hessian matrix cannot be negative definite. Thus, the likelihood function may possibly have more than one local maximum.

3.2. **Estimation of parameters b and c.** Having estimated parameters β_j, $j = 1, \ldots, J$, we can estimate b and c, as the parameters of the regression function given by

$$\beta_j := -b\,(Y - y_j)^c, \quad j = 1, \ldots, J, \quad \text{where} \quad b < 0. \tag{10}$$

Since $\beta_j > 0$, $j = 1, \ldots, J$, nonlinear model (10) can be converted into a linear one in the following way

$$\ln \beta_j = \ln(-b) + c \ln(Y - y_j).$$

Putting

$$\widetilde{\beta}_j := \ln \beta_j, \quad \widetilde{b} := \ln(-b), \text{ and, } \widetilde{y}_j = \ln y_j, \quad j = 1, \ldots, J,$$

we get the linear model

$$\widetilde{\beta}_j = \widetilde{b} + c\widetilde{y}_j.$$

Parameters $\widetilde{b}$ and c can now be estimated using the Least Squares method.

3.3. **Estimation of parameter a.** Estimates of α_j obtained in the first step show great volatility. To obtain smooth values the following third step is performed. Assume that the sequence α_j, $j = 1, \ldots, J$ is a first order autoregressive process of the form

$$\alpha_{j-1} = \frac{1}{\widetilde{a}}\alpha_j + \varepsilon_j, \quad |\widetilde{a}| < 1, \ \widetilde{a} \neq 0, \tag{11}$$

where

$$\alpha_{J+1} := 0,$$

and ε_j are independent random variables such that $\varepsilon_j \sim N(0, \sigma)$.

Having obtained estimates of parameters α_j, $j = 1, \ldots, J$, we can estimate the parameter $\widetilde{a}$ in the above AR model by the Least Squares method. Estimator of a then becomes

$$\hat{a} = \frac{1}{\widetilde{a}}$$

3.4. **The algorithm.** The estimation method described in Sections 3.1, 3.2 and 3.3 allows us to calculate m_j and σ_j, $j = 1, \ldots, J$ in the model (2)-(3). It involves first conducting a preliminary estimation, in which we obtain sequences $\widehat{\alpha}_j$ and $\widehat{\beta}_j$, and then using these sequences when estimating the parameters a, b, and c. This two-step procedure can be given in algorithmic way.

For a fixed country i, the algorithm can be stated as follows.

Step 1. Estimate parameters α_j and β_j, $j = 1, \ldots, J$ in the model (5), using Maximum Likelihood estimators (8) and (9).

Step 2. Estimate parameter a in the AR model (11), using $\widehat{\alpha}_j$, $j = 1, \ldots, J$, obtained in Step 1.

Step 3. Estimate parameters b and c in the regression model (10), using $\widehat{\beta}_j$, $j = 1, \ldots, J$, obtained in Step 1.

4. Preliminary estimation for several EU countries

In this section we analyze data from the National Inventory Reports (NIR)[2]. for a few EU countries: Austria, Belgium, Denmark, Finland, United Kingdom, Ireland, and Sweden.

4.1. **NIR data for Austria.** The data refers to CO_2 emissions in the years 1990 – 2005, and recalculations (revisions), performed every year, from 1999 to 2005.

We start with building a smoothing spline Sp_Y for the most recently revised data – from the year $Y = 2005$ (Figure 2).

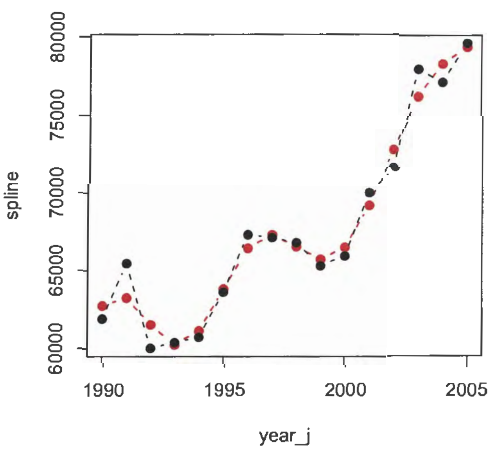

FIGURE 2. Smoothing spline Sp_Y for $Y = 2005$, Austria, $\widehat{\sigma}_Y = 2065.5$

Then we calculate the differences v_j^n, between the constructed spline Sp_Y, and consecutive revisions of the data, carried out in 1999 - 2004. The results are shown in Figure 3.

[2]According to United Nations Framework Convention on Climate Change (UNFCCC), each of the countries which have signed the Convention, is obliged to provide annually data on greenhouse gas inventories.

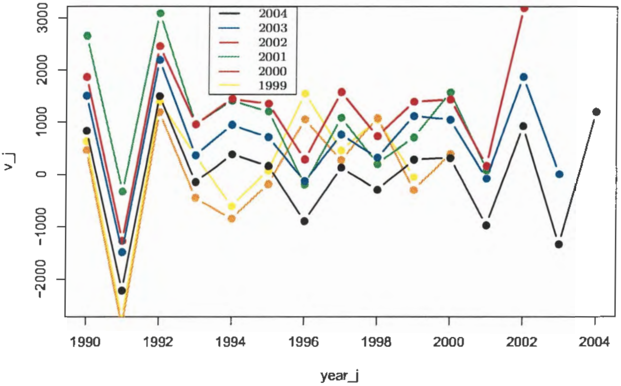

FIGURE 3. Differences v_j^n, 1999-2004, Austria

We proceed according to the algorithm described in Section 3.4. First we estimate parameters α_j and β_j, $j = 1, \dots, J$, in the model (5), using formulas (8) and (9). The values estimated are depicted in Figure 4 and in Table 2.

j	1999	2000	2001	2002	2003	2004	mean	std
$\widehat{\alpha}_j$	36.25	-2.72	260.81	402.24	330.49	-3.21	170.6	182.0
$\widehat{\beta}_j$	-149.16	-190.50	-262.85	-335.42	-582.58	-1094.44	-435.8	357.1

TABLE 2. Estimates of parameters α_j and β_j in the model (5); Austria.

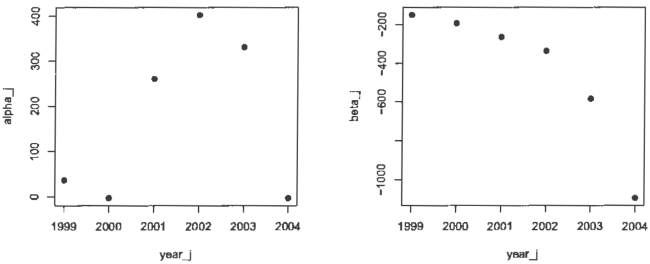

FIGURE 4. Estimates of parameters α_j and β_j in the model (5), Austria.

Then we use $\widehat{\beta}_j$, $j = 1, \dots, J$, to estimate paramcters b and c of the model (3). Fitting the regression function (10) gives $\hat{b} = 1158.6$ and $\hat{c} = -1.11$, where $R^2 = 0.994$, which indicates a good fit. Finally, we use $\widehat{\alpha}_j$, $j = 1, \dots, J$ and estimate parameter $\tilde{a}$ in the AR model (11). We get $\tilde{a} = 0.696$, so $\hat{a} = 1.44$.

Now, it remains only to calculate the values of m_j and σ_j. The results obtained are presented in Figure 5 and Tables 3a – 3b.

j	1999	2000	2001	2002	2003	2004	parameters
m_j	8.62	7.18	5.74	4.31	2.87	1.44	$\hat{a} = 1.44$
σ_j	1119.33	1099.62	1074.95	1042.20	994.20	906.87	$\hat{b} = 1158.6$, $\hat{c} = -1.11$

TABLE 3a. Estimates of m_j and σ_j, model (3), Austria

Parameter	Estimate	Model fit
a	1.44	$\sigma^2 = 43669$
b	1158.6	St.error=0.056, t-test: p-value 0.000000023, $R^2 = 0.99$
c	-1.11	St.error = 0.045, t-test: p-value–0.0000152

TABLE 3b. Estimates of parameters a, b, and c, Austria.

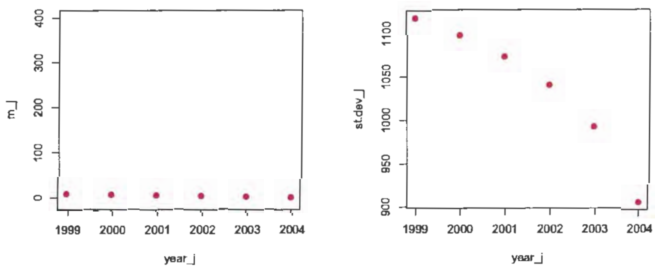

FIGURE 5. Estimated values of m_j and σ_j in the model (3), Austria.

The results show clear decline of the imprecision level of inventories (their standard deviation), and at the same time, practically constant and close to 0 accuracy of the inventory gathering (their mean value).

4.2. **NIR data for several EU countries.** Now, we apply the same procedure to the National Inventory data for a few EU countries (Belgium, Denmark, Finland, Ireland, UK, and Sweden). The results obtained are presented below, in figures and tables (Figures 6 – 23 and Tables 4a – 9b).

4.2.1. *Belgium.*

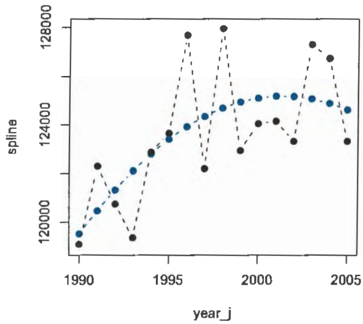

FIGURE 6. Smoothing spline Sp_Y for $Y = 2005$, Belgium, $\widehat{\sigma}_Y = 2400.12$

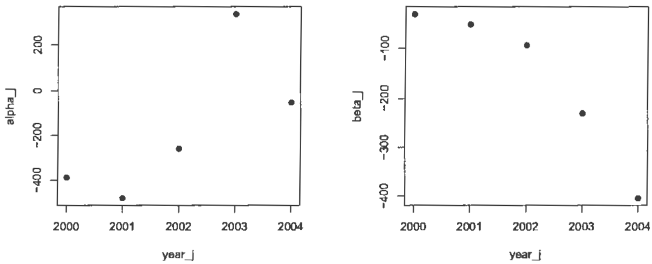

FIGURE 7. Estimates of parameters α_j and β_j in the model (5), Belgium.

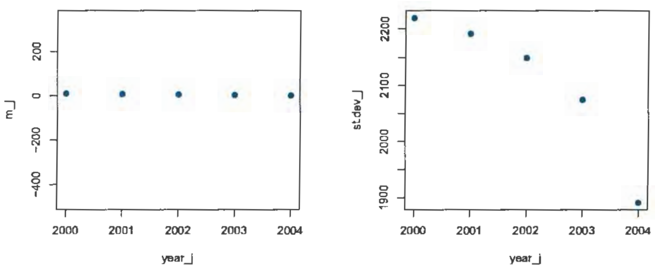

FIGURE 8. Estimates of m_j and σ_j in the model (3), Belgium.

j	2000	2001	2002	2003	2004	mean	std
$\widehat{\alpha}_j$	-388.04	-480.5	-258.48	338.06	-52.45	-168.3	325.5
$\widehat{\beta}_j$	-29.54	-50.41	-93.19	-230.59	-404.29	-161.6	156.6
m_j	10.09	8.07	6.05	4.04	2.02	$\hat{a} = 2.02$	
σ_j	2219.95	2192.08	2149.68	2074.86	1891.61	$\hat{b}$ =508.52, $\hat{c} = -1.64$	

TABLE 4a. Estimates of parameters in the model (3), Belgium

Parameter	Estimate	Model fit
a	2.02	$\sigma^2 = 115171$
b	508.52	St.error=0.240, t-test: p-value=0.000127, $R^2 = 0.95$
c	-1.64	St.error = 0.216, t-test: p-value=0.004829

TABLE 4b. Estimates of parameters a, b, and c, Belgium.

4.2.2. *Denmark.*

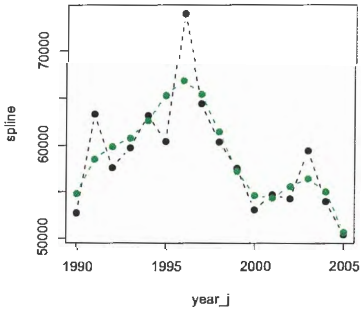

FIGURE 9. Smoothing spline Sp_Y for $Y = 2005$, Denmark, $\widehat{\sigma}_Y = 4969.46$

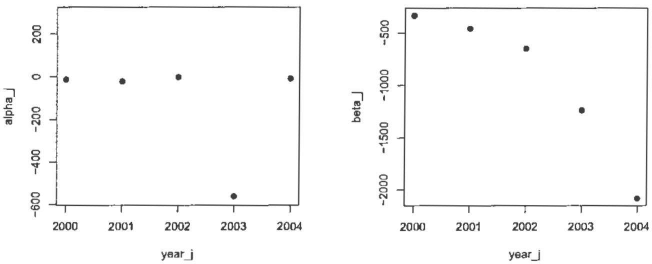

FIGURE 10. Estimates of parameters α_j and β_j in the model (5), Denmark.

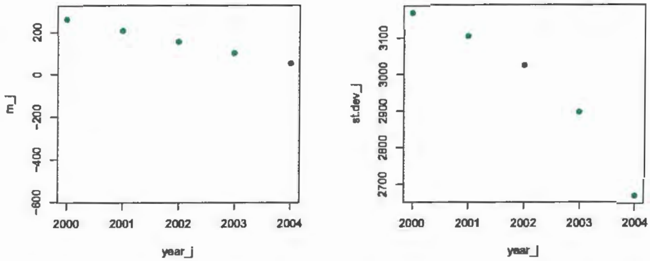

FIGURE 11. Estimates of m_j and σ_j in the model (3), Denmark.

j	2000	2001	2002	2003	2004	mean	std
$\widehat{\alpha}_j$	-12.64	-21.32	-1.76	-560.9	-8.46	-121.0	246.0
$\widehat{\beta}_j$	-336.2	-459.6	-649.8	-1235.1	-2079.8	-952.1	718.5
m_j	260.42	208.33	156.25	104.17	52.08	$\hat{a} = 52.08$	
σ_j	3170.81	3108.55	3025.10	2901.08	2670.49	$\hat{b} = 2298.98$, $\hat{c} = -1.15$	

TABLE 5a. Estimates of parameters in the model (3), Denmark.

Parameter	Estimate	Model fit
a	52.08	$\sigma^2 = 78778$
b	2298.98	St.error=0.109, t-test: p-value=0.0000061, $R^2 = 0.98$
c	-1.15	St.error = 0.098, t-test: p-value=0.00131

TABLE 5b. Estimates of parameters a, b, and c, Denmark.

4.2.3. *Finland.*

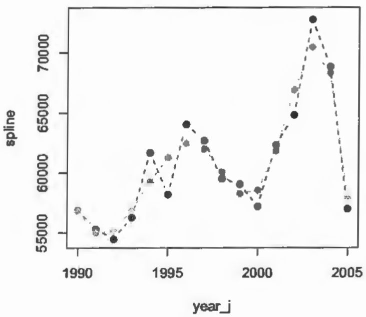

FIGURE 12. Smoothing spline Sp_Y for $Y = 2005$, Finland, $\widehat{\sigma}_Y = 2725.18$

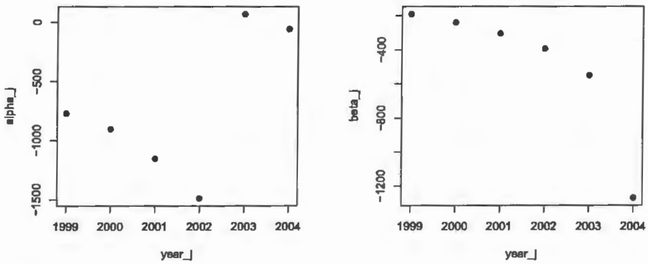

FIGURE 13. Estimates of parameters α_j and β_j in the model (5), Finland.

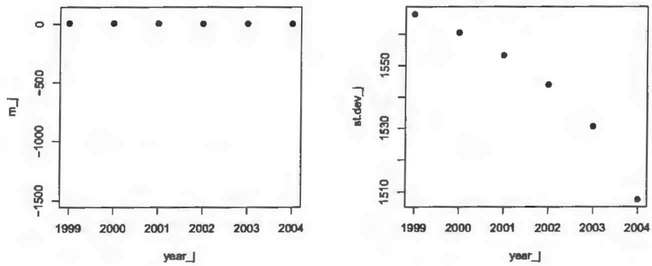

FIGURE 14. Estimated values of m_j and σ_j in the model(3), Finland.

j	1999	2000	2001	2002	2003	2004	mean	std
$\widehat{\alpha}_j$	-768.21	-902.93	-1154.69	-1487.88	67.41	-57.67	-717.3	612.0
$\widehat{\beta}_j$	-189.17	-239.84	-303.95	-392.89	-549.09	-1268.7	-490.6	401.7
m_j	7.83	6.52	5.22	3.91	2.61	1.30	$\hat{a} = 1.30$	
σ_j	1566.31	1560.47	1553.28	1543.94	1530.66	1507.61	$\hat{b} = 1217.6$, $\hat{c} = -1.03$	

TABLE 6a. Estimates of parameters in the model (3), Finland.

Parameter	Estimate	Model fit
a	1.30	$\sigma^2 = 477797$
b	1217.6	St.error=0.045, t-test: p-value=0.00000001, $R^2 = 0.99$
c	-1.03	St.error = 0.036, t-test: p-value=0.000009

TABLE 6b. Estimates of parameters a, b, and c, Finland.

4.2.4. *United Kingdom.*

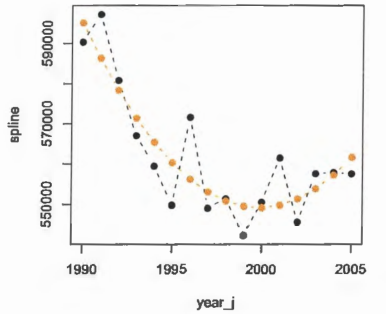

FIGURE 15. Smoothing spline Sp_Y for $Y = 2005$, UK, $\widehat{\sigma}_Y = 8831.51$

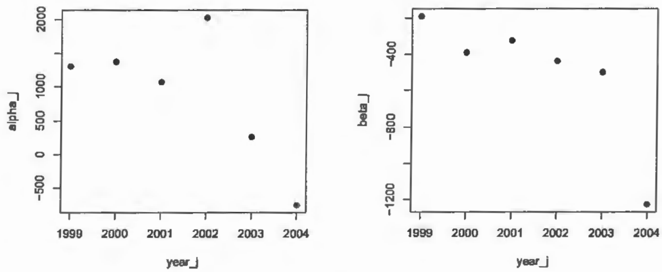

FIGURE 16. Estimates of parameters α_j and β_j in the model (5), UK.

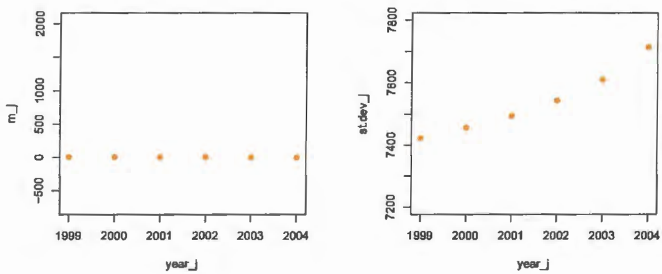

FIGURE 17. Estimated values of m_j and σ_j in the model(3), UK.

j	1999	2000	2001	2002	2003	2004	mean	std
$\widehat{\alpha}_j$	1304.6	1373.9	1068.08	2026.57	265.03	-749.99	881.4	980.7
$\widehat{\beta}_j$	-189.99	-388.40	-322.32	-435.93	-497.76	-1227.35	-510.29	366.78
m_j	8.09	6.74	5.39	4.04	2.70	1.35	$\hat{a} = 1.35$	
σ_j	7423.16	7455.85	7494.82	7543.44	7608.99	7713.40	$\hat{b} = 1118.1$, $\hat{c} = -0.87$	

TABLE 7a. Estimates of parameters in the model (3), UK.

Parameter	Estimate	Model fit
a	1.35	$\sigma^2 = 926521$
b	1118.1	St.error=0.196, t-test: p-value=0.0000036, $R^2 = 0.89$
c	-0.87	St.error = 0.157, t-test: p-value=0.00514

TABLE 7b. Estimates of parameters a, b, and c, UK.

4.2.5. *Ireland.*

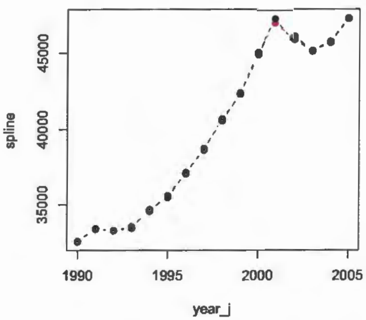

FIGURE 18. Smoothing spline Sp_Y for $Y = 2005$, Ireland, $\widehat{\sigma}_Y = 586.64$

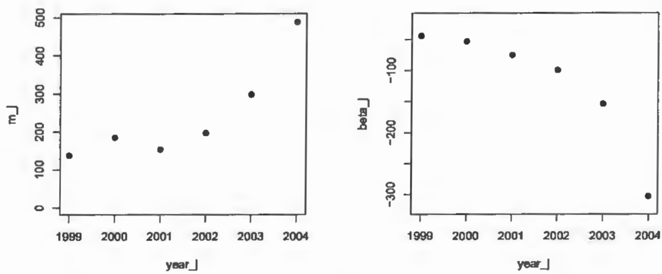

FIGURE 19. Estimates of parameters α_j and β_j in the model (5), Ireland.

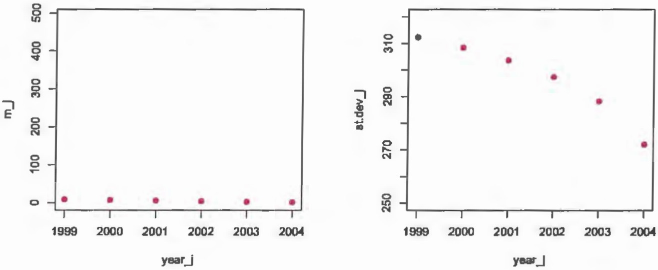

FIGURE 20. Estimated values of m_j and σ_j in the model(3), Ireland.

j	1999	2000	2001	2002	2003	2004	mean	std
$\widehat{\alpha}_j$	137.62	184.75	153.63	196.57	297.97	488.20	243.12	132.50
$\widehat{\beta}_j$	-44.27	-53.13	-74.87	-99.22	-153.63	-302.99	-121.35	97.24
m_j	8.82	7.35	5.88	4.41	2.94	1.47	$\hat{a} = 1.47$	
σ_j	312.39	308.54	303.74	297.44	288.32	272.06	$\hat{b} = 314.6$, $\hat{c} = -1.07$	

TABLE 8a. Estimates of parameters in the model (3), Ireland.

Parameter	Estimate	Model fit
a	1.47	$\sigma^2 = 1636$
b	314.6	St.error=0.041, t-test: p-value=0.000000016, $R^2 = 0.996$
c	-1.07	St.error = 0.032, t-test: p-value=0.0000052

TABLE 8b. Estimates of parameters a, b, and c, Ireland.

4.2.6. *Sweden.*

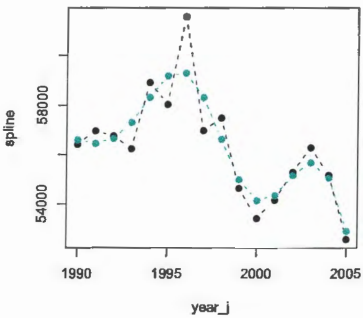

FIGURE 21. Smoothing spline Sp_Y for $Y = 2005$, Sweden, $\widehat{\sigma}_Y = 1503.6$

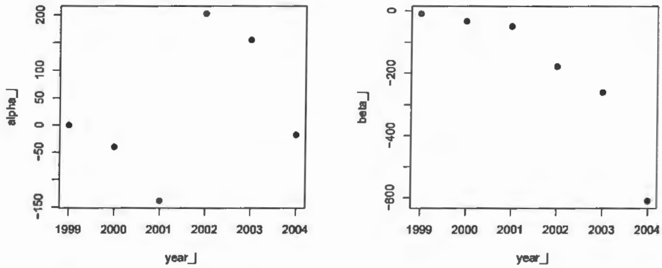

FIGURE 22. Estimates of parameters α_j and β_j in the model (3), Sweden.

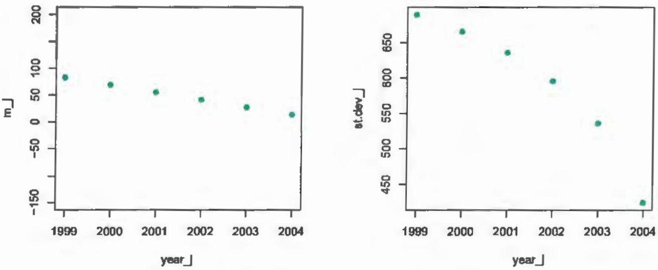

FIGURE 23. Estimated values of m_j and σ_j in the model(3), Sweden.

j	1999	2000	2001	2002	2003	2004	mean	std
$\widehat{\alpha}_j$	-0.57	-40.26	-138.29	202.80	154.98	-17.82	26.81	127.98
$\widehat{\beta}_j$	- 9.85	-33.78	-49.82	-178.15	-260.36	-609.44	-190.23	226.95
m_j	83.22	69.35	55.48	41.61	27.74	13.87	$\hat{a} = 13.87$	
σ_j	689.36	665.64	635.67	595.44	535.55	423.88	$\hat{b} = 1079.7$, $\hat{c} = -1.16$	

TABLE 9a. Estimates of parameters in the model (3), Sweden.

Parameter	Estimate	Model fit
a	13.87	$\sigma^2 = 17089$
b	1079.7	St.error=0.472, t-test: p-value=0.000129, $R^2 = 0.86$
c	-1.16	St.error = 0.375, t-test: p-value=0.000454

TABLE 9b. Estimates of parameters a, b, and c, Sweden.

5. Conclusions

In this report an idea of a model describing the learning process in evaluation of the national emissions is presented. Preliminary calculations for data from several countries and a discussion on choice of the model have been done.

This report improves the results presented in [9] – a new, more general model is presented and much better fit is obtained. The results are compiled in Figures 24 and 25.

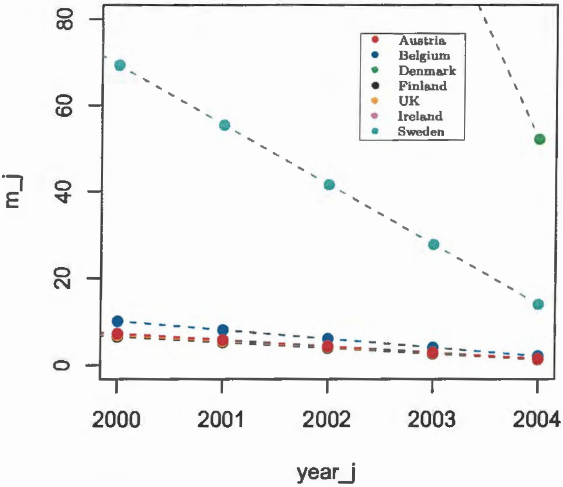

FIGURE 24. Estimated values of m_j in the model (3) for several EU countries.

It can be observed that the estimates of the mean values $\widehat{m}_j$ are changing very little and are practically negligible, even in the case of Denmark and Sweden, for which they take bigger values[3]. Although the estimates of $\widehat{\alpha}_j$ are rather volatile, they are still relatively small.

When it comes to standard deviations $\widehat{\sigma}_j$, one can notice that they rather decrease in time (except for the United Kingdom and Finland where the sequence $\widehat{\sigma}_j$ is slightly increasing). But it can also be been seen, that, their absolute values differ considerably among countries (Figure 25). The greatest values of $\widehat{\sigma}_j$ there are for the United Kingdom, the smallest – for Ireland. However, the situation looks different when the relative values

$$\frac{\widehat{\sigma}_j}{\mathrm{Sp}_j}, \quad \text{where} \quad j \in \{1999, 2000, 2001, 2002, 2003, 2004\}$$

are considered instead (Figure 26). These latter values are not greater than a few percent (see Table 10 for details), and the order of the countries in the figure has changed.

[3]In the case of Denmark, the values of $\widehat{m}_j$ are between 260.42 and 52.08 (see Table 5a), and in the case of Sweden – between 83.22 and 13.87 (see Table 9a).

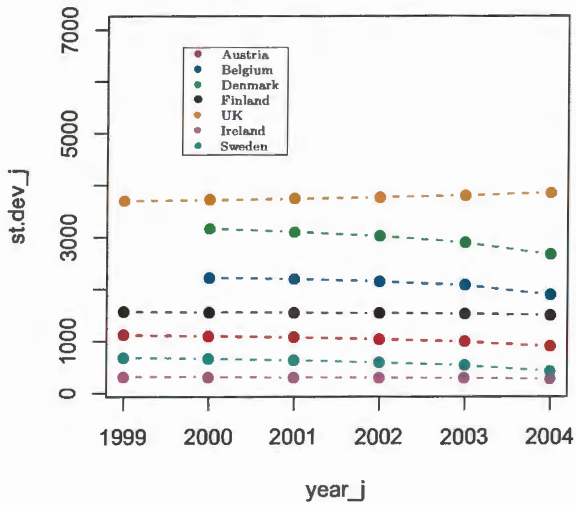

FIGURE 25. Estimated values of σ_j in the model (3) for several EU countries.

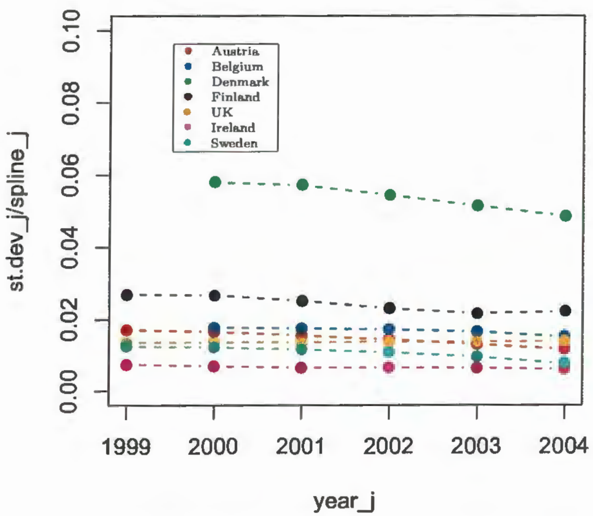

FIGURE 26. Relative values $\frac{\hat{\sigma}_j}{\mathrm{Sp}_j}$ for several EU countries.

Country\Year	1999	2000	2001	2002	2003	2004
Austria	0.017	0.016	0.015	0.014	0.013	0.012
Belgium	-	0.018	0.0175	0.0171	0.0166	0.015
Denmark	-	0.058	0.057	0.054	0.051	0.049
Finland	0.0268	0.0266	0.0251	0.0230	0.0217	0.0221
UK	0.0135	0.0136	0.01363	0.01368	0.0137	0.0138
Ireland	0.0073	0.0068	0.006452	0.00645	0.00638	0.00595
Sweden	0.0125	0.01229	0.0116	0.0108	0.0096	0.0077

TABLE 10. Relative values $\frac{\sigma_j}{\mathrm{Sp}_j}$ for several EU countries.

It is worth mentioning that we conducted the analysis for the NIR data from those countries, which, in accordance with agreements signed, prepared the National Inventory Reports on GHG emissions for nearly twenty years, and where revisions were made frequently.

In the continuation of the project, fitting NIR data for other countries will be attempted, with smaller number of data. Another direction might be calculation for other databases, like provided by CDIAC, IEA, EIA or EDGAR sources. The main problem in using the data from these databases are the discrepancies in the values, due to different methodologies of calculating emissions, see [4, 5].

References

[1] Ch. D. Ghiliani (2010) Adjustment Computations: Spatial Data Analysis. Wiley.

[2] K. Hamal (2010) Reporting GHG emissions: Change in uncertainty and its relevance for detection of emission changes. Interim Report IR-10-003. IIASA, Laxenburg.

[3] N. Höhne, H. Blum, J Fuglestvedt, R.B. Skeie, A. Kurosawa, G. Hu, J. Lowe, L. Gohar, B. Matthews, A.C. Nioac de Salles, C. Ellermann (2011) Contributions of individual countries' emissions to climate change and their uncertainty. *Climatic Change*, 106:359-391.

[4] J. Macknik (2009) Energy and carbon dioxide emission data uncertainties. Interim Report IP-09-032. IIASA, Laxenburg.

[5] J. Macknick (2011) Energy and CO_2 emission data uncertainties. *Carbon Management*, 13(2):189-205.

[6] G. Marland (2008), Uncertainties in accounting for CO_2 from fossil fuels. *Journal of Industrial Ecology*, 12(2):136-139.

[7] G. Marland, K. Hamal, M. Jonas (2009), How uncertain are estimates of CO_2 emissions? *Journal of Industrial Ecology*, 13(1):4-7.

[8] Z. Nahorski, W. Jęda (2007), Processing national CO_2 inventory emission data and their total uncertainty estimates, *Water, Air, and Soil Pollution: Focus*, 7(4-5):513-527.

[9] Z. Nahorski, J. Jarnicka (2010), Modeling Uncertainty Structure of Greenhouse Gases Inventories, RB/11/2010, IBS PAN, Warszawa.