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Kyoto Protocol and the growth of a small country open economy: a simple mathematical model approach [1]

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Abstract

This paper analyzes an impact of introducing the Kyoto protocols on the economic growth in a small economy (illustrated on the case of Polish economy). The analysis is performed using a simple one sector macroeconomic model, which includes economic as well as environmental factors. The endogenization of the environmental costs causes extra charges or benefits related to the trade in the emission permits. The model being considered constitutes a part of a finite horizon optimization problem with the zero end-point constraints on the net import and foreign debt, which are related to the assumed finite time horizon of economic adjustment. The optimal solution determines the optimal structure of investments in two competing technologies. Two simulation scenarios were performed, which differed in the level of permit prices, used as the model parameters.

1. Introduction

Imposing emission limits and trading emission permits is applied to influence the economic calculus of the economic agents accounting for the extra gains or costs related to the emission levels. This way excessive (above the limits) emissions cause increase of cost while unused limits can be traded, thus providing financial gains. Hence, there is a benefit for the relatively more efficient (in terms of emission) economies on the one hand and a stimulus for excessively polluting economies to control the emissions on the other hand. Solution to this problem is a mix of the decisions concerning the output, investment in the production technology and the foreign trade.

The technological transition caused by charging for the excessive pollutant emission should not be confused with the technical change, often called technical progress, which is commonly associated with the desirable changes2 in the production processes. However, in the problem being analyzed here, desirable changes in the abatement of the pollutant emission are connected with the mostly inevitable decrease of the productivity of capital, thus deteriorating direct economic efficiency.

The common tool for the analysis of the changes in the production processes is the production function. In most cases in the macroeconomic modeling these changes are assumed to be disembodied technical changes, see for example (Nordhaus W. and J. Boyer, 1999).

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[2] Conventionally it is assumed that such changes are capital- and/or labor- and/or materials/energy- saving ones.

However, imposing the emission limits forces the economic agents to switch from the commonly acquired capital goods to those, which cause less pollution. In this very case it is necessary to employ models with embodied technical change. On the other hand, the length of the time-period under consideration makes it necessary to employ both categories of the technical changes: embodied and disembodied ones.

The economy being investigated is a small open economy, by what we mean that changes of its emission does not impact the prices of the emission permits.

The aim of this model is to provide an explanation of the processes of the long-term technological transition caused by imposing the emission limits and trade. We focus our attention on the propagation of change via the exchange of capital, the rate of which depends on the depreciation and investment rates. In such an approach the short term adjustments are omitted. It is assumed that the latter are not related to the proper technological changes but to the short-term measures aimed at achieving the short-term goals, in effect such adjustment are economically less efficient.

In particular, we were inspired by the (Kyoto Protocol, 1998) process of greenhouse gas emission reduction. We look for an answer to the question on the interplay of imposed emission reduction curve and requirements on the technological and economic progress. In the greenhouse gas wording, we are interested how quick should be the technological progress in "clean" technologies, like hydro, solar, wind, tidal, wave, nuclear, biomass or other "cleaner", less greenhouse gas emitting energies, to achieve the given restriction on the reduction distributed in time, and on what cost, expressed in degree of slowing the economic growth. This problem agrees well with the ongoing discussion on the technological progress curves (Ang B.W. 2004, MacKenzie J.J. 2003, Riahi K. et al., 2004), as well as on scenarios and modeling of future national emissions (Kaivo-oja J. and J. Luukkanen , 2004, MacKenzie J.J., 2003, Manne A. and R. Richels 2004, McKibbin W.J. and Wilcoxen P.J. 2004).

However, the approach here is different. We consider a small national economy and a simplistic model trying to extract the primary dependences between interesting us variables using optimization tools. In a way, the approach can be considered as an extension of the question addressed in (Horabik J. and Z. Nahorski, 2003): where optimization of emission policy of a country with abatement and permit trade was considered, treated as a one stage static problem. Here our problem is a dynamic one, where we take into account the whole path to achieve the end stage goal. We also embed our problem in a simple macroeconomic model to find answer to a question of influence of solving the burden on country economic parameters.

The model is an optimization one; the results should be treated as a benchmark, as they provide an answer to the question: what would be performance of the economy, if it behaved optimally. This paper omits a discussion if such an economic policy is feasible.

In Section 2 the elements of the model are presented, namely models of technology, production, emission and foreign trade. Section 3 includes the simulation results and final remarks.

2. Technology, output, emission and foreign trade

The output of the model is measured in two ways, by the gross output and the added value. The former is needed because it includes the usage of the intermediary goods, production of which overwhelmingly contributes to the pollutant emissions. The latter is necessary to comply with the convention. All economic variables in the model are expressed in real terms.

The concept of technology in the model is strictly associated with the technological parameters of the capital assets. The technology vector T is defined below;

$$T_t = T[\, a,\ \delta,\ \gamma_t,\ PK^*_t,\ \mu^*_t \,], \tag{1}$$

where:

a - share of the intermediate consumption, constant coefficient;

δ - depreciation rate, constant coefficient;

PK^*_t - average productivity of capital,

γ_t- a long-term impact of technical change on production,

μ^*_t - average unit emission.

The value of the parameter a expresses the intensity of material inputs. Denoting by Q_t the gross output in the year t, to be specified later, the intermediate consumption used by the production sector can be expressed by the following expression:

$$a\, Q_t.$$

The dynamics of the stock of the capital assets is described by the commonly employed relationship:

$$K_t = K_{t-1} + I_t - \delta K_{t-1} = (1 - \delta) K_{t-1} + I_t\,, \tag{2}$$

where I_t denotes investment in the year t while K_t denotes the stock of the capital assets at the end of the year t. The investments made in a given year increases the stock of the capital in the succeeding year.

The capital assets are not assumed to be homogenous. This means that the capital assets in the stock belong to at least two generations of capital assets (characterized by different technology vectors (1)). Values of parameters PK^*_t and μ^*_t, which represent the mean values of the productivity of capital and unit emission, respectively, depend on the structure of the stock of capital assets.

In order to describe the process of determining PK^*_t and μ^*_t, a model is proposed below. As both these parameters will be described by the same model, let us assume that the ρ_t represents the marginal value of a variable of interest, while ρ^*_t represents the mean value of that variable of the entire capital assets.

Assume that agents invest I_t in the year t in such a way that in the year t an amount $I1_t$ is being invested in the technology 1 and $I2_t$ in the technology 2, and, of course, $I_t = I1_t + I1_t$. Under these assumptions the marginal value in the year t can be expressed by the following formula:

$$\rho_t = (I^1{}_t\,\rho^1 + I^2{}_t\,\rho^2) / I_t, \tag{3}$$

which is the weighted average of the values ρ^1 and ρ^2 with weights being the shares of respective technologies in the investment made in the year t?

The mean value ρ^*_t evolves in time according to the following equation[3]:

$$\rho^*_{t+1} = (1 - \lambda_t)\,\rho^*_t + \lambda_t\,\rho_t\,, \tag{4}$$

where the time-varying coefficient λ_t denotes the share of the capital assets obtained from the investment in the year t in the total amount of the capital assets at the end of the year t:

$$\lambda_t = I_t / K_t\,. \tag{5}$$

Note that λ_t, equation (5), is positive (or equal to 0, if there is no investment) and smaller than 1 (or equal to 1, if the end period stock of the capital assets were entirely created by recent investment). It follows from equation (4), that the average value ρ^*_{t+1} is unchanged when there is no investment, while ρ^*_{t+1} assumes the marginal value ρ_t, if the entire stock of the capital assets is created in the previous year.

The above described property of equation (4) is important as it enables for an adequate description of the process of change of the technological parameters, which is usually distributed in time and its rate depends on the rate of investments.

The gross output in year t, Q_t, is determined by the following production function[4]:

[3] This model was proposed in (Gadomski 2003).

$$Q_t = PK^*_t \, K_{t-1}. \tag{6}$$

By substituting PK^*_t for ρ^*_t the dynamics of PK^*_t can be described by equation (4).

Production causes emission E_t, which is the following function of the output and the average unit emission μ^*_t of the installed capital assets:

$$E_t = \mu^*_t \, Q_t. \tag{7}$$

The dynamics of μ^*_t is described by an equation based on (4). Note that in this model the emission is associated solely with the production rate and not with such factors as, for example, consumption.

In the greenhouse gas case, bounds on emission growth are imposed only in chosen commitment periods while the path to achieve the bounds is free. Here, however, we assign a path with a constant year decrement r for a smooth transition to the assigned goal. Thus, it is assumed that the emission norm N_t, set for a country in a year t, follows the following expression:

$$N_t = N_{t0} \{ 1 - \varphi [1 - (1 - r)^t] \}, \tag{8}$$

where N_{t0} denotes the emission in the initial year, φ denotes the planned percent decrease of the emission norm with per annum decrement of r percent. N_t converges to $N_{t0}(1 - \varphi)$. We take, however, a finite interval $T = 2/r$, after which this equality is only approximately true.

Balance of payment of the country depends solely on the trade balance (net import, which can be positive or negative), and the capital outflows related to the principal repayment or inflows when the country is a net creditor. The net foreign debt D_t of a country at the end of the year t is created by the net import M_t and the due repayment:

$$D_t = D_{t-1} + M_t - D_{t-1} / T_D = D_{t-1} (T_D - 1)/ T_D + M_t, \tag{9}$$

where expression D_{t-1} / T_D indicates that the average debt repayment period is T_D years. Note that the debt in this model is the real net debt so that it can assume negative values when a country is a net creditor.

Disposable aggregate supply Y_t accounts for the flows of foreign exchange:

$$Y_t = (1 - a) Q_t + M_t + P_t (N_t - E_t) - D_{t-1} (i + 1/T_D), \tag{10}$$

where P_t denotes the unit gain (when $N_t - E_t \geq 0$) or payment (when $N_t - E_t < 0$) for the excessive emission, and i stands for the real interest rate. Out of the commitment period $P_t = 0$. The last summand in equation (10) represents financial flows related to the repayment of principal D_{t-1} / T_D and interest on debt D_{t-1}.

[4] Production function (6) does not account for the impact of the labor on the output. It is assumed that the labor is abundant.

Consumption is determined as residual of the disposable aggregate demand diminished by the investment:

$$C_t = Y_t - I_t \tag{11}$$

3. Mathematical problem formulation

The aim of the policy in the period t, $t = t_0+1,..,t_0+T$, is to maximize the following function:

$$\max \{S = \sum_{i=1}^{T} Q_{t_0+i}(1+r)^{-i}\} \tag{12}$$

where r stands for the discount rate[5], over the following variables:

* amount of investment I_t in each period t, $t = t_0+1,...,t_0+T$, consisting of the decisions on the structure of investment: $I1_t$, standing for the investment in the capital representing the older technology, and $I2_t$ being an investment in the capital belonging to the technology with smaller emission ($I_t = I1_t + I2_t$)
* net import M_t in each period t, $t = t_0+1,...,t_0+T$

Equality constraints

Capital:

$K_t = K_{t-1} + I_t - \delta K_{t-1} = (1 - \delta) K_{t-1} + I_t$, equation (2);

Changes of the marginal productivity of capital in the technology 1 being the result of the disembodied technical progress with average growth rate r_{PK1} :

$PK1_t = PK1_{to} (1 + r_{PK1})^{t-to}$

Changes of the marginal productivity of capital in the technology 2 being the result of the disembodied technical progress with the average growth rate r_{PK2} :

$PK2_t = PK2_{to} (1 + r_{PK2})^{t-to}$

Output:

$Q_t = PK^*_t K_{t-1}$, equation (6);

Marginal productivity of investment:

$PK_t = (I1_t PK1_t + I2_t PK2_t) / I_t$, equation (3);

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[5] Conventionally the rate used in discounting equals interest rate. Factor $(1 + r)^{-i}$, $i = 1, 2,..,T$, can be also interpreted as a weigh attributed to the output in i-th year. In particular, problem with $r<0$ can be interpreted as a case, when later outputs are assigned greater weighs than the earlier ones.

Average productivity of capital:

$PK^*_t = PK^*_{t-1} + (I_{t-1} / K_{t-1})(PK_t - PK^*_{t-1})$, equations (4) and (5);

Changes of the marginal unit emission in the technology 1 being a result of the disembodied technical progress with the average growth rate $r_{\mu 1}$:

$\mu 1_t = \mu 1_{to} (1 + r_{\mu 1})^{t-to}$

Changes of the marginal unit emission in the technology 2 being a result of the disembodied technical progress with the average growth rate $r_{\mu 2}$:

$\mu 2_t = \mu 2_{to} (1 + r_{\mu 2})^{t-to}$

Marginal unit emission:

$\mu_t = (I1_t\, \mu 1_t + I2_t\, \mu 2_t) / I_t$, equation (3);

Average unit emission:

$\mu^*_t = \mu^*_{t-1} + (I_{t-1} / K_{t-1})(\mu_t - \mu^*_{t-1})$, equations (4) and (5);

Emission:

$E_t = \mu^*_t\, Q_t$. equation (7); equation (3);

Emission norm:

$N_t = N_{to} \{ 1- \varphi [1 - (1 - r)^{t-to}] \}$, equation (8);

Foreign debt/liability:

$D_t = D_{t-1} + M_t - D_{t-1} / T_D = D_{t-1} (T_D - 1)/ T_D$, equation (9);

Disposable aggregate supply:

$Y_t = (1 - \alpha) Q_t + M_t + P_t (N_t - E_t) - D_{t-1} (i + 1/T_D)$, equation (10);

Consumption:

$C_t = Y_t - I_t$, equation (11);

Inequality constraints

Scenario independent inequality constraints.

Minimum consumption (securing social stability; too high investment rate can cause social unrest):

$C_t \geq c_{min}\, Y_t$, (c$_{min}$ is the minimum value of the average propensity to consume)

Minimum investment (enforcing investment rate greater than that providing simple capital reproduction by a margin rate r_l):

$I_t \geq (1 + r_l) dK_{t-1}$

Balance of payment stability constraint:

$D_t \leq DPR_{max} Y_t$, (DPR_{max} stands for the maximum value of the admissible debt-to-GDP ratio)

Border constraints.

End period constraint 1:

$D_t = 0$, for $t \geq t_0 + k$; $1<k\leq T$

End period constraint 2:

$M_t = 0$, for $t \geq t_0 + k$; $1<k\leq T$.

The last two end period constraints provide foreign exchange balance condition at the end of the period, thus imposing the time limit for the adjustment policy.

4. Simulation results

As an example illustrating application the presented above model a transformation of the Polish economy to the bounds imposed in the Kyoto Protocol was considered. To better show changes and technological adaptations of the economy in the transition period it was assumed that the emission norms are valid bounds on emissions in every year. In the consequence, also emission permits were assumed to be traded on the yearly basis. Initial conditions for the model were set for 2001, and parameters characterizing Polish economy were estimated from the data from 1995 to 2000.

Capital assets $K_{2001} = 1732\ 10^9$ PLN.

Average productivity of the capital $PK*_{2001} = 1.007$.

Average unit emission $\mu*_{2001} = 1$.

Marginal productivity of the capital in the technology 1, $PK1_{2001}= 1.007$.

Marginal productivity of the capital in the technology 2, $PK1_{2001}= 0.80$.

Marginal unit emission in the technology 1, $\mu 1_{2001} = 1$.

Marginal unit emission in the technology 2, $\mu 2_{2001} = 0.75$.

Emission, $E_{2001} = 1.456 \cdot 10^8$ tC.

Emission norm, $N_{2001} = 3.734 \cdot 10^8$ tC.

Debt, $D_{2001} = 0$.

Two developed scenarios were based on different prices for the unit emission: the first with a low price (60 zł/tC), denoted as lp, and the second with a higher price (600 zł/tC), denoted as hp. Lower price means that a country having a surplus of emission permits is less sensitive to this stimulus on the one hand, but on the other hand receives less from the sale of the emission permits.

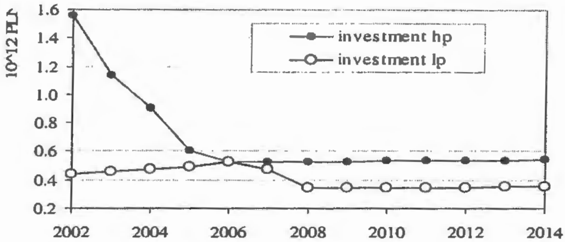

Fig.1. Total investment in Scenarios hp and lp.

Results

Investment rates in both scenarios optimized by the model are shown in Fig.1. In Scenario hp the whole investment is located in the capital belonging to the less productive, but also less emitting technology. Higher permit prices enable for a high investment rate at the beginning of the period contributing to a higher accumulation of capital. Comparison of the investment and the depreciation rates is presented in Fig.2.

From the year 2006 on the investment equals the minimum investment bound. Further investment growth is an effect of the technical progress (which in both scenarios is set at 1% per year).

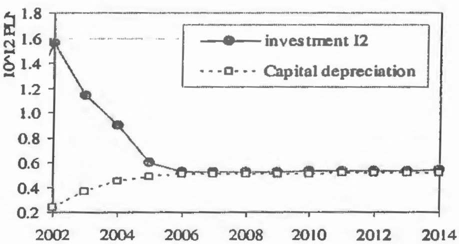

Fig.2. Investment and capital depreciation in Scenario hp.

In Scenario lp provided by optimisation, contrary to Scenario hp, all capital invested is located in the more productive and more emitting technology. The investment and depreciation rates in Scenario lp are presented in Fig.3.

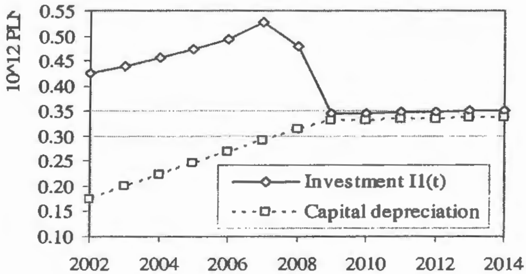

Fig.3. Investment and capital depreciation in Scenario lp.

Similarly to Scenario hp, in Scenario lp from the year 2009 on the investment is determined by the minimum bound. Also in this case further increase of investment is due to the technical progress. The time-path of the debt in both scenarios is depicted in Fig.4.

Adjustment to the imposed restrictions in the scenarios considered is considerably different. In Scenario lp an effort to adjust requires external support in the form of a credit. In Scenario hp adjustment is mainly financed by higher revenues from sales of the emission permits, although in the years 2004 and 2005 loans are drawn. But these loans are small in comparison to total revenues from the sales of emission permits.

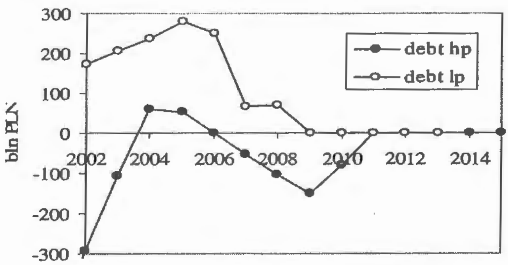

Fig.4. Debts in Scenarios hp and lp.

The overall performance of the economy in both scenarios is presented in Fig.5. The figure shows the advantage of the Scenario hp over Scenario lp in the considered case. In the former the economy reveals much higher dynamics at the initial period, what enables greater investments and, in consequence, greater production.

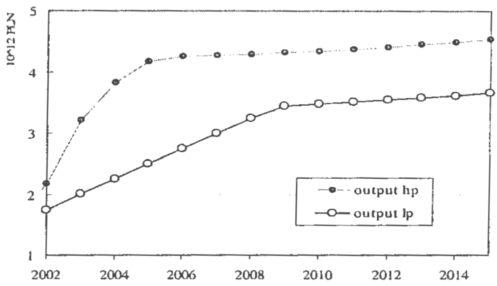

Fig.5. Output in Scenarios hp and lp.

5. Conclusions

The optimizations performed allowed us to make the following observations:

1. A small country endowed with excessive emission permits has mid-term benefits from the emission permit trade. These benefits increase with the price of permits.
2. Introduction of the emission control imposes adjustment. In both scenarios considered it caused increased investment. Adjustment requires a financial effort.
3. In the case of the lower permission prices no change in the production technology occurred. Increased investment was financed solely by the foreign loans. Further economic growth is determined by the rate of the technical progress.
4. In the case of the higher permission prices most investment was financed by the revenues from the trade in the emission permits. There occurred a change of the production technology; all invested capital belonged to the new less productive but less emitting technology.
5. In both analyzed scenarios the period of a fast economic growth is succeed by the period where further economic growth is determined by the rate of the technical progress.

It is perhaps worth to add that the results have rather an illustrative character, both because of the assumptions taken and because the parameters used for computations were only roughly estimated from the unsure data.

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