of the degrees $m, n, p, q, \ldots$ respectively, and U_1, V_2, &c., are the corresponding functions $(a, \ldots \between x_1, y_1)^m$, $(a', \ldots \between x_2, y_2)^n$, &c., the values of A, B, C being here

$$A = m - \beta - \gamma - \delta - \ldots,$$

$$B = n - \gamma - \alpha - \varepsilon - \ldots,$$

$$C = p - \alpha - \beta - \zeta - \ldots;$$

the theorem expresses that the covariants $\overline{12}\Omega$, $\overline{23}\Omega$, $\overline{31}\Omega$, are linearly connected together; or, writing it in the form $(A\,\overline{12} - C\,\overline{13} + B\,\overline{23})\,\Omega = 0$, we have the proper linear combination $A\,\overline{12}\Omega - C\,\overline{13}\Omega$ of the two covariants $\overline{12}\Omega$ and $\overline{13}\Omega$, equal to $-B\,\overline{23}\Omega$, a determinate multiple of $\overline{23}\Omega$. Speaking roughly, we say that the *difference* of the covariants $\overline{12}\Omega$ and $\overline{13}\Omega$ is equal to $\overline{23}\Omega$.

ON THE NONEXISTENCE OF A SPECIAL GROUP OF POINTS.

By Prof. *Cayley*.

It is well known that, taking in a plane any eight points, every cubic through these passes through a determinate ninth point: it is interesting to show that there is no system of seven points such that every cubic through these passes through a determinate eighth point.

Assuming such a system: first, no three of the points can be in a line, for, if they were, then among the cubics through the seven points we have the line through the three points and an arbitrary conic through the remaining four points, and these composite cubics have no common eighth point of intersection.

Secondly, no six of the points can be on a conic, for, if they were, then among the cubics through the seven points we have the conic through the six points and an arbitrary line through the remaining point, and these composite cubics have no common eighth point of intersection.

Taking now the points to be 1, 2, 3, 4, 5, 6, 7; among the cubics through these we have the composite cubics (A, P), (B, Q), (C, R), where A, B, C are the lines 67, 75, 56, and P, Q, R the conics 12345, 12346, 12347 respectively; by what precedes, the points 5, 6, 7 do not lie on a line, and the points (6, 7), (7, 5) and (5, 6) neither of them lie on the conics P, Q, R respectively.

The common eighth point of intersection, if it exists, must be

$$(A, B, C)\,;\ (A, Q, R),\ (B, R, P),\ (C, P, Q)\,;$$
$$(P, B, C),\ (Q, C, A),\ (R, A, B)\,;\ \text{or}\ (P, Q, R).$$

There is no point (A, B, C).

There is no point (A, Q, R), for Q, R intersect only in the points 1, 2, 3, 4, no one of which lies on A; and similarly there is no point (B, R, P) or (C, P, Q).

B, C intersect in 5 which is a point on P, and thus 5 is the only point (P, B, C). Similarly 6 is the only point (Q, C, A) and 7 is the only point (R, A, B).

P, Q, intersect in 1, 2, 3, 4, which are each of them on R; hence 1, 2, 3, 4 are the only points (P, Q, R).

Hence the points 1, 2, 3, 4, 5, 6, 7 present themselves each once, and only once, among the intersections of the three cubics, and there is no common eighth point of intersection.

ON WARING'S FORMULA FOR THE SUM OF THE *m*th POWERS OF THE ROOTS OF AN EQUATION.

By Prof. *Cayley*.

The formula in question, Prob. I. of Waring's *Meditationes Algebraicæ*, Cambridge, 1782, making therein a slight change of notation, is as follows: viz. the equation being

$$x^n + bx^{n-1} + cx^{n-2} + dx^{n-3} + \ldots = 0,$$

then we have

$$
\begin{array}{rl}
(-)^m S_m = & \\
 & b^m \\
-mc & b^{m-2} \\
+md & b^{m-3} \\
\left.\begin{array}{r} -me \\ +\tfrac{1}{2}m\,.\,m-3\,.\,c^2 \end{array}\right\} & b^{m-4} \\
\left.\begin{array}{r} +mf \\ -m\,.\,m-4\,.\,cd \end{array}\right\} & b^{m-5} \\
\left.\begin{array}{r} -m\,.\,g \\ +m\,.\,m-5\,.\,ce \\ +\tfrac{1}{2}m\,.\,m-5\,.\,d^2 \\ -\tfrac{1}{6}m\,.\,m-4\,.\,m-5\,.\,c^3 \end{array}\right| & b^{m-6} \\
\left.\begin{array}{r} +m\,.\,h \\ -m\,.\,m-6\,.\,cf \\ -m\,.\,m-6\,.\,de \\ +\tfrac{1}{2}m\,.\,m-5\,.\,m-6\,.\,c^2d \end{array}\right\} & b^{m-7} \\
\left.\begin{array}{r} +m\,.\,i \\ +m\,.\,m-7\,.\,cg \\ +m\,.\,m-7\,.\,df \\ +\tfrac{1}{2}m\,.\,m-7\,.\,e^2 \\ -\tfrac{1}{2}m\,.\,m-6\,.\,m-7\,.\,c^2e \\ -\tfrac{1}{2}m\,.\,m-6\,.\,m-7\,.\,cd^2 \\ +\tfrac{1}{24}m\,.\,m-5\,.\,m-6\,.\,m-7\,.\,c^4 \end{array}\right\} & b^{m-8}. \\
+\,\&\text{c.}, &
\end{array}
$$