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A Market for Pollution Emission Permits with Low Accuracy of Emission Estimates

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Abstract

Uncertainties of pollution inventories are often high due to low precision of emission quantity assessments for many emitting sources. A good example is emission of greenhouse gases, where uncertainty of some sources may be as high as 40-100%, while uncertainty of other sources is as low as 2-3%. This discrepancy in uncertainty should be accounted for in compliance as well as in emissions trading, because the traded commodities have different quality. The compliance and emissions trading rules have been discussed in earlier papers by the present authors (Nahorski et al. 2007; Nahorski and Horabik 2007; Nahorski and Horabik 2008; Nahorski and Horabik 2010). In this report we focus on presentation of the idea of a market for emissions with so highly scattered uncertainties.

Keywords: Pollution emission, Uncertainty, Interval calculus, Compliance, Emission permit trading

1 Introduction

Emissions of environmental pollutions are often too difficult to be measured directly. Therefore, they are calculated indirectly by inventorying activities causing emissions, and estimating their influence on the final emissions. These estimates are due to different errors, like low precision of amount of source materials used, their quality, insufficient knowledge of processes emitting pollutions, etc. A good example of this kind of procedure is emission of greenhouse gases. Although emissions of carbon dioxide from large professional power plants can be estimated with quite satisfactory uncertainty of 2-5%, uncertainty of methane emissions may reach up to 40%, and uncertainty of nitrous oxide inventories is even more than 100%. There arises the problem how to evaluate compliance with imposed limits on such uncertain emissions, and how to trade emission permits.

The problem how to treat uncertain greenhouse gases emissions has been discussed already for some years (Winiwarter 2004; Gillenwater et al. 2007; Jonas et al. 2007; Jonas and Nilsson 2007; Lieberman at al. 2007; Hurteau et al. 2009; Mignone et al. 2009; Jonas et al. 2010). In particular, some ways of solving the permit trading were proposed. For example, Monni et al. (2007) proposed to exclude uncertain emissions from trade, or to trade emissions of similar uncertainty on separate markets.

In this paper we shall focus on so-called undershooting approach (Godal et al. 2003; Nahorski et al. 2003; Nahorski et al. 2007; Nahorski and Horabik 2008; Nahorski and Horabik 2010). Our aim is to outline the market for emission permits with highly diversified emission uncertainties.

In section 2 we discuss the problem of evaluation of compliance for uncertain emissions with different uncertainty distributions. In section 3 we deal with the asymmetric interval uncertainty and we derive conditions for checking compliance in such a case. In section 4 we present so-called effective emissions, which can be traded directly, without taking into account emission uncertainty. In section 5 the market with effective emissions as additional instruments is discussed, while section 6 considers the market with solely effective emissions used in trading. The market rules are given and market properties are derived. Section 7 concludes.

2 Uncertainty and evaluation of compliance

In the case without uncertainty, for a given upper limit L imposed on emissions, the actual emissions x must satisfy the condition

$$x \leq L \tag{1}$$

In some cases, a reduction of inventory is given in percents. Denoting x_c as an emission inventory at the end of reduction period, x_b as an emission inventory in the beginning of the reduction period, and ρ as a required fraction of emission reduction, then the compliance condition is $x_c \leq \rho x_b$. It can be transformed to $x_c - \rho x_b \leq 0$. This way, the condition has the form (1), with $x = x_c - \rho x_b$ and $L = 0$. To simplify argumentation, only the condition of type (1) will be considered in the sequel.

The problem arises when the emissions x are not known with a satisfactory accuracy. For example, this is the case when emissions are estimated from an approximate inventory, as it is for emissions of greenhouse gases. In order to highlight uncertainty of such an estimate, this value is denoted in the sequel by $\hat{x}$. Moreover, distributions of uncertainty for different gases as well as for national inventories may be asymmetric (Ramirez et al. 2006; Winiwarter and Muik 2007).

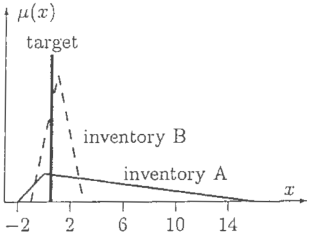

Figure 1: Distributions of two inventories considered in the text.

For better illustration of the problems related to dealing with uncertain emissions, let us look at simplified distributions $\mu(x)$ of two inventories, A and B, presented in Figure 1, shifted to zero at the limit L, often called the target. The calculated inventories $\hat{x}$, called here the nominal values, correspond to the highest values of the distributions. Taking into account only the nominal values, the party with inventory A fulfills the emission condition (1), while the party with inventory B does not. However, if the distributions are interpreted as the probability distributions, one can see that the probability that the real inventory does not fulfill the limit (the area under the distribution for the positive values of x) is higher than the probability that it does not (the area for the negative values of x). One may ask the question which criterion is better suited to order uncertain inventories.

The impression that the inventory A is not necessarily better that B is actually questioned by many techniques used to compare uncertain values, see Graves at al. (2009). Here we mention few of them.

The most elementary technique of ordering uncertain values is based on *the mean value and the variance* (MV). According to this technique, the smaller the mean value and the variance are, the better the inventory is. For the example presented in Figure 2, the respective values are depicted in Table 1. Although the nominal value of the inventory A is smaller than that of B, the mean value of A is greater than the mean value of B. The same is true for the standard deviations. Thus, even this simple criterion shows that an inventory of the party B should be considered smaller than an inventory of the party A. This is contrary to the result for nominal values, which ignores uncertainty.

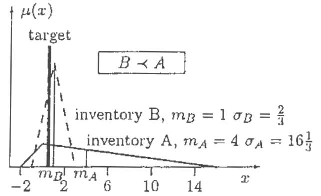

Figure 2: Comparison of means and variances.

A large group of techniques uses the notion of *critical probability* (CP), proposed already in 1952, Roy (1952). The methods in this group require knowledge of respective probability distribution $\mu(x)$. The measure used to compare inventories is the probability of surpassing the target L

$$crp = \int_L^{\infty} \mu(x)dx \tag{2}$$

A smaller value of crp indicates better inventory. According to Table 1, again, an inventory of the party B is evaluated as the smaller one.

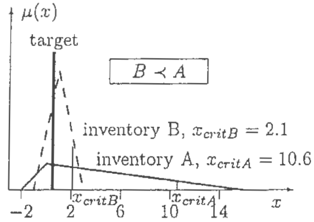

Figure 3: Calculation of critical values.

In other related methods, as *the Baumol's risk measure* and *the value at risk* (VaR), the probability of inventory x to be above a critical value x_{crit}

is fixed, and then the value x_{crit} is calculated, see Figure 3. Without going into details, an inventory is smaller when x_{crit} is smaller. In our example, fixing probability to 0.1, the inventory B is chosen as the smaller one.

In conclusion, decision about fulfillment of an obligation, which is based on deterministic (nominal value) comparison of an inventory with a target, contradicts the already existing scientific knowledge on ordering uncertain projects using the stochastic approach.

Table 1: Criteria values for comparison of inventories A and B.

Method	Criterion value for A	Criterion value for B	Inventory chosen
MV	$m_A = 4$ $\sigma_A = 16\frac{1}{3}$	$m_B = 1$ $\sigma_B = \frac{2}{3}$	B
MSV	$s_{SA}^2 = 13.45$	$s_{SB}^2 = 0.35$	B
CP	$crp_A = \frac{8}{9}$	$crp_B = \frac{7}{8}$	B
risk	$c_{critA} = 10.6$	$c_{critB} = 2.1$	B

A technique similar in spirit to the CP and risk measures has been proposed to ensure a reliable compliance in the context of greenhouse gases. It is called *undershooting* (Gillenwater et al., 2007; Godal et al., 2003; Nahorski and Horabik, 2010; Nahorski et al., 2007; Nahorski et al., 2003), and it is illustrated in Figure 4. In this approach, it is required that only a small enough α-th part of an inventory distribution may lie above a target. This idea, when used for ordering inventories, becomes equivalent to the CP technique.

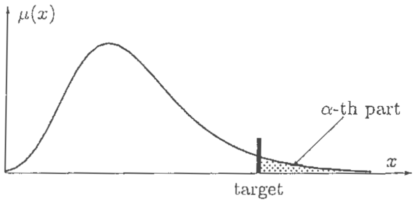

Figure 4: Illustration of compliance in the undershooting approach.

The undershooting technique is used throughout the paper to design a market for the emissions with highly diversified uncertainties by introducing comparable quotas which depend on the uncertainty levels.

3 Compliance

Let us denote the lower spread of the uncertainty interval by d^l and the upper spread by d^u. Then, the actual (unknown) emission x is situated in the intervals

$$x \in [\hat{x} - d^l, \hat{x} + d^u]$$

The limit L is known exactly. To be completely sure that a party fulfills the limit, its emission inventory $\hat{x}$ should satisfy the following condition, see Figure 5 (a).

$$\hat{x} + d^u \leq L \tag{3}$$

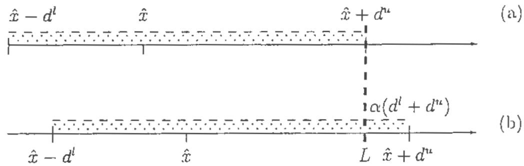

Figure 5: Full compliance (a) and compliance with risk α (b) in the interval uncertainty approach.

As the bounds can be quite large, a weaker condition will be used, see Nahorski et al. (2007).

Definition 1. *A party is compliant with the risk α if its emission inventory satisfies the condition*

$$\hat{x} + d^u \leq L + \alpha(d^l + d^u) \tag{4}$$

Here the risk is understood as a likelihood that a party may not fulfill the agreed obligation regarding the emission limit, due to uncertainty of the emission inventory.

The condition (4) means that the αth part of the party's emission estimate (inventory) uncertainty interval is allowed to lie above the limit L, see Figure 5 (b). After some algebraic manipulations the condition (4) can be also written in the following form

$$\hat{x} + \left[1 - \left(1 + \frac{d^l}{d^u}\right)\alpha\right] d^u \leq L \tag{5}$$

The above condition shows that a part of the upper spread of the uncertainty interval is added to the emission estimate before compliance is checked. This can be also interpreted to mean that an unreported emission, due to uncertainty, is included in the condition to reduce the risk of non-compliance. Let us denote by $R^u = d^u/\hat{x}$ and $R^l = d^l/\hat{x}$ the relative upper and lower spreads of the uncertainty intervals, respectively. Denoting the fraction of the unreported emission in the emission estimate as

$$u(\alpha) = \left[1 - \left(1 + \frac{d^l}{d^u}\right)\alpha\right] R^u \tag{6}$$

the equation (5) can be also written as

$$\hat{x}[1 + u(\alpha)] \leq L \tag{7}$$

Definition 2. *We call the left hand side value in (5) or (7) the expanded estimated emission and denote it as*

$$\hat{l} = \hat{x} + \left[1 - \left(1 + \frac{d^l}{d^u}\right)\alpha\right] d^u = \hat{x}[1 + u(\alpha)] \tag{8}$$

4 Effective emissions

The above compliance-proving policy can be used to modify the rules of emission trading. The main idea presented in earlier papers (Nahorski et al. 2007; Nahorski and Horabik 2008; Nahorski and Horabik 2010) involves transferring the uncertainty of the seller's emissions to the buyer's emissions together with the quota of traded emissions, and then including it in the buyer's emission balance.

Let us denote by $\hat{E}^S$ the amount of estimated seller emission allocated for trade. Emission $\hat{E}^S$ is associated with the lower and upper spreads of the uncertainty intervals $\hat{E}^S R^{lS}$ and $\hat{E}^S R^{uS}$, respectively.

Before a transaction the buyer has to satisfy the condition (5), which is reformulated to

$$\hat{x}^B + d^{uB} - (d^{lB} + d^{uB})\alpha \leq L^B$$

After buying $\hat{E}^S$ units of emissions from the seller and including the corresponding uncertainty in the formula, the buyer's condition becomes

$$\hat{x}^B - \hat{E}^S + d^{uB} + \hat{E}^S R^{uS} - (d^{uB} + \hat{E}^S R^{uS} + d^{lB} + \hat{E}^S R^{lS})\alpha \leq L^B \tag{9}$$

Two above conditions differ in the value defined below.

Definition 3. *The value*

$$E_{eff} = \hat{E}^S\Big\{1 - \big[1 - (1 + \frac{d^{lS}}{d^{uS}})\alpha\big]R^{uS}\Big\} = \hat{E}^S\big[1 - u^S(\alpha)\big] \qquad (10)$$

is called the effective emission *(Nahorski et al. 2007).*

Note that the effective emission is smaller than the estimated emission. The bigger the relative upper spread of the uncertainty interval of the seller is, the smaller the effective emission. Effective emissions depend also on the ratio d^{uS}/d^{lS}, and obviously on α.

5 Market with effective emissions

The market for uncertain inventories has been discussed in Nahorski et al. (2007); Nahorski and Horabik (2008); Bartoszczuk and Horabik (2007); Ermolieva at al. (2010). It was formulated as an optimization problem with minimization of the sum of costs to achieve the common limit on emissions, subject to compliance with the risk α. This simulation was, however, quite far from the real market conditions. First of all, in the real market parties take decisions without knowledge of the cost characteristics of the partner in trading. A rough cost evaluation is done in a process of price negotiation between parties. The idea of the trading prices was introduced in Stańczak and Bartoszczuk (2010), however, prices were only drawn randomly from the feasible region, and they were not negotiated. Some elements of negotiations were used in (Nahorski at al. 2010), but uncertainty was not considered there. Here we discuss organization of the market with uncertainties taken into account, and particularly with the effective emissions.

The basic assumption of this market is that selling/buying emissions is combined with simultaneous transferring of the corresponding uncertainty. The amount of the traded emissions is connected with the effective emissions. We denote the values before trade by the subscript $_0$, and those after the transaction number t by the subscript $_t$. Let us assume that the amount of $\hat{E}_t^S$ is sold from the seller S to the buyer B. The lower e_t^{lS} and the upper e_t^{uS} uncertainty spreads related with this amount are

$$e_t^{lS} = R_0^{lS}\hat{E}_t^S = \frac{\hat{E}_t^S}{\hat{x}_0^S}d_0^{lS} \qquad e_t^{uS} = R_0^{uS}\hat{E}_t^S = \frac{\hat{E}_t^S}{\hat{x}_0^S}d_0^{uS} \qquad (11)$$

Thus, after the transaction we have

$$\hat{x}_t^S = \hat{x}_{t-1}^S + \hat{E}_t^S \qquad \hat{x}_t^B = \hat{x}_{t-1}^B - \hat{E}_t^S \qquad (12)$$

Here we consider only two trading parties, and therefore both their estimated emissions change in transactions. When more parties are involved, their estimated emissions, besides the ones for the trading parties, do not change after transaction t, which is formally written by taking for them $\hat{E}_t^S = 0$. According to the rules of interval algebra we have

$$d_t^{lS} = d_{t-1}^{lS} + e_t^{lS} \qquad d_t^{uS} = d_{t-1}^{uS} + e_t^{uS} \tag{13}$$

$$d_t^{lB} = d_{t-1}^{lB} + e_t^{lS} \qquad d_t^{uB} = d_{t-1}^{uB} + e_t^{uS} \tag{14}$$

In the usual condition, the seller's estimated emission is distinctly less than the limit, while the buyer's is distinctly higher. Thus, the transaction helps the buyer to achieve his limit.

Theorem 1. *A reasonable amount of traded estimated emissions in a transaction t between a seller S and a buyer B is given by*

$$\hat{E}_t^S \le \min\left\{\frac{\hat{l}_{t-1}^B - L^B}{1 - u_t^S(\alpha)}, \frac{L^S - \hat{l}_{t-1}^S}{1 + u^S(\alpha)}\right\}$$

and of effective emissions by

$$E_{eff,t}^S = \min\left\{\hat{l}_{t-1}^B - L^B, \frac{1 - u_t^S(\alpha)}{1 + u_t^S(\alpha)}(L^S - \hat{l}_{t-1}^S)\right\}$$

Proof. It is easy to calculate how many units of permits $\hat{E}_t^S$ the buyer should buy to become compliant with the risk α. Let us denote the recalculated expanded estimated emission of the buyer after the transaction $t-1$ as

$$\hat{l}_{t-1}^B = \hat{x}_{t-1}^B + \left[1 - \alpha\left(1 + \frac{d_{t-1}^{lB}}{d_{t-1}^{uB}}\right)\right] d_{t-1}^{uB} = \hat{x}_{t-1}^B + d_{t-1}^{uB} - \alpha(d^{uB} + d_{t-1}^{lB})$$

After the transaction t, the condition, which the buyer has to satisfy, becomes

$$\hat{x}_{t-1}^B - \hat{E}_t^S + \left[1 - \alpha\left(1 + \frac{d_{t-1}^{lB} + e_t^{lS}}{d_{t-1}^{uB} + e_t^{uS}}\right)\right](d_{t-1}^{uB} + e_t^{uS}) \le L^B$$

where L^B is the buyer's limit. After simple algebraic manipulations the above inequality can be transformed to

$$\hat{x}_{t-1}^B + d_{t-1}^{uB} - \alpha(d_{t-1}^{uB} + d_{t-1}^{lB}) - [\hat{E}_t^S - e_t^{uS} + \alpha(e_t^{uS} + e_t^{lS})] \le L^B$$

Taking into account that from (11)

$$\frac{e_t^{lS}}{e_t^{uS}} = \frac{d_t^{lS}}{d_t^{uS}} = \frac{d^{lS}}{d^{uS}}$$

and introducing a simple analogy to the effective emission defined in (10)

$$E_{eff,t}^{S} = \hat{E}_t^{S}\Big\{1 - \big[1 - (1 + \frac{d_0^{lS}}{d_0^{uS}})\alpha\big]R_0^{uS}\Big\} = \hat{E}_t^{S}\big[1 - u^{S}(\alpha)\big] \qquad (15)$$

we obtain

$$\hat{l}_{t-1}^{B} - E_{eff,t}^{S} \leq L^{B} \qquad (16)$$

The necessary amount of bought permits to achieve the compliance with the risk α is now

$$\hat{E}_t^{S} \geq \frac{\hat{l}_{t-1}^{B} - L^{B}}{1 - u^{S}(\alpha)}$$

But actually it is not optimal for the buyer to purchase more emissions than it is necessary. Thus, he will rather buy at most the amount equal to the right hand side

$$\hat{E}_t^{S} \leq \frac{\hat{l}_{t-1}^{B} - L^{B}}{1 - u^{S}(\alpha)} \qquad (17)$$

The buyer may, however, prefer to buy, and certainly to pay, for the effective emissions. They have to satisfy the following simple condition to achieve the buyer's limit

$$E_{eff,t}^{S} \leq \hat{l}_{t-1}^{B} - L^{B} \qquad (18)$$

Let us again repeat that this bound gives only a preferable amount of emissions to be bought in order to achieve a compliance with the risk α. If not enough emissions is bought in this transaction, the remaining emissions may be possibly bought in subsequent transactions.

Considering the seller, let us denote

$$\hat{l}_{t-1}^{S} = \hat{x}_{t-1}^{S} + \Big[1 - \alpha\big(1 + \frac{d_{t-1}^{lS}}{d_{t-1}^{uS}}\big)\Big]d_{t-1}^{uS}$$

After selling the estimated emissions, the seller should not exceed its limit, that is, it should hold

$$\hat{x}^{S} + \hat{E}_t^{S} + \Big[1 - \alpha\big(1 + \frac{d_{t-1}^{lS} + e_t^{lS}}{d_{t-1}^{uS} + e_t^{uS}}\big)\Big](d_{t-1}^{uS} + e_t^{uS}) \leq L^{S}$$

which can be transformed to

$$\hat{l}^S_{t-1} + \hat{E}^S_t + e^{uS}_t - \alpha(e^{uS}_t + e^{lS}_t) \leq L^S$$

Taking into account the relations (11) we can write

$$\hat{E}^S_t + e^{uS}_t - \alpha(e^{uS}_t + e^{lS}_t) = \hat{E}^S_t[1 + u^S(\alpha)]$$

from where we get the condition on the maximum amount of estimated emissions to be sold

$$\hat{E}^S_t \leq \frac{L^S - \hat{l}^S_{t-1}}{1 + u^S(\alpha)} \tag{19}$$

Taking into account definition (15), the condition (19) can be formulated in terms of the effective emissions as

$$E^S_{eff,t} \leq \frac{1 - u^S(\alpha)}{1 + u^S(\alpha)}(L^S - \hat{l}^S_{t-1}) \tag{20}$$

This bound is more important than the bound of the buyer, in the sense that if it is not satisfied, the seller looses the compliance with the risk α.

Now, combining (16) and (19), and then (18) and (20) we get the theorem thesis. □

6 Market in effective emissions

Two kinds of emissions exist in the market outlined in the previous section, i.e. the estimated emissions and effective emissions. They have to be recalculated to each other during the trade. Here we propose a market with only one kind of emissions, which are the effective emissions.

Definition 4. *The value*

$$\tilde{L} = \frac{L}{1 + u(\alpha)}$$

is called the corrected limit.

Definition 5. *The value*

$$\tilde{l} = \frac{\hat{l}}{1 + u(\alpha)}$$

is called the corrected estimated emission.

Then, let us consider a market in effective emissions, acting according to the following principles.

- When trading, the effective emissions and corrected limits are used.
- After a transaction, the seller adjusts his corrected estimated emission according to the rule

$$\tilde{l}_t^S = \tilde{l}_{t-1}^S + \frac{E_{eff,t}^S}{1 - u^S(\alpha)} \tag{21}$$

- After a transaction, the buyer adjusts his corrected estimated emission according to the rule

$$\tilde{l}_t^B = \tilde{l}_{t-1}^B - \frac{E_{eff,t}^S}{1 + u^B(\alpha)} \tag{22}$$

These definitions allow us to formulate simple bounds for a reasonable amount of effective emissions to be traded in a transaction.

Theorem 2. *A reasonable amount of the effective emissions to be traded in a transaction t between a seller S and a buyer B is*

$$E_{eff,t}^S \leq \min\left\{(1 + u^B(\alpha))(\tilde{l}_{t-1}^B - \tilde{L}^B), (1 - u^S(\alpha))(\tilde{L}^S - \tilde{l}_{t-1}^S)\right\} \tag{23}$$

Proof. Multiplying and dividing the right hand side of (18) by $1 + u^B(\alpha)$ we get

$$E_{eff,t}^S \leq \frac{1 + u^B(\alpha)}{1 + u^B(\alpha)}(\tilde{l}_{t-1}^B - L^B)$$

Then, from Definitions (4) and (5), and using Theorem 1 we obtain the relations of the theorem. □

Then we derive a basic property of the market.

Theorem 3. *A party is compliant with the risk α after transaction t, if and only if its corrected estimated emission is not greater than its corrected limit*

$$\tilde{l}_t \leq \tilde{L}$$

Proof. Let us first consider the seller. From (21), we have

$$\tilde{l}_t^S = \tilde{l}_0^S + \frac{\sum_{i=1}^t E_{eff,t}^S}{1 - u^S(\alpha)}$$

Then, from the Definition 5, (15), and (8), taking also into account that $\hat{l}_0^S = \hat{l}^S$, we get

$$\tilde{l}_t^S = \frac{\hat{l}_0^S}{1+u^S(\alpha)} + \sum_{i=1}^{t} \hat{E}_t^S = \frac{(\hat{x}_0 + \sum_{i=1}^{t} \hat{E}_t^S)[1+u^S(\alpha)]}{1+u^S(\alpha)}$$

As the seller is compliant with the risk α after transaction t, that is he satisfies the appropriate inequality similar to (7), then, using additionally Definition 4

$$\tilde{l}_t^S \leq \frac{L^S}{1+u^S(\alpha)} = \tilde{L}^S$$

This proves the "if" part of the theorem for the seller. To prove the "only if" part let us notice that the reasoning can be easily reversed. So, the theorem is true for the seller.

Let us now consider the buyer. Similarly as above, we have

$$\tilde{l}_t^B = \tilde{l}_0^B - \frac{\sum_{i=1}^{t} E_{eff,t}^B}{1+u^B(\alpha)} = \frac{[1+u^B(\alpha)]\hat{x}_0^B - [1-u^S(\alpha)]\sum_{i=1}^{t} \hat{E}_t^S}{1+u^B(\alpha)}$$

Now, it is easy to notice that the numerator on the right hand side above, is equal to the left hand side of (9). Thus

$$\tilde{l}_t^B \leq \frac{\hat{L}^B}{1+u^B(\alpha)} = \tilde{L}^B$$

So, the theorem is also true for the buyer.

In the general case, we can order the buying transactions as the first $K < t$ transactions, without loosing generality. Then, considering only the first K transactions we know that the theorem is true. Treating now the estimated emissions and uncertainty spreads after first K transactions as a new starting point, and considering then the selling transactions we conclude from the former part of the proof that the theorem is true. This completes the proof of the theorem. □

In conclusion, the organization of the market is as follows.

1. Before starting the trade the inventories are recalculated to the expanded estimated emissions $\hat{l}$ according to (8), and then to the corrected estimated emissions $\tilde{l}$ according to Definition 5. The limits are recalculated to the corrected limits $\tilde{L}$ according to Definition 4.

2. In the trade the parties negotiate the trading conditions taking into account the effective emissions E_{eff}. The number of effective emission

possessed by a party is calculated as $E_{eff} = [1 - u(\alpha)]\tilde{L}$. After terminating the transactions both the seller and the buyer adjust their corrected estimated emissions according to (21) and (22), respectively.

3. To check the compliance, the present corrected estimated emissions are compared with the corrected limits.

In simulation of the trade it is convenient to express emission reduction cost curves in terms of the corrected estimated emissions. It would be helpful for comparison of trading prices with marginal prices of the parties.

7 Conclusions

The paper deals with the problem of trading of pollutant emissions in the case when the observed emission values are highly uncertain with asymmetric uncertainty distributions. Asymmetric uncertainty of national greenhouse gases inventories is evidenced by recent investigations, and particularly by Monte Carlo simulations of uncertainty distributions.

In the market proposed in the present paper the inventories and limits are converted to so-called corrected estimated emissions and corrected limits, which are smaller than original emissions and limits. This is due to inclusion of unreported emissions related to uncertainty. The organization of the market in effective emissions is presented and its basic properties are proved. The market operates almost the same way as a usual market. The difference is that after each transaction the effective emissions have to be appropriately converted in order to adjust the corrected estimated emissions of the trading parties.

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