The common eighth point of intersection, if it exists, must be

$$(A,\ B,\ C)\ ;\ (A,\ Q,\ R),\ (B,\ R,\ P),\ (C,\ P,\ Q)\ ;$$
$$(P,\ B,\ C),\ (Q,\ C,\ A),\ (R,\ A,\ B)\ ;\ \text{or}\ (P,\ Q,\ R).$$

There is no point $(A, B, C)$.

There is no point $(A, Q, R)$, for $Q$, $R$ intersect only in the points 1, 2, 3, 4, no one of which lies on $A$; and similarly there is no point $(B, R, P)$ or $(C, P, Q)$.

$B$, $C$ intersect in 5 which is a point on $P$, and thus 5 is the only point $(P, B, C)$. Similarly 6 is the only point $(Q, C, A)$ and 7 is the only point $(R, A, B)$.

$P$, $Q$, intersect in 1, 2, 3, 4, which are each of them on $R$; hence 1, 2, 3, 4 are the only points $(P, Q, R)$.

Hence the points 1, 2, 3, 4, 5, 6, 7 present themselves each once, and only once, among the intersections of the three cubics, and there is no common eighth point of intersection.

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# ON WARING'S FORMULA FOR THE SUM OF THE *m*th POWERS OF THE ROOTS OF AN EQUATION.

By Prof. *Cayley*.

The formula in question, Prob. I. of Waring's *Meditationes Algebraicæ*, Cambridge, 1782, making therein a slight change of notation, is as follows: viz. the equation being

$$x^n + bx^{n-1} + cx^{n-2} + dx^{n-3} + \ldots = 0,$$

then we have

$$(-)^m S_m =$$

$$\begin{array}{rl}
 & b^m \\
-mc & b^{m-2} \\
+md & b^{m-3} \\
\left.\begin{array}{r} -me \\ +\tfrac{1}{2}m.m-3.c^2 \end{array}\right\} & b^{m-4} \\
\left.\begin{array}{r} +mf \\ -m.m-4.cd \end{array}\right\} & b^{m-5} \\
\left.\begin{array}{r} -m.g \\ +m.m-5.ce \\ +\tfrac{1}{2}m.m-5.d^2 \\ -\tfrac{1}{6}m.m-4.m-5.c^3 \end{array}\right| & b^{m-6}
\end{array}
\qquad
\begin{array}{rl}
\left.\begin{array}{r} +m.h \\ -m.m-6.cf \\ -m.m-6.de \\ +\tfrac{1}{2}m.m-5.m-6.c^2d \end{array}\right\} & b^{m-7} \\
\left.\begin{array}{r} +m.i \\ +m.m-7.cg \\ +m.m-7.df \\ +\tfrac{1}{2}m.m-7.e^2 \\ -\tfrac{1}{2}m.m-6.m-7.c^2e \\ -\tfrac{1}{2}m.m-6.m-7.cd^2 \\ +\tfrac{1}{24}m.m-5.m-6.m-7.c^4 \end{array}\right\} & b^{m-8}. \\
+\ \&c., &
\end{array}$$

where, reckoning the weights of $b, c, d, e$ ... as 1, 2, 3, 4, ..., respectively, the several terms are all the terms of the weight $m$, or (what is the same thing) in the coefficient of $b^{m-\theta}$ we have all the combinations of $c, d, e, \ldots$, (or say all the nonunitary combinations) of the weight $\theta$, and where the numerical coefficient of

$$b^{m-\theta} c^{\mathrm{c}} d^{\mathrm{d}} e^{\mathrm{e}} \ldots (\mathrm{c}+\mathrm{d}+\mathrm{e}+\ldots=\theta),$$

is

$$=(-)^{\mathrm{c}+\mathrm{e}+\mathrm{g}+\ldots} \frac{m \,.\, m-(\theta-\delta+1) \,.\, m-(\theta-\delta+2) \ldots m-(\theta-1)}{\Pi\mathrm{c} \,.\, \Pi\mathrm{d} \,.\, \Pi\mathrm{e} \ldots}.$$

Thus for the term $b^{m-8}c^2e^1$, $\theta=8$; c, d, e $=2, 4, 1$ respectively (the other exponents each banishing), and the coefficient is

$$(-)^3 \frac{m \,.\, m-6 \,.\, m-7)}{1.2.1}, = -\tfrac{1}{2} m \,.\, m-6 \,.\, m-7,$$

as above; and so in other cases.

For the Mc Mahon form

$$1+bx+\frac{cx^2}{1.2}+\ldots=(1-\alpha x)(1-\beta x) \ldots, \text{ or say}$$

$$y^n+\frac{b}{1} y^{n-1}+\frac{c}{1.2} y^{n-2}+\ldots=(y-\alpha)(y-\beta) \ldots,$$

we must for $b, c, d, \ldots$, write $b, \frac{c}{1.2}, \frac{d}{1.2.3}, \ldots$ respectively: we thus have

$$\begin{aligned}
(-)^m . S_m = {} & b^m \\
& - m \frac{c}{1.2} \qquad\qquad b^{m-2} \\
& + m \frac{d}{1.2.3} \qquad\qquad b^{m-3} \\
& \left.\begin{aligned} & - m \frac{e}{1.2.3.4} \\ & + \tfrac{1}{2} m \,.\, m-3 \left(\frac{c}{1.2}\right)^2 \end{aligned}\right\} \qquad b^{m-4} \\
& + \&c.,
\end{aligned}$$

or say

$$
\begin{aligned}
(-)^m \Pi(m-1)\, S_m = \Pi(m-1)\ & b^4 \\
- \Pi m \frac{c}{1\,.\,2}\, b^{m-2} & \\
+ \Pi m \frac{d}{1\,.\,2\,.\,3} \qquad\qquad & b^{m-3} \\
\left.\begin{array}{l} - \Pi m \dfrac{e}{1\,.\,2\,.\,3\,.\,4} \\ + \Pi m \dfrac{m-3}{2}\left(\dfrac{c}{1\,.\,2}\right)^2 \end{array}\right\} \qquad & b^{m-4} \\
+ \&c., &
\end{aligned}
$$

the numerical coefficient of

$$b^{m-\theta} c^{\mathrm{c}} d^{\mathrm{d}} e^{\mathrm{e}} \ldots \ (\mathrm{c}+\mathrm{d}+\mathrm{e}+\ldots=\theta)$$

being

$$(-)^{\mathrm{c+e+g+}\ldots} \frac{\Pi m\,.\,m-(\theta-\delta+1)\,.\,m-(\theta-\delta+2)\ldots m-(\theta-1)}{\Pi\mathrm{c}\,.\,\Pi\mathrm{d}\,.\,\Pi\mathrm{e}\ldots(\Pi 2)^{\mathrm{c}}\,(\Pi 3)^{\mathrm{d}}\,(\Pi 4)^{\mathrm{e}}\ldots}\,.$$

It is convenient to write down the literal terms in alphabetical order (*A O.*), calculating and affixing to each term the proper numerical coefficient, thus

$$1 + bx + c\frac{x^2}{1\,.\,2} + \ldots = (1-\alpha x)(1-\beta x)(1-\gamma x)\ldots,$$

we find

$$
\begin{array}{rll}
-120\, S_6 = & g & .\ \ \ \ \ \ 1 \\
& bf & .-\ \ \ 6 \\
& ce & .-\ \ 15 \\
& d^2 & .-\ \ 10 \\
& b^2e & .+\ \ 30 \\
& bcd & .+120 \\
& c^3 & .+\ \ 30 \\
& b^3d & .-120 \\
& b^2c^2 & .-270 \\
& b^4c & .+360 \\
& b^6 & .-120 \\
\hline
& & .\pm 541
\end{array}
$$

this expression, as representing the value of the nonunitary function $S_6$, being in fact a seminvariant.

It is to be remarked that the foregoing expression for the sum of the $m^{\text{th}}$ powers of the roots of the equation

$$x^n + bx^{n-1} + cx^{n-2} + \ldots = 0$$

is in fact the series for $x^m$ continued so far only as the exponent of $b$, is not negative: see as to this Note XI of Lagrange's *Équations Numeriques.* For the *à posteriori* verification, observe that we have

$$x + b + \frac{c}{x} + \frac{d}{x^2} + \ldots = 0,$$

or writing for a moment $u = -b$, say this is

$$x = u + fx, \text{ where } fx = -\frac{c}{x} - \frac{d}{x^2} - \frac{e}{x^3} - \&c.$$

Hence, by Lagrange's theorem,

$$\begin{aligned} x^m = \; & u^m \\ & - mu^{m-1}\left(\frac{c}{u} + \frac{d}{u^2} + \frac{e}{u^3} + \ldots\right) \\ & + \left\{mu^{m-1}\left(\frac{c}{u} + \frac{d}{u^2} + \frac{e}{u^3} + \ldots\right)^2\right\}' \frac{1}{1.2} \\ & - \left\{mu^{m-1}\left(\frac{c}{u} + \frac{d}{u^2} + \frac{e}{u^3} + \ldots\right)^3\right\}'' \frac{1}{1.2.3} \\ & + \&c., \end{aligned}$$

where the accents denote differentiations in regard to $u$. This is

$$\begin{aligned} & = u^m \\ & - m\{cu^{m-2} + du^{m-3} + eu^{m-4} + fu^{m-5} + gu^{m-6} + \ldots\} \\ & + \tfrac{1}{2}m\{(m-3)c^2u^{m-4} + (m-4)\,2cdu^{m-5} + (m-5)(d^2 + 2ce)u^{m-6} + \ldots\} \\ & - \tfrac{1}{6}m\{(m-4)(m-5)c^3u^{m-6} + \ldots\} \\ & + \&c. \end{aligned}$$

$$\begin{aligned} = \; & u^m \\ & + u^{m-2}. - mc \\ & + u^{m-3}. - md \\ & + u^{m-4}. - me + \tfrac{1}{2}m.m-3.c^2 \\ & + u^{m-5}. - mf + \tfrac{1}{2}m.m-4.2cd \\ & + u^{m-6}. - mg + \tfrac{1}{2}m.m-5.(d^2 + 2ce) - \tfrac{1}{6}m.m-4.m-5.c^3 \\ & + \&c., \end{aligned}$$

which, putting therein $u=-b$ and multiplying each side by $(-)^m$, is the before-mentioned formula for $(-)^m.S\alpha^m$, only in that formula the series is to be continued only so far as the exponent of $b$ is not negative.

I notice also that we cannot easily by means of the known formula $S\alpha^m\beta^p = S\alpha^m . S\alpha^p - S\alpha^{m+p}$, deduce an expression for $S\alpha^m\beta^p$: in fact, forming the product of the series for $S\alpha^m$, $S\alpha^p$ respectively, this product is identically equal to the series for $S\alpha^{m+p}$, or we seem to obtain $0 = S\alpha^m S\alpha^p - S\alpha^{m+p}$; to obtain the correct formula, we have to take each of the three series only so far as the exponent of $b$ therein respectively is not negative: and it is not easy to see how the resulting formula is to be expressed.

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# A PROPERTY OF THE EQUATION OF CONDUCTION OF HEAT.

By *J. Brill, M.A.*, St. John's College.

The theorem to which I desire to call attention is as follows:

If $f(x, y, z, t)$ be a solution of the equation

$$\frac{\partial V}{\partial t} = a^2\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}\right),$$

then the expression

$$t^{-\frac{3}{2}} e^{-\frac{x^2+y^2+z^2}{4a^2t}} f\left(\frac{x}{t},\ \frac{y}{t},\ \frac{z}{t},\ -\frac{1}{t}\right)$$

is also a solution.

I sent this as a question to the Mathematical Column of the *Educational Times*, but as a considerable time has elapsed without the appearance of a solution, I thought it desirable to publish my own.

Writing $V = uv$ in the above equation, we have

$$v\frac{\partial u}{\partial t} + u\frac{\partial v}{\partial t} = a^2\left\{v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + u\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)\right.$$
$$\left. + 2\left(\frac{\partial u}{\partial x}\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\frac{\partial v}{\partial y} + \frac{\partial u}{\partial z}\frac{\partial v}{\partial z}\right)\right\};$$