*The functions $E_i$ and $I_i$, § 19.*

§ 19. I have not worked out the corresponding formulæ for $E_i$ and $I_i$, which are considered by Mr. Kleiber in his paper. The series for $\frac{2E}{\pi}$ and $\frac{2I}{\pi}$ in equations (170) and (171) are obviously incorrect, and it would seem that they do not, as in the case of equations (164) and (167), form part of the required expansions.* The formulæ (168) and (169) are therefore also inaccurate. It may be added that the relations (123) and (124) need some modification, as when $i = 0$, they are intended to reduce to

$$\frac{dE}{dh} = \frac{I}{2h}, \quad \frac{dI}{dh} = -\frac{E}{2h'}.$$

It should be stated that Mr. Kleiber's paper was not put into type until after his death, so that he did not see any part of it in print. The paper has been printed from the manuscript without alteration, except that slips of the pen, when noticed, were corrected.

## NOTE ON A PROBLEM IN THE THEORY OF NUMBERS.

By *W. W. Rouse Ball.*

THE elegant theorem on the resolution of numbers of a certain form into factors, which was given by Mr. Birch in the number of the *Messenger* for August (pp. 52–55), may be applied to determine the factors of the number 100895598169.

The partition of this number was proposed to Fermat by Mersenne; and, in a letter dated April 7, 1643, Fermat wrote to Mersenne, "Vous me demandez si le nombre 100895598169 est premier ou non, et une méthode pour découvrir dans l'espace d'un jour s'il est premier ou composé. A cette question, je réponds que ce nombre est composé et se fait du produit de ces deux: 898423 et 112303, qui sont premiers." The discovery of the method by which Fermat arrived at this result has been one of the puzzles of higher arithmetic.

Mr. Birch's theorem on the factors of a number $N$ depends on the proposition that, if numbers $x$ and $y$ (such that $N>x>y$) can be found to satisfy the equation

$$x^2 = Ny + 1,$$

* The series for $E$ and $I$ were given on p. 149 of Vol. XXI.

then $x/(N-x)$ can be expressed as a continued fraction of the form

$$\frac{1}{a_0-1}\,+\,\frac{1}{a_1}\,+\ldots+\,\frac{1}{a_{n-1}}\,+\,\frac{1}{a_n}\,+\,\frac{1}{a_{n-1}}\,+\ldots+\,\frac{1}{a_1}\,+\,\frac{1}{a_0};$$

and $N$ is equal to the continuant

$$(a_0, a_1, \ldots, a_{n-1}, a_n, a_{n-1}, \ldots, a_1, a_0),$$

of which $(a_0, a_1, \ldots, a_{n-1})$ is a factor.

In Mersenne's problem, $N = 100895598169$. If we take $y=32$ and $x=1796847$, then $x^2=Ny+1$, and we find

$$\frac{x}{N-x}=\frac{1796847}{100893801322}=\frac{1}{56150}\,+\,\frac{1}{2}\,+\,\frac{1}{7}\,+\,\frac{1}{2}\,+\,\frac{1}{56151},$$

and $$N=(56151,\ 2,\ 7,\ 2,\ 56151).$$

Hence a factor of $N$ is the continuant (56151, 2), which is equal to

$$\begin{vmatrix} 56151, & 1 \\ -1, & 2 \end{vmatrix}$$

that is, 112303. Hence a factor of $N$ is known.

The question proposed to Fermat and his answer seem to have escaped the attention of many writers on the theory of numbers; but, as far as I know, no solution of the problem has been hitherto published, and therefore it is particularly interesting to find that it is covered by the theorem given by Mr. Birch.

*September* 22, 1892.

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# NOTE ON PSEUDO-ELLIPTIC INTEGRALS.

By *W. Burnside.*

Any integral which while apparently elliptic may really be reducible to a logarithm is expressible as the sum of a number of terms of the form

$$\int \frac{(x-a)\,dx}{(x-b)\sqrt{\{f_4(x)\}}},$$

where $f_4(x)$ is a rational integral quartic function. For the case in which the integral consists of a single term I propose