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On some methodology for economic system modelling

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On some methodology for economic system modelling

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1. Introduction

We asked ourselves the question: ***What rules govern the world of economics?*** We did not exclude the need to consider economic phenomena as universal principles. Perhaps economy is ruled by other laws, maybe the laws of ***physics***.

Our proposed approach to modelling the world of economy may be an alternative and extension of existing models. However, the approach may come across certain limitations which may be difficult to overcome. Firstly, how to convince decision makers to system thinking that has been propagated for many years by Peter M. Senge (*The Fifth Discipline*) [Senge/1990], Jay W. Forrester (*Industrial Dynamics*) [Forrester/1961] and others. Secondly, the proposed model requires access to reliable data. It is difficult to say whether such data exist and are available.

Mathematical models are widely used in economics theory. One could say that all the used models operate on aggregated data, e.g. consumption, investments, etc. The result of such analyses and process simulations are rather qualitative conclusions, while quantitative conclusions are very approximate [Barczyk/1990] [Berg/1991] [Czaja/1991] [Fleming/1962] [Kalecki/1935] [Keynes/1985] [Strotz/1953].

A detailed description of economic models can be found in [Jakimowicz/2005]] and [Garbicz/1999].

2. Geometric interpretation of balance and income statement elements

The idea of developing a universal economic model is directly derived from the accounting practices used in management of a single enterprise. To a large extent, the model is based on the balance and income statement (profit and loss statement) of the enterprise [Sachs/1993], [Solow/1956], [Garbicz/1999], [Green/1993].

It should be stressed, that elements of the model were tested in practice by financial management of enterprises located in the Tarnobrzeg Special Economic Zone in Poland. The practice fully confirmed the functionality of the model.

Accounting is a system of business records, the system monitors all processes that occur in the company, including production processes, services, distribution processes, sales and purchases, as well as business relations with others. Accounting is a formalized system that reflects all economic processes included in the monetary meter and therefore all monetary streams [Lange/1977], [Burda/1990], [Mundel/1962], [Stevenson/1998], [Theil/1971].

It seems that the basic section existing in accounting is financial reporting, the aim of which is to prepare information on economic condition of an enterprise. On the basis of economic data we can illustrate the material, financial and income situation of the enterprise by compiling:

- **Balance sheet,**

- **Income Statement.**

One of the universal principles of accounting – double entry principle – says that every operation needs to be credited to two different ledger accounts, which enables to preserve balance and forces balancing of assets and liabilities at any point in time. Hence, accounting principles force a balance of assets and liabilities in every accounting period [Burda/1995].

In a large simplification a balance sheet consists of:

- **Assets**
  - **Inventory**
  - **Receivables**
- **Liabilities**
  - **Capital**
  - **Payments**.

Balance is presented in the following graphical form:

On the left of Figure 1 there is a symbolic representation of **Assets**, on the right – **Total Liabilities**, with deliberately different colours of arrows, on the right – **Total Liabilities**, both **Capital** and **Payments** are presented in red, as they have the same **FINancial** nature.

On the right **Assets** are distinguished in red, **Receivables**, as they also are **FIN**ancial in nature, while blue denotes **Investments** and **Inventory** in order to distinguish other **REAL** character of these balance elements.

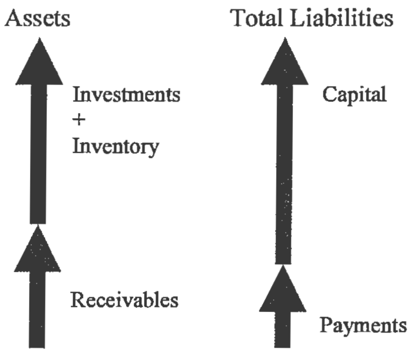

Figure 1. Graphical description of a balance.

3. Real sphere and financial sphere

Based on the principles used in the systems theory, systems research and systems analysis, on the basis of the balance and income statement, we will present the structure of new division of balance elements into two spheres, REAL and FINancial. Such division provides the basis for the development of a new flow model in economic systems.

In Figure 2 we can distinguish four vectors represented by four major components of assets and liabilities. We relocated the liabilities vector in such a way that the beginning of the **Payments** vector was aligned with the end of the **Receivables** vector. Now we will treat the **Investments** + **Inventory** vectors and the **Capital**, **Receivables** and **Payments** vectors separately.

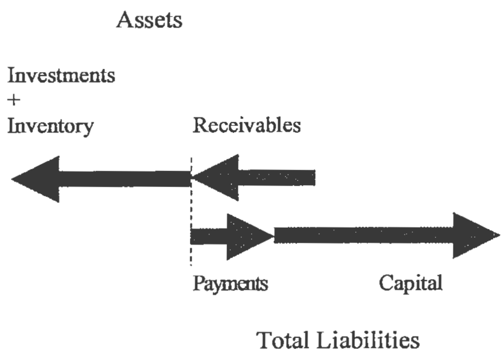

Figure 2. A process of splitting the balance into REAL and FINancial spheres.

On the other hand the income statement can also be treated as part of the reserve or elements of further consumption sold or bought in the given reporting period. It is clear that the sale reduces reserves and purchase increases it.

We can therefore attach two vectors to the **Inv** vector **Selling** and **Buying** known from the income statement. Next we draw aside both part of the vector graph in a way shown below:

In this way we obtained a new model of economic system, in which we can distinguish:

REAL sphere and

FINancial sphere.

REAL sphere consists of the three following vectors:

Inv = Investments + Inventory

Buy = Buying

Sel = Selling

the FINancial sphere also consists of three vectors:

Cap = Capital

Pay = Payments

Rec = Receivables.

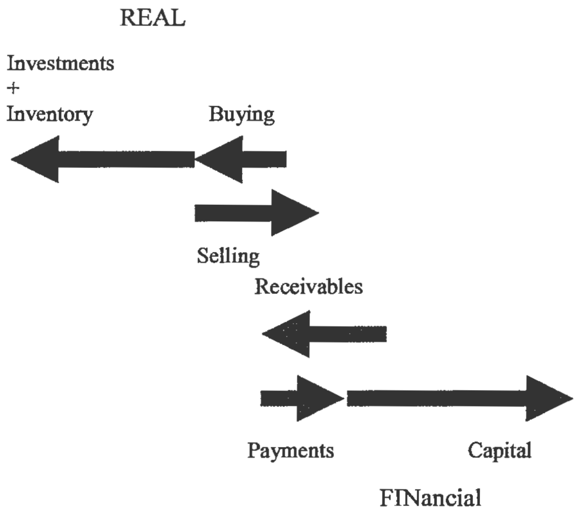

Figure 3. Elements of REAL sphere and FINancial sphere.

4. Matrix of REAL and FIN sphere flows

Let us consider an example economic model of a single state. Let us suppose that we distinguish three group economic entities that we denote as follows:

- ***H*** – a group economic entity of aggregated householders,
- ***P*** – a group economic entity of aggregated producers of goods and services,
- ***G*** – a group economic entity representing government.

The mutual relations between entities (economic circular flow) in a matrix form for REAL and FIN sphere. On the main diagonal of the matrix REAL description of economic flows there are investment goods and stocks **Inv**. Outside the main diagonal is the mutual exchange – the flows **Sel** and **Buy** between entities. The exchange of goods and commodities between economic life entities occurs in the REAL sphere matrix outside the main diagonal. Let us assume the following labels referring to the volume of REAL goods and services lying outside the main diagonal of the flow matrix $\mathrm{Buy}_{ij} = \mathrm{Sel}_{ji}$, where $i = H$ or $i = P$ or $i = G$ and $j = H$ or $j = P$ or $j = G$, while $i \neq j$.

It is understandable that all sales in the REAL sphere causes a change in receivables in the FINancial sphere in the receivables ledger section and in the liabilities section of another economic entity, to which the first economic entity is selling goods and/or services. This means that a change of one element in the REAL sphere flow matrix causes a change of one (corresponding) element in the FIN sphere flow matrix.

Similarly, during the purchase there is a shift of liabilities for the second economic entity and in receivables for the first economic entity. This causes change of one element in the FINancial sphere. Thus, the payment for goods and services is only a change of liabilities of the economic entity into liabilities of a bank issuing money.

Let us assume the following labels regarding the quantity outside the main diagonal of the FIN financial flow matrix $\mathrm{Rec}_{ij} = \mathrm{Pay}_{ji}$, where $i = H$ or $i = P$ or $i = G$ and $j = H$ or $j = P$ or $j = G$, while $i \neq j$.

$$\begin{bmatrix} \mathrm{Inv}_H & \mathrm{Sel}_{HP} & \mathrm{Sel}_{HG} \\ \mathrm{Sel}_{PH} & \mathrm{Inv}_P & \mathrm{Sel}_{PG} \\ \mathrm{Sel}_{GH} & \mathrm{Sel}_{GP} & \mathrm{Inv}_G \end{bmatrix} \qquad \begin{bmatrix} \mathrm{Cap}_H & \mathrm{Rec}_{HP} & \mathrm{Rec}_{HG} \\ \mathrm{Rec}_{PH} & \mathrm{Cap}_P & \mathrm{Rec}_{PG} \\ \mathrm{Rec}_{GH} & \mathrm{Rec}_{GP} & \mathrm{Cap}_G \end{bmatrix} \tag{1}$$

It goes without saying there are flows within each matrix REAL and FIN in periods t_i-t_{i-1}, for $i = 1, 2, ..., \mathrm{T}$, in periods when the balance sheet and the income statement are usually prepared. However there are flows between matrices REAL and FIN. There flows between REAL(t_{i-1}) matrix and REAL(t_i) matrix, as well as between FIN(t_{i-1}) matrix and FIN(t_i) matrix. In a similar way there are flows between FIN(t_{i-1}) matrix and FIN(t_i) matrix, and between REAL(t_{i-1}) matrix and REAL(t_i) matrix.

5. Identification of economic events in REAL and FIN matrices

Circular motion in the economy generally consists of the following elements [Burda/1995], [Allen/1961], [Alien/1975]:

- starting from the initial state,
- sales of goods and services,
- increase in receivables,
- purchase of goods and services,
- increase in liabilities (decrease in receivables).

We can present the circular motion as a series of events occurring between the REAL flow matrix and the FINancial flow matrix (actually between elements of the matrices) in successive periods, recorded at the end of the periods. Let us consider two periods: the first period begins at the t_1 moment and ends at the t_2 moment, the second period begins at the t_2 moment and ends at the t_3 moment. Let us present the circular motion as a series of operations related to the t_1, t_2, and t_3 moments.

The circular motion can be presented as a series of operations in REAL and FIN matrices.

$$\mathrm{FIN}(t_1) \rightarrow \mathrm{REAL}(t_1) \rightarrow \mathrm{FIN}(t_2) \rightarrow \mathrm{REAL}(t_2) \rightarrow \mathrm{FIN}(t_3) \rightarrow \mathrm{REAL}(t_3)$$

We assume that the matrix time series consisting of REAL and FIN matrices in individual moments that designate reporting periods contains full information on the whole economic process. There remains a problem of finding the laws governing changes in time of the REAL and FIN matrices, i.e. mutual relations between REAL and FIN matrices in successive moments of time.

We now introduce definitions of operators in the square matrix elements (2), with the dimensions $n \times n$, elements of which are real numbers.

$$\mathrm{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \tag{2}$$

Definition 1.

Divergence operator of element a_{kk} in matrix A is denoted DIV' a_{kk} and defined as follows:

$$DIV'a_{kk} = \sum_{\substack{i=1 \\ i \neq k}}^{n} a_{ik} - \sum_{\substack{j=1 \\ i \neq k}}^{n} a_{kj}\text{, for } k = 1,2,\ldots,n \tag{3}$$

In general, the term *divergence* means sourceability, variance, in mathematics and physics *divergence* is a term used in classical field theory and is the operator for the vector field called *scalar field* defining sources and outlets of the field [Feynman/1970].

Divergence operator (3) defined in this way has analogous meaning to the flow matrix, which has features of a vector field. Divergence operator (3) has the following meaning, namely the divergence of a_{kk} matrix element is equal to the sum of all elements in the k matrix column (inflow to a_{kk} *cell* reduced by a sum of all elements of the k matrix verse (outflow from a_{kk} *cell).* In other words, it is the difference between what flows in of the k column to element a_{kk} and what flows out of the a_{kk} element to verse k. For all a_{kk}, $k = 1,2,...,n$ elements we obtain n value of the divergence.

Definition 2.

DIV'A divergence of matrix A is defined as follows:

$$\mathrm{DIV'A} = \begin{bmatrix} DIV'a_{11} & 0 & \cdots & 0 \\ 0 & DIV'a_{22} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & DIV'a_{nn} \end{bmatrix} \tag{4}$$

It seems that it is a good definition of changes in *sources* or *cells* (elements of matrix A, as a particular field, as *inflow* and *outflow* from that source.

Definition 3.

Rotation operator of A matrix elements is denoted ROT' A_{ij} and defined as follows:

$$\mathrm{ROT'A}_{ij} = a_{ij} - a_{ji}, \text{ for } i,j = 1,2,...,n \tag{5}$$

In general, the term *rotation* means *vorticity.* In mathematics and physics *rotation* is a term used in *field theory* and means the operator for a vector field *another vector field* defining the rotating force.

Definition 4.

ROT'A rotation of matrix A is defined as follows:

$$\mathrm{ROT'A} = \begin{bmatrix} 0 & \mathrm{ROT'A}_{21} & \cdots & \mathrm{ROT'A}_{1n} \\ \mathrm{ROT'A}_{21} & 0 & \cdots & \mathrm{ROT'A}_{2n} \\ \vdots & \vdots & & \vdots \\ \mathrm{ROT'A}_{n1} & \mathrm{ROT'A}_{n2} & \cdots & 0 \end{bmatrix} \tag{6}$$

It seems that it is a good definition of "forces" between upper part and lower part of the matrix, "forces" trying to turn around the matric along the main diagonal.

Let us apply yhe above introduced definition for matrices REAL and FIN.

Investment and stocks are presented in the REAL flow matrix in the main diagonal, i.e. Inv_H, Inv_P and Inv_G. There are four main causes of change in **Inv** states, namely:

- reduced and increased consumption and
- reduced and increased internal production,
- reduced and increased sales and
- reduced and increased purchases.

Using the designations introduced earier, the REAL sphere flow matrix divergence can be represented as follows:

$$\mathrm{DIV'REAL} = \mathrm{DIV'Sel} = \begin{bmatrix} \mathrm{DIV'Sel}_H & 0 & 0 \\ 0 & \mathrm{DIV'Sel}_P & 0 \\ 0 & 0 & \mathrm{DIV'Sel}_G \end{bmatrix} \tag{7}$$

where

$$\mathrm{DIV'Sel}_H = \mathrm{Sel}_{PH} + \mathrm{Sel}_{GH} - \mathrm{Sel}_{HP} - \mathrm{Sel}_{HG}$$

$$\mathrm{DIV'Sel}_P = \mathrm{Sel}_{HP} + \mathrm{Sel}_{GP} - \mathrm{Sel}_{PH} - \mathrm{Sel}_{PG}$$

$$\mathrm{DIV'Sel}_G = \mathrm{Sel}_{PG} + \mathrm{Sel}_{HG} - \mathrm{Sel}_{GP} - \mathrm{Sel}_{GH}$$

Capital is presented in the FIN financial flow matrix on the main diagonal of the matrix, i.e. Cap_H, Inv_P and Inv_G. The state of capital **Cap** is changed by two main causes:

- decreased and increased capital outflow and
- decreased and increased capital revenue.

Using the designations introduced earier, the FIN sphere flow matrix divergence can be represented as follows:

$$\mathrm{DIV'FIN} = \mathrm{DIV'Rec} = \begin{bmatrix} \mathrm{DIV'Rec}_H & 0 & 0 \\ 0 & \mathrm{DIV'Rec}_P & 0 \\ 0 & 0 & \mathrm{DIV'Rec}_G \end{bmatrix} \tag{8}$$

where

$$\mathrm{DIV'Rec}_H = \mathrm{Rec}_{PH} + \mathrm{Rec}_{GH} - \mathrm{Rec}_{HP} - \mathrm{Rec}_{HG}$$

$$\mathrm{DIV'Rec}_P = \mathrm{Rec}_{HP} + \mathrm{Rec}_{GP} - \mathrm{Rec}_{PH} - \mathrm{Rec}_{PG}$$

$$\mathrm{DIV'Rec}_G = \mathrm{Rec}_{PG} + \mathrm{Rec}_{HG} - \mathrm{Rec}_{GP} - \mathrm{Rec}_{GH}$$

The **Sel** and **Buy** goods exchange is presented in the REAL flow matrix outside the main diagonal of the matrix. The flow of sales and purchase exchange is changed by two main causes:

- reduced and increased by financial possibilities,
- reduced and increased by supply control.

The introduced rotation operator (6) describes economic turnover, i.e. increasing and decreasing the exchange flow between elements positioned symmetrically in relation to the REAL matrix main diagonal, i.e. between the **Sel** and **Buy** elements.

Using the designations introduced earier, the REAL sphere flow matrix rotation can be represented as follows:

$$\mathrm{ROT'Sel} = \begin{bmatrix} 0 & \mathrm{ROT'Sel}_{HP} & \mathrm{ROT'Sel}_{HG} \\ \mathrm{ROT'Sel}_{PH} & 0 & \mathrm{ROT'Sel}_{PG} \\ \mathrm{ROT'Sel}_{GH} & \mathrm{ROT'Sel}_{GP} & 0 \end{bmatrix} \quad (9)$$

where

$$\mathrm{ROT'Sel}_{HP} = \mathrm{Sel}_{HP} - \mathrm{Sel}_{PH}$$

$$\mathrm{ROT'Sel}_{HG} = \mathrm{Sel}_{HG} - \mathrm{Sel}_{GH}$$

$$\mathrm{ROT'Sel}_{PH} = \mathrm{Sel}_{PH} - \mathrm{Sel}_{HP}$$

$$\mathrm{ROT'Sel}_{PG} = \mathrm{Sel}_{PG} - \mathrm{Sel}_{GP}$$

$$\mathrm{ROT'Sel}_{GH} = \mathrm{Sel}_{GH} - \mathrm{Sel}_{HG}$$

$$\mathrm{ROT'Sel}_{GP} = \mathrm{Sel}_{GP} - \mathrm{Sel}_{PG}$$

The exchange of money or receivables **Rec** and liabilities **Pay** is described in the FINancial flow matrix outside its main diagonal. The flow of receivables and liabilities exchange is changed by two main causes:

- reduced and increased by profit opportunities in sales
- reduced and increased by supply control.

We see that the operator that increases or reduces the flow of money exchange is the rotation of the FIN matrix. For the considered FIN matrix we can determine the rotation operator in the following manner:

$$\mathrm{ROT'Rec} = \begin{bmatrix} 0 & \mathrm{ROT'Rec}_{HP} & \mathrm{ROT'Rec}_{HG} \\ \mathrm{ROT'Rec}_{PH} & 0 & \mathrm{ROT'Rec}_{PG} \\ \mathrm{ROT'Rec}_{GH} & \mathrm{ROT'Rec}_{GP} & 0 \end{bmatrix} \tag{10}$$

where

$$\mathrm{ROT'Rec}_{HP} = \mathrm{Rec}_{HP} - \mathrm{Rec}_{PH}$$

$$\mathrm{ROT'Rec}_{HG} = \mathrm{Rec}_{HG} - \mathrm{Rec}_{GH}$$

$$\mathrm{ROT'Rec}_{PH} = \mathrm{Rec}_{PH} - \mathrm{Rec}_{HP}$$

$$\mathrm{ROT'Rec}_{PG} = \mathrm{Rec}_{PG} - \mathrm{Rec}_{GP}$$

$$\mathrm{ROT'Rec}_{GH} = \mathrm{Rec}_{GH} - \mathrm{Rec}_{HG}$$

$$\mathrm{ROT'Rec}_{GP} = \mathrm{Rec}_{GP} - \mathrm{Rec}_{PG}$$

ROT'Rec rotation operator shows the intensity of financial turnover between the ***H***, ***P*** and ***G*** economic zones.

6. REAL and FIN matrixes as time functions

Let us consider the REAL and FIN matrix pair in two adjacent moments in time.

$$t = t_1 \quad \begin{bmatrix} \mathrm{Inv}_H & \mathrm{Sel}_{HP} & \mathrm{Sel}_{HG} \\ \mathrm{Sel}_{PH} & \mathrm{Inv}_P & \mathrm{Sel}_{PG} \\ \mathrm{Sel}_{GH} & \mathrm{Sel}_{GP} & \mathrm{Inv}_G \end{bmatrix} \qquad \begin{bmatrix} \mathrm{Cap}_H & \mathrm{Rec}_{HP} & \mathrm{Rec}_{HG} \\ \mathrm{Rec}_{PH} & \mathrm{Cap}_P & \mathrm{Rec}_{PG} \\ \mathrm{Rec}_{GH} & \mathrm{Rec}_{GP} & \mathrm{Cap}_G \end{bmatrix} \tag{11}$$

$$t = t_2 \quad \begin{bmatrix} \mathrm{Inv}_H & \mathrm{Sel}_{HP} & \mathrm{Sel}_{HG} \\ \mathrm{Sel}_{PH} & \mathrm{Inv}_P & \mathrm{Sel}_{PG} \\ \mathrm{Sel}_{GH} & \mathrm{Sel}_{GP} & \mathrm{Inv}_G \end{bmatrix} \qquad \begin{bmatrix} \mathrm{Cap}_H & \mathrm{Rec}_{HP} & \mathrm{Rec}_{HG} \\ \mathrm{Rec}_{PH} & \mathrm{Cap}_P & \mathrm{Rec}_{PG} \\ \mathrm{Rec}_{GH} & \mathrm{Rec}_{GP} & \mathrm{Cap}_G \end{bmatrix} \tag{12}$$

The REAL and FIN matrices in the t_1 moment present the picture of economy of the considered system (11). In the $t_2 - t_1$ period there were various flows in the REAL and FIN matrices and at the end of the period in moment t_2 we receive the current picture of economy (212).

Change in investment and stocks, i.e. values lying on the REAL matrix diagonal, in the $t_2 - t_1$ period, which we denote $\frac{\Delta \mathrm{Inv}}{\Delta t}$, was caused by sales and purchases. Therefore we can write the following statement:

$$\frac{\Delta \mathrm{Inv}}{\Delta t} = -\Delta_1 \mathrm{Sel} + \mathrm{C'Inv} \tag{13}$$

which means that the increase of investments and resources in time $\frac{\Delta \mathrm{Inv}}{\Delta t}$ occurred as a result of:

- "spatial" changes Δ_1Sel in the REAL matrix and

- possible C'Inv decisions affecting Inv.

It is assumed that C'Inv decisions may be used to control **Inv** changes and have an impact on production, services and consumption. C'Inv is a decisive variable. The adopted convention of sales inflow and outflow directions determines the use of the " – " sign in formula (13) and others.

Spatial changes Δ_1Sel influencing investments and stocks, i.e. the content of elements lying on the main diagonal of REAL flow matrix were defined by the divergence of the matrix, equation (7). In this manner we can obtain the following equation describing investment and resources changes in time:

$$\frac{\Delta \mathrm{Inv}}{\Delta t} + \mathrm{DIV'Sel} = \mathrm{C'Inv} \tag{14}$$

Now let us consider changes in time of the REAL matrix elements that lie outside the main diagonal, i.e. the changes of purchases **Buy** and sales **Sel**. These changes are influenced by mutual relations of liabilities **Pay** and obligations **Rec** which occur in the FINancial flow matrix. Let us write such statement in the following expression:

$$\frac{\Delta \mathrm{Sel}}{\Delta t} = \Delta_2 \mathrm{Rec} + \mathrm{C'Sel} \tag{15}$$

By $C'Sel$ we marked possible decisions that influence changes in purchases and sales, decisions influencing increase or reduction of sales, in other words, the demand. $C'Sel$ is a decisive variable.

By $\Delta_2 Rec$ we denoted other "spatial" changes occurring in the FIN matrix, changes describing relations of liabilities and receivables which occur symmetrically to the main diagonal of the FIN matrix. To describe this kind of spatial changes in matrices we introduced the rotation operator and for the specific FIN matrix it is expressed with the relation (10). The equation (15) can therefore be presented in the following form:

$$\frac{\Delta Sel}{\Delta t} - ROT' Rec \qquad = C'Sel \tag{16}$$

Next let us consider obligation changes in time, i.e. elements of FIN matrix lying outside the main diagonal in the matrix, the change of receivables **Rec** and liabilities **Pay**. The changes are caused by the mutual relation of purchases **Buy** and sales **Sel** occurring in the REAL flow matrix, the purchases process generates receivables and the sales process generates liabilities. Let us write such statement in the following expression:

$$\frac{\Delta Rec}{\Delta t} = -\Delta_2 Sel + C'Rec \tag{17}$$

With $C'Rec$ we denoted possible decisions affecting changes in liabilities and receivables, decisions that may change relations between liabilities and receivables, e.g. decisions to pay for goods and services. C'Rec is a decisive variable.

In the (17) formula there is the $\Delta_2\text{Sel}$ segment which denotes "spatial" changes that occur in the REAL matrix, changes describing relations of purchases and sales. These changes occur symmetrically towards the main diagonal of the REAL matrix. The changes are the same in character as the spatial changes occurring in the (16) formula. Such changes are described as rotation operator with the presented formula (9). Formula (17) takes the following form:

$$\frac{\Delta\text{Rec}}{\Delta t} + \text{ROT'Sel} = \text{C'Rec} \tag{18}$$

In the case of FIN matrix elements lying on the main diagonal we proceed similarly to the REAL matrix. The change of capital **Cap** occurs as a result of receivables and liabilities flow, increasing or reducing. Therefore we can write the following statement:

$$\frac{\Delta\text{Cap}}{\Delta t} = -\Delta_1\text{Rec} + \text{C'Cap} \tag{19}$$

where $\Delta_1\text{Rec}$ denotes "spatial" changes in FIN matrix that consist of contributions to the capital (lying on the main diagonal of the FIN matrix) reduced by liabilities and increased by retrieving receivables, while C'Cap denotes possible decisions associated e.g. with money issue. $\Delta_1\text{Rec}$ spatial changes are described with changes in the value of elements lying on the main diagonal of the FIN matrix, which is the divergence operator denoted with the (8) equation. Equation (19) thus assumes the following form:

$$\frac{\Delta\text{Cap}}{\Delta t} + \text{DIV'Rec} = \text{C'Cap} \tag{20}$$

For the considered example economic system consisting of three entities – households ***H***, producers ***P*** and government ***G*** equation (14) has 3 elements, equation (16) has 6 elements, equation (18) has 6 elements and equation (20) has 3 elements.

The above equations contain decisive variables which influence the economic processes and which can be used to control the economic system. Decisive variables can be arranged in two matrices. One matrix is associated with the REAL flow sphere, the other with the FINancial flow sphere.

Definition 5.

The matrix C'REAL defines controls, or decisive variables.

$$\mathrm{C'REAL} = \begin{bmatrix} \mathrm{C'Inv}_H & \mathrm{C'Sel}_{HP} & \mathrm{C'Sel}_{HG} \\ \mathrm{C'Sel}_{PH} & \mathrm{C'Inv}_P & \mathrm{C'Sel}_{PG} \\ \mathrm{C'Sel}_{GH} & \mathrm{C'Sel}_{GP} & \mathrm{C'Inv}_G \end{bmatrix} \tag{21}$$

Its values influence changes in the REAL flow matrix.

Definition 6.

The matrix C'FIN defines controls, or decisive variables.

$$\mathrm{C'FIN} = \begin{bmatrix} \mathrm{C'Cap}_H & \mathrm{C'Rec}_{HP} & \mathrm{C'Rec}_{HG} \\ \mathrm{C'Rec}_{PH} & \mathrm{C'Cap}_P & \mathrm{C'Rec}_{PG} \\ \mathrm{C'Rec}_{GH} & \mathrm{C'Rec}_{GP} & \mathrm{C'Cap}_G \end{bmatrix} \tag{22}$$

Its values influence changes in the FIN flow matrix.

Knowing the C'REAL and C'FIN matrices and the REAL and FIN matrices for the previous moment in time t_1 we can calculate the value of REAL and FIN matrices for the next moment in time t_2 as time gains resulting from the previous state and calculated behaviours.

In this way, using the proposed model of economic systems we can distinguish:

- **analysis process** of the economic system, i.e. by knowing the REAL and FIN matrices for various moments in time we can determine what they were in controlling periods,
- **synthesis process** of the economic system, i.e. by knowing the REAL and FIN matrices for a chosen moment in time we can change controls and then generate REAL and FIN matrices for successive moments in time and compare with real values of the matrices.

In order to illustrate the applied operators on the elements of both matrices we will consider two simple numerical examples - one example of economic system analysis and one of economic system synthesis. The examples show that the developed model of economic system is working properly.

Example

We consider REAL and FIN for different moments in time:

t=	REAL			FIN		
0	20	4	0	20	4	0
	4	20	0	4	20	0
	0	0	0	0	0	0
1	21	5	0	20	5	0
	4	20	0	3	20	0
	0	0	0	0	0	0

The information contained in REAL and FIN matrices lets us determine model parameters, i.e. the following operators:

$$\mathrm{DIV'Sel} = \begin{bmatrix} \mathrm{DIV'Sel}_H & 0 & 0 \\ 0 & \mathrm{DIV'Sel}_P & 0 \\ 0 & 0 & \mathrm{DIV'Sel}_G \end{bmatrix}$$

$$\mathrm{DIV'Rec} = \begin{bmatrix} \mathrm{DIV'Rec}_H & 0 & 0 \\ 0 & \mathrm{DIV'Rec}_P & 0 \\ 0 & 0 & \mathrm{DIV'Rec}_G \end{bmatrix}$$

$$\mathrm{ROT'Sel} = \begin{bmatrix} 0 & \mathrm{ROT'Sel}_{HP} & \mathrm{ROT'Sel}_{HG} \\ \mathrm{ROT'Sel}_{PH} & 0 & \mathrm{ROT'Sel}_{PG} \\ \mathrm{ROT'Sel}_{GH} & \mathrm{ROT'Sel}_{GP} & 0 \end{bmatrix}$$

$$\mathrm{ROT'Rec} = \begin{bmatrix} 0 & \mathrm{ROT'Rec}_{HP} & \mathrm{ROT'Rec}_{HG} \\ \mathrm{ROT'Rec}_{PH} & 0 & \mathrm{ROT'Rec}_{PG} \\ \mathrm{ROT'Rec}_{GH} & \mathrm{ROT'Rec}_{GP} & 0 \end{bmatrix}$$

and controlling matrices C'REAL and C'FIN:

t=	C'REAL	REAL	FIN	C'FIN
1		$\begin{bmatrix} 20 & 4 & 0 \\ 4 & 20 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 20 & 4 & 0 \\ 4 & 20 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	
2	$\begin{bmatrix} \text{[redacted]} & \text{[redacted]} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 21 & 5 & 0 \\ 4 & 20 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 20 & 5 & 0 \\ 3 & 20 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & \text{[redacted]} & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Let us note that in the considered example there are two controls in the REAL flow sphere and one in the FINancial sphere, namely

$$C'Inv_H = 1, \;\; C'Sel_{HP} = 1, \;\; C'Cap_P = 1 \tag{23}$$

which shows undertaken decisions (controls).

Above we introduced a differential model of economic system, let us quote the equations again:

$$\frac{\Delta Inv}{\Delta t} + DIV'Sel = C'Inv \tag{24}$$

$$\frac{\Delta Sel}{\Delta t} - ROT'Rec = C'Sel \tag{25}$$

$$\frac{\Delta Rec}{\Delta t} + ROT'Sel = C'Rec \tag{26}$$

$$\frac{\Delta Cap}{\Delta t} + DIV'Rec = C'Cap \tag{27}$$

Let us consider, in a nutshell, another mathematical approach used in the *field theory* and describe the notation of the *linear polivector* representing economic system as an element of Clifford's algebra [Jadczyk/2012].

Let us treat the elements of REAL flow matrix as *real* elements of the paravector, elements of the FINancial flows matrix as *imaginary* elements of the paravector describing the whole sphere of the economy.

In this way we write the basic elements occurring in financial statements in the form of a polivector in the space of geometric algebra Cl (3,0) [Feynman/1970]:

$$E = \mathrm{Inv} + \mathrm{Sel} + i\ \mathrm{Rec} + i\ \mathrm{Cap} \tag{28}$$

and the coercion polivector containing fixed elements of changes of basic elements of the polivector in time as:

$$Ce = C'\mathrm{Inv} + C'\mathrm{Sel} + i\ C'\mathrm{Rec} + i\ C'\mathrm{Cap} \tag{29}$$

Let us write a simple equation of flow:

$$D E = Ce \tag{30}$$

Subjecting polivector E to the actions of operator D we obtain the following equations:

$$\frac{d\,\mathrm{Inv}}{dt} \qquad\qquad + \mathrm{DIV\,Sel} = \mathrm{C'Inv} \tag{31}$$

$$\frac{d\,\mathrm{Sel}}{dt} - \mathrm{ROT\,Rec} \qquad\qquad = \mathrm{C'Sel} \tag{32}$$

$$\frac{d\,\mathrm{Rec}}{dt} + \mathrm{ROT\,Sel} \qquad\qquad = \mathrm{C'Rec} \tag{33}$$

$$\frac{d\,\mathrm{Cap}}{dt} \qquad\qquad + \mathrm{DIV\,Rec} = \mathrm{C'Cap} \tag{34}$$

The operators occurring in the above equations $\frac{d\,\mathrm{Inv}}{dt}$, $\frac{d\,\mathrm{Sel}}{dt}$, $\frac{d\,\mathrm{Rec}}{dt}$, $\frac{d\,\mathrm{Cap}}{dt}$ are equivalent to interest rates and operators $\mathrm{DIV\,Sel}$, $\mathrm{DIV\,Rec}$ are divergences [Feynman/1970]. [Feynman/1970]. Terms occurring on the right side of the equations (31 - 34) are control vectors $\mathrm{C'Inv}$, $\mathrm{C'Sel}$, $\mathrm{C'Rec}$, $\mathrm{C'Cap}$.

At this point we wish to quote the famous equations of Maxwell for electromagnetic field [Feynman/1970]:

$$0 \qquad\qquad + \mathrm{div\,E} = \rho_e \tag{35}$$

$$\frac{d\mathrm{E}}{dt} - \mathrm{rot\ B} \qquad\qquad = \mathrm{j}_e \tag{36}$$

$$\frac{d\mathrm{B}}{dt} + \mathrm{rot\ E} \qquad\qquad = 0 \tag{37}$$

$$0 \qquad\qquad + \mathrm{div\,B} = 0 \tag{38}$$

Without going into details of particular equations parts' meanings (35 – 38), it is not difficult to see that the equations modelling flows in the economic system (31, 32, 33, 34) that we obtained resemble the structure of Maxwell's equations - which are a bit easier but in different space!

The conclusion seems to be as follows:

Generating economic flow resembles generating the electromagnetic field.

When referring to the Maxwell's equation perhaps it is worth mentioning that Robert Grosseteste in his 13^{th} century works predicted the meaning of the light theory.

However, it seems that the similarity of Maxwell's electromagnetism theory and the mathematical model describing economic life presented in this paper is not a coincidence. The economic micro-world runs randomly (decisions of individual economic entities are independent), but the micro-world generates a determined macro-world, where deterministic relations can be noticed. Just like electromagnetic waves is a deterministic description of a non-deterministic micro-world.

Another conclusion that arises when comparing results of this paper with Maxwell's equations is that perhaps the world of economy can be described with equation of physics.

If so, we can apply methods developed by areas in which Maxwell's equations are used for modelling, i.e. in electrical engineering to economic modelling.

7. Conclusions

Two flow spheres were distinguished in the model:

- the REAL flow matrix

- the FINancial flow matrix

The spheres are described with matrix time sequence of matrix pairs or an equivalent matrix time sequence of paravectors.

For the REAL and FIN matrix time sequence matrices we introduced operators which link the matrices in various moments in time.

For discrete data it was shown that the flows in the economic system are governed by laws described with differential equations and a differential flow model in the economic system was obtained.

The nature of the equations shows that the laws governing economy are similar in nature to the laws of electromagnetic fields in physics.

We believe that the same methodology can be applied to the modelling of flows in other economic systems, e.g. for an enterprise, a group of enterprises, a region, a country, as well as the group of countries.

The resulting law of economic flows was written in the form of a system of differential equations. The equations, apart from REAL and FINancial sphere flows take into account the conjugations occurring between these spheres in different moments, as well as delays generated by the FINancial flows sphere.

In order to conduct the analysis and synthesis of the economic system, properly prepared data are needed. The data must me aggregated in the manner required by detail or complexity of the considered problems. It is a natural assumption that decision-makers have data, on the basis of which they wish to make decisions or simulate various results of the decisions made.

It is also possible to present the task of economic system optimization. Knowing the model parameters we can identify results of control values change (C'REAL and C'FIN matrices) and then optimize the economic system by minimizing or maximizing a chosen quality criterion.

Therefore we believe that precise calculation of control based on models can only provide hints for decision-makers, as it is more important to detect trends and assume a certain margin of error of optimal decisions.

However, it seems that the similarity of Maxwell's electromagnetism theory and the mathematical model describing economic life is not a coincidence. As the accidental economic micro-world (decisions of economic entities) generates the determined macro-world, we pay attention to deterministic relationships (similarly to many physical problems).

Isn't the unity of reality visible, regardless of whether we speak of the micro- or the macro-world? The possibility of using the laws of physics for the attempt to optimize economic processes cannot be excluded. Assuming that the laws are universal, perhaps one should attempt to imitate the laws noticed in nature? Just as the electric and magnetic fields are produced and transferred as an electromagnetic wave, perhaps we should design production and transfer of goods, services and receivables as *the economic field.*

It is worth noting that the REAL sphere matrix is an analogon of electric field and the FINancial sphere matrix is an analogon of magnetic field. It is expected that the known effect of wave mismatch in the electromagnetic field has corresponding analogons in the economic space. This effect is manifested by the fact that the inappropriate time and place of intervention in the economic system may cause even counterproductive effects. These results are against common sense.

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