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Abstract

In the paper we propose a fuzzy version of the Kendall control chart introduced by Hryniewicz and Szediw originally for monitoring production processes. This chart is a relatively simple tool for looking for dependencies between consecutive observations of the process. The proposed fuzzy version of the Kendall control chart is based on the fuzzy Kendall's τ statistic and possibility and necessity measures used in the possibility theory and fuzzy sets.

Keywords: Control chart, Fuzzy data, Kendall τ.

1 Introduction

Statistical process control (SPC) aims at the goal of achieving continuous improvement in quality. This improvement may be attained by removal of all possible causes (known as assignable causes) of process deterioration. In order to accomplish that goal the SPC procedures have been devised to assure continuous monitoring of the process under study and quickly detect the occurrences of assignable causes of its deterioration. The Shewhart control chart, and other control charts - like CUSUM, MAV, and EWMA - are the most popular SPC methods used to detect whether observed process is under statistical control, i.e if it is not deteriorated. One may note, however, that the exactly same procedures may be used for the monitoring of other processes. It is sufficient to note that the concept of 'being under statistical control' is equivalent to another frequently used concept of 'being stable'. In the SPC context stability means that the results of all process measurements are governed by the same probability distribution.

Statistical procedures of SPC may be also used for the monitoring of other processes. In all such applications the concept of 'stability' has to be the same. However, the concept of the 'assignable cause' may be quite different. The detection of the presence of an 'assignable cause' may be interpreted, for example, as the influence of a certain event on the results of public opinion polls. In all such or similar cases the statistical procedure remains the same, but its interpretation may depend upon a particular context.

Originally all SPC statistical tools have been designed under the assumption that process measurements are described by independent and identically distributed random variables. Moreover, it is also assumed that all measurements are precisely reported, and the only source of their uncertainty is due to their randomness. In the majority of practical cases these assumptions are fulfilled at least approximately. However, there exist processes where consecutive observations may be correlated in an obvious way. Consider, for example, consecutive observations coming from public opinion polls. When these observations are made in short time intervals, and their values are influenced by the same event, it seems to be unwise to assume that they are mutually independent. In such situation they usually form a dependent time series whose stability cannot be evaluated using statistical procedures developed under the assumption of independence.

In the area of statistical quality control this problem drew attention of researchers in the late 1970s. First, the phenomenon of autocorrelated observations was noticed for processes described by individual observations (e.g. chemical processes). Later, its importance was recognized for the case when samples taken from processes are independent, but measurements within each sample are autocorrelated. The most general, and the most difficult to be addressed, case of the autocorrelation both within samples and between samples has not been treated yet in a sufficiently practical way.

The design of control charts under the assumption of autocorrelation has attracted many researchers during the last more than twenty years. One of the most frequently used approaches was published in Alwan and Roberts [?] who proposed to chart so called residuals, i.e. differences between actual observations and their predicted, in accordance with a pre-specified mathematical model, values. This approach has its roots in the statistical analysis of time series used in automatic control, originated by the famous book of Box and Jenkins [2]. In other approaches, the effect of autocorrelation was taken into account in the design of control limits of a control chart. A comprehensive overview of pertinent literature can be found in a recent publication of Hryniewicz and Szediw [8].

The common feature of all SPC procedures proposed for dealing with autocorrelated data is their high level of complexity. As a matter of fact, shop-floor practitioners are usually unable to work with autocorrelated data without an assistance of specialists. Moreover, in many cases it is necessary to use specialized software. Therefore, there is a practical need to detect autocorrelation in data as quickly as possible. Statistical tools available for such analysis are available, but generally they have been developed for dealing with normally distributed autoregression processes. In practice, however, we usually do not know whether the investigated process can be described by the normal autoregressive model. Thus, there is a need to develop a simple (for practitioners) non-parametric (distribution-free) tool that would be useful for the detection of autocorrelation in data. Such a tool - a Kendall control chart - has been proposed recently by Hryniewicz and Szediw [8]. We present this procedure in the second section of this paper. In the third section we present the extension of the results of Hryniewicz [7], who developed a fuzzy version of the Kendall of test independence for imprecise (fuzzy) data. The results presented in the second and third sections allow us to propose a fuzzy version of the Kendall control chart. This new result is presented in the fourth section of the paper. The paper is concluded in its last section.

2 Kendall control chart for testing lack of autocorrelation in case of crisp data

Dependence or autocorrelation of statistical data has great influence on the behaviour of statistical procedures. Practical consequences of such dependence may be quite different. In some cases certain types of dependence may be considered natural and even desirable, but in the majority of practical cases the existence of autocorrelation between observed statistical data is considered as an unwanted feature. In any case, however, decision processes based on dependent statistical data are difficult to be designed, as probabilities of erroneous decisions, whose knowledge is necessary for the proper design of the procedure, are difficult to be computed. When we look at statistical textbooks we may find that the coefficient of autocorrelation is the main statistic proposed for testing independence of serial data. However, its usage is well justified only for normally distributed data. When the assumption of normality is not valid, statistical tests based on the coefficient of autocorrelation may not be as effective as in the normal case. Therefore, we need to use methods that are not dependent upon the type of probability distribution, i.e. distribution-free non-parametric statistical methods.

Hryniewicz and Szediw in their paper [8] propose to use for this purpose the well known Kendall's τ statistic, which is a fundamental statistical measure of association.

Let $Z_1, Z_2, \dots, Z_n$ denote a random sample of n consecutive process observations. These observations can be transformed into two-dimensional vector (X_i, Y_i), where $X_i = Z_i$ and $Y_i = Z_{i+1}$ for $i = 1, 2, \dots, n-1$. Then, the Kendall's τ sample statistic which measures the association between random variables X and Y is given by the following formula

$$\tau_n = \frac{4}{n-1} \sum_{i=1}^{n-1} V_i - 1. \tag{1}$$

where

$$V_i = \frac{card\{(X_j, Y_j) : X_j < X_i, Y_j < Y_i\}}{n-2}, i = 1, \dots, n-1. \tag{2}$$

Kendall's τ_n given by (1) can be represented as a function of the number of disconcordances M, i.e. the number of pairs (Z_i, Z_{i+1}) and (Z_j, Z_{j+1}) that satisfy either $Z_i < Z_j$ and $Z_{i+1} > Z_{j+1}$ or $Z_i > Z_j$ and $Z_{i+1} < Z_{j+1}$. In these terms we have

$$\tau_n = 1 - \frac{4M}{(n-1)(n-2)}, \tag{3}$$

where

$$M = \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} I(Z_i < Z_j, Z_{i+1} > Z_{j+1}), \tag{4}$$

and $I(A)$ represents the indicator function of the set A. When the vectors $(X_1, X_2, \dots, X_n)$ and $(Y_1, Y_2, \dots, Y_n)$ are mutually independent, the pairs of observations (X_i, Y_i), $i = 1, 2, \dots, n-1$ are also independent, and the probability distribution of (1) is well known. However, in case of time series, even in the case of mutual independence of individual observations in the series $Z_1, Z_2, \dots, Z_n$, pairs of observations

(X_i, Y_i) are dependent, and the probability distribution of τ_n for small values of n has been obtained only recently [5]. Ferguson et al. [5] present a table with precise probabilities $P_n(M \leq m)$ for $n = 3, \ldots, 10$, and with approximate probabilities for $n > 10$. The analysis of these probabilities presented in [8] shows that it is not possible to construct a one-sided statistical test of independence against the alternative of positive dependence with the same probability of false alarms as in the case of a Shewhart control chart. Thus, for small values of the number of consecutive observations n it is in principle impossible to make precise comparisons of control charts based on Kendall's τ with classical three-sigma Shewhart control charts. Moreover, the close investigation of the probability distribution of M presented in [8] shows that due to a discrete nature of the Kendall's τ this situation is the same in even for larger values of n.

The application of Kendall's τ requires the usage of tables with critical values of this statistic. This is rather impractical, so Hryniewicz and Szediw [8] proposed a new procedure designed as a Shewhart-type control chart based on τ_n with the following control limits: the lower limit

$$K_L = max(E(\tau_n) - k\sigma(\tau_n), -1) \tag{5}$$

and the upper limit

$$K_U = min(E(\tau_n) + k\sigma(\tau_n), 1). \tag{6}$$

They called that procedure *the Kendall control chart.* To calculate the limits of the Kendall control chart we use the following formulae for the expected value and the variance of τ_n given in [5]:

$$E(\tau_n) = -\frac{2}{3(n-1)}, n \geq 3. \tag{7}$$

$$V(\tau_n) = \frac{20n^3 - 74n^2 + 54n + 148}{45(n-1)^2(n-2)^2}, n \geq 4. \tag{8}$$

Hryniewicz and Szediw [8] noticed that the probability distribution of τ_n is not symmetric for small values of n. Therefore, the properties of the proposed control chart for testing independence of consecutive observations from a process may be improved by using control lines that are asymmetric around the expected value of τ_n. However, for sake of simplicity, they decided not to consider this possibility, and investigated the properties of the Kendall's control chart in its symmetric version.

3 Kendall test for fuzzy data

The situation becomes much more difficult to analyze when the observed data points are imprecisely defined, and described by fuzzy sets. Hryniewicz [7] considered this problem for the case of fuzzy binomial data. However, nothing precludes consideration of a more general case when the series of consecutive observations is given by a vector of fuzzy data $(\tilde{Z}_1, \tilde{Z}_2, \ldots, \tilde{Z}_n)$. It is rather unlikely that this fuzzy time series forms an unequivocal ordering of all data points. Therefore, the value of the Kendall's τ statistic for the fuzzy data has to be also fuzzy.

In order to compute the fuzzy version of the Kendall's τ statistic for the considered fuzzy time series let us assume that each fuzzy observation is described by a membership function $\mu_i(z)$, $i = 1, \ldots, n$. Now, we can use the Zadeh's extension principle for the definition of the fuzzy equivalent of the Kendall's τ statistic in the considered fuzzy case. First, let us rewrite (2) in the following form

$$V_i = \frac{card_{j \neq i}\{(Z_j, Z_{j+1}) : Z_j < Z_i, Z_{j+1} < Z_{i+1}\}}{n-2}, i = 1, \ldots, n-1. \quad (9)$$

We have to write the fuzzy form of (9) for the fuzzy vector $(\tilde{Z}_1, \tilde{Z}_2, \ldots, \tilde{Z}_n)$. Let us notice now that each fuzzy data point $\tilde{Z}_i$ is completely defined by the set of its α-cuts $[Z^{\alpha}_{i,L}, Z^{\alpha}_{i,U}]$, $0 < \alpha \leq 1$. Hence, the fuzzy equivalent of V_i, denoted by $\tilde{V}_i$, is defined by the set of its α-cuts $[V^{\alpha}_{i,L}, V^{\alpha}_{i,U}]$, $0 < \alpha \leq 1$, where

$$V^{\alpha}_{i,L} = \min_{\substack{z_i \in [Z^{\alpha}_{i,L}, Z^{\alpha}_{i,U}] \\ i=1,\ldots,n}} \frac{card_{j \neq i}\{(z_j, z_{j+1}) : z_j < z_i, z_{j+1} < z_{i+1}\}}{n-2} \quad (10)$$

and

$$V^{\alpha}_{i,U} = \max_{\substack{z_i \in [Z^{\alpha}_{i,L}, Z^{\alpha}_{i,U}] \\ i=1,\ldots,n}} \frac{card_{j \neq i}\{(z_j, z_{j+1}) : z_j < z_i, z_{j+1} < z_{i+1}\}}{n-2} \quad (11)$$

for $i = 1, \ldots, n-1$. Having the α-cuts $[V^{\alpha}_{i,L}, V^{\alpha}_{i,U}]$, $0 < \alpha \leq 1$ for all $i = 1, \ldots, n$ we can straightforwardly calculate the α-cuts of the fuzzy Kendall's τ statistic $[\tau^{\alpha}_L, \tau^{\alpha}_U]$, $0 < \alpha \leq 1$, and thus we can obtain its membership function.

Despite the compact notation of (10) and (11) the calculation of the membership function of the fuzzy Kendall's τ statistic may be, in a general case, a difficult and computationally intensive task. For this purpose

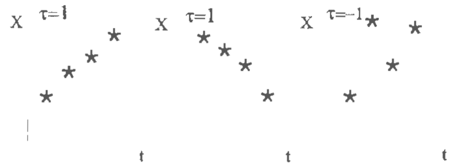

Figure 1: Configurations of observations of a time series for the values of the Kendall's τ statistic are either the largest or the lowest

we can use a general methodology proposed by Denoux et al. [3] for the calculation of fuzzy rank statistics. However, for the particular case of the fuzzy Kendall control chart we can use a simple approximate algorithm. The construction of this algorithm will be apparent if we consider the influence of the *pattern* of consecutive observations on the value of Kendall's τ. On Figure 1 taken from [7] we present three distinctive patterns of crisp data which lead to limiting values of Kendall's τ, namely 1 and -1. The maximal value of Kendall's τ , equal to 1, is attained when consecutive points form a monotonically increasing or decreasing series. On the other hand, the minimal value of Kendall's τ , equal to -1, is attained when consecutive points numbered by even numbers form a monotonically decreasing series and simultaneously consecutive points numbered by odd numbers form a monotonically increasing series, and vice versa. In both cases the increasing and decreasing series should not intersect.

For the given value of α the largest value of τ is attained for a series of values $z_i^L \in [z_{i,L}^{\alpha}, z_{i,U}^{\alpha}]$, $i = 1, \ldots, n$, $0 < \alpha \leq 1$ that form a monotone (or nearly monotone) increasing (decreasing) series. To find such a series we can start with the series $z_i^* = z_{i,L}^{\alpha}$, $i = 1, \ldots, n$, $0 < \alpha \leq 1$. In the next step we can increase certain values of this series in order to arrive at a monotone (or nearly monotone) increasing series. The same procedure should be repeated in search of a monotone (or nearly monotone) decreasing series. In this case we can start with the series $z_i^* = z_{i,U}^{\alpha}$, $i = 1, \ldots, n$, $0 < \alpha \leq 1$, and in the next step we should decrease certain values of this series in order to arrive at a monotone (or nearly monotone) decreasing series.

The lowest value of τ is attained for a series of values $z_i^U \in [z_{i,L}^{\alpha}, z_{i,U}^{\alpha}]$, $i = 1, \ldots, n$, $0 < \alpha \leq 1$ that form an alternating series of values such that the observations with odd (even) indices form a decreasing series, and the observations with even (odd) indices form an increasing series. To find such a series we can start with the series $z^\alpha_{1,L}, z^\alpha_{2,U}, z^\alpha_{3,L}, \ldots$ or with the series $z^\alpha_{1,U}, z^\alpha_{2,L}, z^\alpha_{3,U}, \ldots$. In the next step we can increase certain values initially defined by the lower limits of the α-cuts and decrease certain values initially defined by the upper limits of the α-cuts in order to arrive at an alternating (or nearly alternating) series.

Let us describe that heuristic algorithm in a more formal way. For notational convenience we omit the symbol α which refers to a chosen α-cut.

The upper limit of the α-cut for the fuzzy value of Kendall's τ is computed according to Algorithm 1.

Algorithm 1

begin
set ε *to a small value*
$k = 0$
$z^*_{k+1} = z_{k+1,L}$

*loop*1 : $k = k + 1$
if $[(z_{k+1,L} \geq z^*_k)$ *or* $(z_{k+1,U} < z^*_k)]$ *then* $z^*_{k+1} = z_{k+1,L}$
else $z^*_{k+1} = z^*_k + \varepsilon$
if $k < n - 1$ *goto loop*1
use $(z^*_1, \ldots, z^*_n)$ *for the calculation of* $\tau_{U,1}$
$k = 0$
$z^*_{k+1} = z_{k+1,U}$

*loop*2 : $k = k + 1$
if $[(z_{k+1,L} \geq z^*_k)$ *or* $(z_{k+1,U} < z^*_k)]$ *then* $z^*_{k+1} = z_{k+1,U}$
else $z^*_{k+1} = z^*_k - \varepsilon$
if $k = n - 1$ *stop*
*else goto loop*2
use $(z^*_1, \ldots, z^*_n)$ *for the calculation of* $\tau_{U,2}$
$\tau_U = \max(\tau_{U,1}, \tau_{U,2})$
end

The lower limit of the α-cut for the fuzzy value of Kendall's τ is computed in a more complicated way, as the pattern of consecutive points for which τ attains its minimal value is more complicated (see

Algorithm 2

$begin$
$set\ \varepsilon\ to\ a\ small\ value$
$k = 1$
$z^*_k = z_{k,U}$

$loop1:$ $if\ [(z_{k+2,L} \geq z^*_k)\ or\ (z_{k+2,U} < z^*_k)]\ then\ z^*_{k+2} = z_{k+2,U}$
$else\ \ z^*_{k+2} = z^*_k - \varepsilon$
$k = k + 2$
$if\ k < n - 2\ goto\ loop1$
$k = 2$
$z^*_k = z_{k,L}$

$loop2:$ $if\ [(z_{k+2,L} \geq z^*_k)\ or\ (z_{k+2,U} < z^*_k)]\ then\ z^*_{k+2} = z_{k+2,L}$
$else\ \ z^*_{k+2} = z^*_k + \varepsilon$
$k = k + 2$
$if\ k < n - 2\ goto\ loop2$
$use\ (z^*_1, \ldots, z^*_n)\ for\ the\ calculation\ of\ \tau_{L,1}$
$k = 2$
$z^*_k = z_{k,U}$

$loop3:$ $if\ [(z_{k+2,L} > z^*_k)\ or\ (z_{k+2,U} < z^*_k)]\ then\ z^*_{k+2} = z_{k+2,U}$
$else\ \ z^*_{k+2} = z^*_k - \varepsilon$
$k = k + 2$
$if\ k < n - 2\ goto\ loop3$
$k = 1$
$z^*_k = z_{k,L}$

$loop4:$ $if\ [(z_{k+2,L} \geq z^*_k)\ or\ (z_{k+2,U} < z^*_k)]\ then\ z^*_{k+2} = z_{k+2,L}$
$else\ \ z^*_{k+2} = z^*_k + \varepsilon$
$k = k + 2$
$if\ k < n - 2\ goto\ loop4$
$use\ (z^*_1, \ldots, z^*_n)\ for\ the\ calculation\ of\ \tau_{L,2}$
$\tau_L = \min(\tau_{L,1}, \tau_{L,2})$
end

Fig.1 for an example).

The application of both algorithms does not guarantee that the computed pair (τ_L, τ_U) is the true α-cut for the fuzzy value of Kendall's τ. However, it gives a very good approximation or may be used as a starting point for further search of both limits of the α-cut.

4 Kendall fuzzy control chart

The fuzzy version of the Kendall τ statistic may be now used for the construction of the Kendall control chart for fuzzy data. In contrast to the case of crisp data, we cannot plot points on a control chart, and thus we cannot determine in an unequivocal way whether the observed pattern of measurement points indicates the existence of associations (dependencies) between consecutive values of the observed process. There are two general possibilities to cope with this problem: either to find a crisp representation of the fuzzy value of $\tilde{\tau}_n$ (i.e. to defuzzify this statistic) or to introduce an additional requirement which will be used for making decisions. In this paper we apply the possibilistic approach proposed in Hryniewicz [6]. According to this approach our decisions will be made after the calculation of possibility and necessity measures that the observed fuzzy value $\tilde{\tau}_n$ falls outside the control limits given by (5) and (6).

For deciding whether consecutive fuzzy observations on the Kendall control chart are dependent (associated) we propose to use two indices proposed by Dubois and Prade [4], namely the *Possibility of Dominance* and *Necessity of Strict Dominance* indices. For two fuzzy numbers $\tilde{A}$ and $\tilde{B}$ the *Possibility of Dominance (PD)* index is calculated from the formula

$$PD = Poss(\tilde{A} \geq \tilde{B}) = \sup_{x,y:x \geq y} \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)\}. \tag{12}$$

The PD index gives the measure of *possibility* that the fuzzy number $\tilde{A}$ is not smaller than the fuzzy number $\tilde{B}$. Positive value of this index tells the decision maker that there exists even slightly evidence that the relation $\tilde{A} > \tilde{B}$ is true.

The degree of *conviction* that the relation $\tilde{A} > \tilde{B}$ is true is reflected by the *Necessity of Strict Dominance (NSD)* index defined as

$$\begin{aligned} NSD &= Ness(\tilde{A} > \tilde{B}) = 1 - \sup_{x,y:x \leq y} \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)\} \\ &= 1 - Poss(\tilde{B} \geq \tilde{A}). \end{aligned} \tag{13}$$

The NSD index gives the measure of *necessity* that the fuzzy number $\tilde{A}$ is greater than the fuzzy number $\tilde{B}$. Positive value of this index tells the decision maker that there exists rather strong evidence that the relation $\tilde{A} > \tilde{B}$ is true.

In our case the calculation of PD and NSD is quite easy as the limits we compare the fuzzy values to are not fuzzy. Hence, for the upper and lower control limits we can calculate the following values of the NSD index:

$$NSD_U = Ness(\tilde{\tau}_n > K_U) = 1 - \alpha_{N,U}, \tag{14}$$

where

$$\alpha_{N,U} = \inf_{\alpha}(\alpha : \tau^{\alpha}_{n,L} \geq K_U) \tag{15}$$

and

$$NSD_L = Ness(K_L > \tilde{\tau}_n) = 1 - \alpha_{N,L}, \tag{16}$$

where

$$\alpha_{N,L} = \sup_{\alpha}(\alpha : \tau^{\alpha}_{n,U} \leq K_L). \tag{17}$$

In order to make an unequivocal decision we have to set a required minimal value of NSD_U and NSD_L. If the calculated value of the NSD index is greater than or equal to this minimal value we can say that the observed fuzzy data reveal a pattern characteristic for dependent statistical data.

In classical statistical process control we also use warning signals. In the case of the Kendall control chart for fuzzy data for the generation of warning signals we propose to use the PD index defined by (12). Similarly to the previously considered case of the NSD index, the calculation of the PD index for both limits of the control chart is the following:

$$PD_U = Poss(\tilde{\tau}_n > K_U) = 1 - \alpha_{P,U}, \tag{18}$$

where

$$\alpha_{P,U} = \sup_{\alpha}(\alpha : \tau^{\alpha}_{n,U} \geq K_U) \tag{19}$$

and

$$PD_L = Poss(K_L > \tilde{\tau}_n) = 1 - \alpha_{P,L}, \tag{20}$$

where

$$\alpha_{P,L} = \inf_{\alpha}(\alpha : \tau_{n,L}^{\alpha} < K_L). \tag{21}$$

In order to make an unequivocal decision we have to set a required minimal value of PD_U and PD_L. If the calculated value of the PD index is greater than or equal to this minimal value we can say that there is a warning that the observed fuzzy data reveal a pattern characteristic for dependent statistical data. It is worth to note that the value of the PD index equal to one shows that the value of the respective NSD index has to be greater than zero.

5 Conclusions

The Kendall control chart for crisp data was introduced in order to give practitioners a relatively easy to use statistical tool for looking for dependencies between consecutive measurements displayed on control charts such as e.g. the Shewhart control chart. The statistical characteristics of the Kendall chart do not depend on the probability distribution of observed data. Therefore, it can be safely used during initial investigations of considered processes when the available information is not sufficient for a recise description of the process, and thus for choosing a better statistical tool. In the considered in this paper case of imprecise data this important feature is still valid. However, due to the imprecise character of data necessary computations become much more involved. The proposed algorithm for the calculation of a α-cuts of the fuzzy Kendall's statistic τ_n is rather simple but it may give only approximate values of the α-cut limits.

Further investigations of the Kendall fuzzy control chart should address such problems as the efficient computations of the precise limits of α-cuts. This problem is really important as the number of observations that is necessary for good discrimination between independent and dependent (autocorrelated) processes may be quite large. Thus, the optimization problems defined by (10)–(11) may require significant computational times. Another interesting and important problem which is still waiting for its solution is the influence of fuzziness of data on important characteristics of a control chart, such as e.g. the average run length ARL (the average time to signal).

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