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Abstract

The problem of the Bayes estimation of the failure rate is considered when the reliability data are presented in a vague form. It is also assumed that the prior information about the estimated failure rate is given in the form of the gamma distribution with imprecisely defined parameters. Fuzzy sets are used to model the lack of precision. The formulae are given for the fuzzy Bayes estimator of the failure rate.

Keywords:F ailure rate; Bayes estimator; Fuzzy reliability data; Fuzzy prior information.

1. Introduction

In reliability testing decision-makers are faced with data of a random nature. Various statistical methods have been developed during the last 200 years in order to cope with random data. However, in nearly all cases it is assumed that available data are described precisely, usually by real numbers. Thus, in the majority of statistical papers it is assumed that the only source of uncertainty is the randomness of data. In many circumstances of a real life, however, the data are not precise, and we often have to analyse not only exact numbers, but vague statements as well. Thus, classical methods are sometimes not sufficient, and there is a need to apply some other theories which, at least in statistics, are considered as non-standard. The theory of fuzzy sets is a theory that has been successfully applied in many cases where we deal with vague data, and when the results of a rigorous mathematical analysis have to be presented to people in a plain language.

In the majority of practical cases the available life-time data are not sufficient for precise estimation of reliability characteristics. Usually, however, there exists additional information that can be merged with the information obtained from reliability tests. In such a case Bayes statistical methods are used both for reliability estimation and decision making. The ready-to-use results are described in numerous books and papers for the case of precise information about reliability data, prior distributions, and related costs. However, in many cases this information is also imprecise.

In the paper we consider the problem of the Bayes analysis of life data when both statistical data and additional information may be expressed in a vague form. In the second section of the paper we present basic notions of the fuzzy statistical Bayes analysis. The general results from the second section we illustrate in the third section with the application from Bayes reliability analysis – the problem of the estimation of the constant failure rate. Theoretical results presented in the paper are illustrated with a numerical example.

2. Bayes statistical analysis of reliability data with imprecise information

Bayes statistical analysis is a special case of a more general problem of making decisions presented in many textbooks such as e.g. Raiffa and Schlaifer [7], and DeGroot [2]. According to the basic model typically used for the problems of the reliability analysis the decision maker can specify the following data defining his decision problem:

- space of terminal decisions (acts): $A = \{a\}$.
- state space: $\Theta = \{\theta\}$.
- sample space: $Z = \{z\}$.
- utility function: $u(\cdot,\cdot,\cdot)$ on $A \times Z \times \Theta$.

Note, that in the case of the Bayes statistical analysis of reliability data it is usually assumed that the decision space is the same as the state space Θ. The decision maker evaluates a utility $u(z,\theta)$ of making a certain decision when the result of this experiment is *z*, and the true state of nature is θ. In order to find appropriate (hopefully optimal) decisions the decision maker has also to specify a joint probability measure $P_{\theta,z}(\cdot,\cdot)$ for a Cartesian product $\Theta \times Z$. The knowledge of this probability measure means that we know the joint probability distribution of observing the statistical data *z* when the *random* state of nature is described by θ. Knowing this joint probability distribution we can calculate some important marginal and conditional probability distributions. In particular, we are usually interested in three distributions:

- the marginal distribution on the state space Θ describing our *prior* information about possible states of nature;
- the conditional distribution on the sample space *Z* for given state of nature θ;
- the conditional distribution on the state space Θ for given result of the experiment *z* describing our *posterior* information about possible states of nature.

Note, that we may know only these particular distributions, as their knowledge is equivalent to the knowledge of the joint probability distribution on $\Theta \times Z$.

Let us consider the simplest case of the general model when there is no statistical data. In such a case, the only information we need is the probability distribution $\pi(\theta)$ defined on the state space Θ. We call this distribution *the prior distribution* of the parameter (parameters) describing the unknown state of nature. If we know the utility function $u(a,\theta)$ defined on $A \times \Theta$ we may calculate *the expected utility* assigned to a particular action (decision) *a* from a simple formula

$$u(a) = \int_{\Theta} u(a,\theta)\pi(\theta)d\theta. \qquad (1)$$

If we use *a loss function* $L(a,\theta)$ for the description of potential consequences of taking decision *a* we may calculate *the expected loss* (usually called *a risk*) from an equivalent formula

$$\rho(a) = \int_{\Theta} L(a,\theta)\pi(\theta)d\theta. \qquad (2)$$

Having calculated the expected utilities for *all* possible decisions we can find the optimal one which is related to the maximal expected utility (or the minimal risk). The expected risk associated with the optimal decision is called a Bayes risk. This procedure is in principle very simple. However, in many practical cases (when the number of possible decisions is sufficiently large) it may require the usage of sophisticated optimisation methods.

When the decision maker has an additional information about the state of nature in a form of observations $\mathbf{z} = (z_1, z_2, \ldots, z_n)$ of a random vector described by a probability distribution $f(\mathbf{z},\theta)$ we may calculate *the expected utility* assigned to a particular action (decision) *a* from a formula

$$u(a,\mathbf{z}) = \int_{\Theta} u(a,\theta)g(\theta \mid \mathbf{z})d\theta, \qquad (3)$$

where

$$g(\theta \mid \mathbf{z}) = \frac{f(\mathbf{z} \mid \theta)\pi(\theta)}{\int\limits_{\Theta} f(\mathbf{z} \mid \theta)\pi(\theta)d\theta} \quad (4)$$

is the posterior distribution of the parameter θ which describes the state of nature. In such a case the expected utility attributed to each decision is calculated from

$$u(a \mid \mathbf{z}) = \int\limits_{\Theta} u(a,\theta)g(\theta \mid \mathbf{z})d\theta, \quad (5)$$

and the respective risk from the formula

$$\rho(a \mid \mathbf{z}) = \int\limits_{\Theta} L(a,\theta)g(\theta \mid \mathbf{z})d\theta, \quad (6)$$

The procedure for finding the optimal decision is exactly the same as in the case described previously.

Suppose now that the parameter p of the prior distribution $\pi(\theta; p)$ and the statistical data $\mathbf{z}$ are defined imprecisely. Let us assume that our imprecise knowledge about their possible values is represented by fuzzy sets. A fuzzy set $\widetilde{X}$ is defined using the membership function $\mu_{\widetilde{X}}(x)$ which in the considered in this paper context describes the grade of possibility that a fuzzy parameter, say $\widetilde{X}$, has a specified value of x. Each fuzzy set may be also represented by its α-cuts defined as ordinary sets

$$X^{\alpha} = \{x \in R : \mu_{\widetilde{X}}(x) \geq \alpha\}, 0 \leq \alpha \leq 1. \quad (7)$$

From the representation theorem for fuzzy sets we know that each membership function may be equivalently represented as

$$\mu_{\widetilde{X}}(x) = sup\{\alpha I_{\widetilde{X}^{\alpha}}(x) : \alpha \in [0,1]\}. \quad (8)$$

Let us denote the imprecisely defined parameter p by $\widetilde{p}$, and the vaguely described life data statistics $\mathbf{z}$ by $\widetilde{\mathbf{z}}$, respectively. Moreover, assume that fuzzy values of $\widetilde{p}$ and $\widetilde{\mathbf{z}}$ are represented by their α-cuts, and that these α-cuts are given in a form of closed intervals $[p_L^{\alpha}, p_U^{\alpha}]$ and $[\mathbf{z}_L^{\alpha}, \mathbf{z}_U^{\alpha}]$, respectively. The knowledge of these α-cuts let us calculate fuzzy equivalents of the expected utility or the expected loss (risk). In general, it is possible to assume that the loss function $L(*)$ may be also imprecisely defined. However, to make the presentation simple we assume that decisions are based on the knowledge of the vague posterior distribution $g(\theta \mid \widetilde{\mathbf{z}}, \widetilde{p})$ and the precisely defined loss function $L(\theta)$. As the posterior distribution function is the function of imprecise fuzzy parameters, it is also fuzzy, and may be denoted as $\widetilde{g}(\theta \mid \widetilde{\mathbf{z}}, \widetilde{p})$.

Now, let us rewrite formula (6) as

$$\widetilde{\rho} = \int\limits_{\Theta} L(a,\theta)\widetilde{g}(\theta \mid \widetilde{\mathbf{z}}, \widetilde{p})d\theta. \quad (9)$$

The risk calculated from (9) is now an imprecisely defined *fuzzy number* whose membership function may be calculated using Zadeh's extension principle (see Klir and Yuan [6], or any other textbook on fuzzy sets, for a reference). It is easy to show that the fuzzy risk $\widetilde{\rho}$ is now represented by its α-cuts $[\rho_L^{\alpha}, \rho_U^{\alpha}]$, where

$$\rho_L^{\alpha} = \inf_{\substack{\mathbf{z} \in [\mathbf{z}_L^{\alpha}, \mathbf{z}_U^{\alpha}] \\ p \in [p_L^{\alpha}, p_U^{\alpha}]}} \widetilde{\rho}(\mathbf{z}, p) \quad (10)$$

and

$$\rho_U^\alpha = \sup_{\substack{\mathbf{z}\in\left[\mathbf{z}_L^\alpha,\mathbf{z}_U^\alpha\right] \\ p\in\left[p_L^\alpha,p_U^\alpha\right]}} \widetilde{\rho}(\mathbf{z}, p). \tag{11}$$

Thus, for every possible decision we may find a fuzzy risk $\widetilde{\rho}$ or a fuzzy expected utility $\widetilde{u}$ which may be calculated in the same way.

3. Bayes estimator of the constant failure rate in the presence of imprecise information

One of the most important problems of Bayes analysis of reliability data is the estimation of the constant failure rate. The constant failure rate characterises the exponentially distributed time to failure Z whose density function is given by

$$f(z;\lambda)=\begin{cases} 0, & z<0 \\ \lambda e^{-\lambda z}, & z\geq 0 \end{cases} \tag{12}$$

Let n items be placed on a reliability test, and $z_1 \leq z_2 \leq \cdots \leq z_d,\, d \leq n$ be the observed times to failure. In the simplest case of censored life-time data the total time on test is given by

$$T=\sum_{i=1}^{d} z_i+(n-d)z_d \tag{13}$$

It can be found in any textbook on the statistical analysis of life data that (d,Z) is the observed value of the test statistics that is sufficient for the estimation of the failure rate λ, and the observed value of the maximum likelihood estimator of λ is given by

$$\lambda^* = \frac{d}{T} \tag{14}$$

When the prior information about the actual value of λ is available in a form of a prior distribution $\pi(\lambda)$ we can apply the Bayes methodology described in the previous section and find the Bayes estimator of λ. In the classical book of Raiffa and Schlaifer [7] it has been shown that the most appropriate prior distribution of λ is the gamma distribution given by the density function

$$\pi(\lambda)=\begin{cases} 0, & \lambda<0 \\ \dfrac{\gamma^\delta \lambda^{\delta-1}e^{-\gamma\lambda}}{\Gamma(\delta)}, & \lambda\geq 0' \end{cases} \quad \gamma\geq 0, \delta\geq 0 \tag{15}$$

The gamma distribution with the parameters (γ,δ) is the conjugate distribution to the exponential distribution. Thus, the posterior distribution of λ is also the gamma distribution with the parameters $(\gamma+T, \delta+d)$.

In the Bayes estimation we assume that the space of decisions A is the same as the space of parameters. Therefore, the Bayes estimator of λ is such a decision $a=\lambda_B$ that minimises (6) for a given loss function $L(\lambda_B,\lambda)$. This is a well known fact (see, e.g., DeGroot [2] for a reference) that for a quadratic loss function, i.e. when $L(\lambda_B,\lambda)=c(\lambda-\lambda_B)^2, c>0$, the Bayes estimator of the constant failure rate λ is equal to the expected value in the posterior distribution of λ, i.e.

$$\lambda_B = \frac{d+\delta}{T+\gamma}, \tag{16}$$

and the associated Bayes risk is equal to the variance in the posterior distribution of λ, i.e.

$$\rho = \frac{d+\delta}{(T+\gamma)^2}. \qquad (17)$$

Let us assume now that both the life-time data and the prior distribution are described in a vague form. The vagueness of life-time data coming from the users has, as it has been pointed out in Grzegorzewski and Hryniewicz [3], many different sources. We could divide these sources into three groups:

- vagueness caused by subjective and imprecise perception of failures by a user,
- vagueness caused by imprecise records of reliability data,
- vagueness caused by imprecise records of the rate of usage.

First source of vagueness is typical for so called parametric failures. A parametric failure occurs when at least one value of functional parameters of an item under investigation falls beyond specification limits. In practice, however, a user has only a perception of the values of these parameters, and is not able to define precisely the moment of a failure. For example, if there exists a requirement for an admissible level of noise, it usually cannot be measured by a user, but only assessed in a subjective, and therefore imprecise, way. As the result, we obtain an imprecise information about the real life-time. Moreover, this type of vagueness causes situations in which even at the end of a test (i.e. at a censoring time) a user is not sure whether the tested item has failed or not. In such a case, we have not only imprecise values of life-times, but we have imprecise information about the number of observed failures as well. This type of imprecise reliability data was considered in Hryniewicz [5], and Grzegorzewski and Hryniewicz [4].

Second source of vagueness is typical for retrospective data. Users do not record precisely the moments of failures, especially when they are not sure if they observed a real failure (see above). So when they are asked about failures which occurred some time ago, they often provide an imprecise information. Another case of vagueness of this type arises when a user knows exactly the time of a failure but does not know the precise length of the time to failure. This often happens in the case of a reliable equipment when failures occur after years of exploitation. In such cases users very often cannot precisely recall the moment when the failed equipment begun its exploitation.

Third source of vagueness is related to the fact that users, who report their data in days (weeks, months), use the tested items with different intensity. Depending on the value of this intensity two items that failed after the same period of time may have completely different time to failure expressed in hours of continuous work. In practice, the users are asked about the intensity of usage (for example, in hours per day), and their responses are usually imprecise.

The lack of precision of reliability field data comes from all these sources, and in many cases cannot be even identified. Moreover, vagueness described above (especially from first two sources) is rather of an epistemic character. Therefore, its description in terms of probabilities is rather doubtful. Even in the case when this could have been done, precise probability models are very often impractical, because of many parameters which are either unknown or difficult to estimate. Thus, when we deal with really vague data expressed by imprecise words, and it is the only source of information which can be used for the verification of hypotheses about the mean life-time of tested items, we need to use another formalism that is more suitable for the description of imprecise data. We believe, that the formalism of the theory of fuzzy sets provides us with well established and easy to use means of the formal description of imprecise (linguistic) information.

Let us assume that instead of exact time to failures $z_1, z_2, \ldots, z_d$ we observe fuzzy times $\tilde{z}_1, \tilde{z}_2, \ldots, \tilde{z}_d$ (note that an exact time to failure is a special case of a fuzzy one). Suppose that the membership function of the observed fuzzy time to failure $\tilde{z}_i, i = 1, \ldots, d$ is defined using the set of α-cuts $\left(z_{i,L}^{\alpha}, z_{i,U}^{\alpha}\right)$ $i = 1,\ldots,d$. The total time on test T becomes now a fuzzy number $\widetilde{T}$ whose membership function $\mu(T)$ may be defined using the set of α-cuts $\left(T_L^{\alpha}, T_U^{\alpha}\right)$ where

$$T_L^{\alpha} = \inf_{z_i \in \left(z_{i,L}^{\alpha}, z_{i,U}^{\alpha}\right)} \left\{\sum_{i=1}^{d} z_i + (n-d) z_i\right\} = \sum_{i=1}^{d} z_{i,L} + (n-d) z_{i,L} \tag{18}$$

and

$$T_U^{\alpha} = \sup_{z_i \in \left(z_{i,L}^{\alpha}, z_{i,U}^{\alpha}\right)} \left\{\sum_{i=1}^{d} z_i + (n-d) z_i\right\} = \sum_{i=1}^{d} z_{i,U} + (n-d) z_{i,U} \tag{19}$$

Now, let us discuss the second source of vagueness considered in this paper, namely the vagueness of the prior distribution. One can argue that the prior distribution itself is the only possible description of this vagueness. However, in order to define a prior distribution we have to indicate exact values of its parameters. Then a new problem arises, how to evaluate the values of these parameters when we do not know them precisely. According to an orthodox Bayes approach we have to define their prior distributions, and to proceed this way till the moment when all necessary values are exactly known. Another way to cope with this problem is to find a natural interpretation of these parameters or their functions, and to assess them subjectively. In such a case we often face problems of the precise assessment of subjectively perceived quantities. In the considered in this paper case of the gamma distribution we have to give the values of two parameters δ and γ. Knowing any two characteristics of the prior distribution of the parameter λ, such as the expected value, the mode, the coefficient of asymmetry or the variance, we can easily write two equations whose solution gives us the required values of δ and γ. However, when at least one of those characteristics is known imprecisely we arrive at vague values of δ and γ.

In the general case we may assume that the parameters of the prior distribution are described by fuzzy numbers $\widetilde{\delta}$ and $\widetilde{\gamma}$ whose α-cuts are given in a form of closed intervals $\left(\delta_L^{\alpha}, \delta_U^{\alpha}\right)$ and $\left(\gamma_L^{\alpha}, \gamma_U^{\alpha}\right)$, respectively. We can now define the fuzzy version of the Bayes estimator of the failure rate as

$$\widetilde{\lambda}_B = \frac{d + \widetilde{\delta}}{\widetilde{T} + \widetilde{\gamma}}, \tag{20}$$

The membership function of $\widetilde{\lambda}_B$ can be found using Zadeh's extension principle (see, e.g., Klir and Yuan [6]). In the simplest case, when both parameters δ and γ are assessed independently, the membership function $\mu(\lambda_B)$ of $\widetilde{\lambda}_B$ may be defined using the set of α-cuts $\left(\lambda_{B,L}^{\alpha}, \lambda_{B,U}^{\alpha}\right)$, where

$$\lambda_{B,L}^{\alpha} = \frac{d + \delta_L^{\alpha}}{T_U^{\alpha} + \gamma_U^{\alpha}} \tag{21}$$

$$\lambda_{B,U}^{\alpha} = \frac{d + \delta_U^{\alpha}}{T_L^{\alpha} + \gamma_L^{\alpha}} \tag{22}$$

Let us consider the case when the shape parameter δ of the gamma prior distribution is known exactly. The shape of the prior distribution is not directly related to any vague concept, so we can assume that the decision maker is able to give the exact value of δ. Now, let us assume that the decision maker has some vague opinion either about the expected value of the failure rate $\widetilde{E}_{\lambda}$ or about the most plausible value of the failure rate equal to the mode of the prior distribution denoted by $\widetilde{D}_\lambda$. The fuzzy Bayes estimators of the failure rate are now given either by

$$\widetilde{\lambda}_B = \frac{d+\delta}{\widetilde{T} + \delta / \widetilde{E}_\lambda}, \tag{23}$$

or, by

$$\widetilde{\lambda}_B = \frac{d+\delta}{\widetilde{T} + (\delta - 1) / \widetilde{D}_\lambda}, \delta > 1,, \tag{24}$$

respectively. Using Zadeh's extension principle it is possible to find the α-cuts for the fuzzy Bayes estimator $\widetilde{\lambda}_B$ also in these cases.

When the exact value of the estimated hazard rate is needed we can use one of many methods for the defuzzification of fuzzy numbers. One of these methods, ω-average, has been proposed by Campos and Gonzalez [1]. Campos and Gonzalez in their paper [1] call this concept as the λ-average. However, in the reliability context λ usually denotes the constant hazard rate, so in this paper we propose to name this concept as the ω-average. When $\widetilde{X}$ is a fuzzy number (fuzzy set) described by the set of its α-cuts $\left[X_L^\alpha, X_U^\alpha\right]$ such that the support of $\widetilde{X}$ is a closed interval, then its ω-average value is defined by Campos and Gonzalez [1] as

$$V_S^\omega\left(\widetilde{X}\right) = \int_0^1 \left[\omega X_U^\alpha + (1-\omega) X_L^\alpha\right] d\alpha, \quad \omega \in [0,1]. \tag{25}$$

Thus, the ω-average value of $\widetilde{X}$ can be viewed as its defuzzified value. The parameter ω in (25) is a subjective degree of decision maker's optimism (pessimism). In the case of fuzzy risks $\omega = 0$ reflects his highest optimism as the minimal values of all α-cuts (representing the lowest possible risks) are taken into consideration. On the other hand, by taking $\omega = 1$ the decision maker demonstrates his total pessimism. If the decision maker takes $\omega = 0{,}5$ his attitude may described as neutral.

4. Numerical example

To illustrate the concepts introduced in this paper let us consider a simple practical example. Suppose that the reliability field data from n=10 pieces of an infrequently used equipment have been collected. The users were asked to give the times to first failures, or the times of exploitation in the case of the equipment that has not failed yet. In case of troubles with providing precise information on those times, the users were asked to provide the data in a form of time intervals [t_{min}, t_{max}]. Note, that this is the simplest form of fuzzy data, when the membership function of a fuzzy number $\widetilde{T}$ has a rectangular form

$$\mu_T(t) = \begin{cases} 1 & t_{min} \le t \le t_{max} \\ 0 & \text{otherwise} \end{cases} \tag{26}$$

The life-time data provided by the users are the following:

- two users gave exact time to failures: 524 hrs, and 634 hrs.;
- two users gave imprecise time to failures in a form of intervals: [450 hrs. , 500 hrs.], [650 hrs. , 700 hrs.];
- four users gave exact times of exploitation without a failure: 700 hrs. 805 hrs., 950 hrs., 1010 hrs.;

- two users gave imprecise times of exploitation without a failure in a form of intervals: [550 hrs. , 600 hrs.], [750 hrs. , 800 hrs.].

Hence, d=4 failures have been observed, and the total fuzzy time on test $\widetilde{T}$ is described by a rectangular membership function (see (18) and (19) for justification) of the following form:

$$\mu_T(t) = \begin{cases} 1 & t_{min} \le t \le t_{max} \\ 0 & \text{otherwise} \end{cases}, \qquad (27)$$

where $t_{\min}$ = 7023 hrs., and $t_{\max}$ = 7223 hrs. Hence, the expected time to failure is given in a form of the interval [1755,75 hrs. , 1805,75 hrs.]. It is also easy to show that the fuzzy maximum likelihood estimator of the hazard rate $\widetilde{\lambda}$ has a form of the rectangular fuzzy number with the support given by the interval [5,538*10^{-4} hrs^{-1} , 5,696*10^{-4} hrs^{-1}].

Now let us assume that there exists some prior information about the failure rate of the considered equipment. Let the most plausible value of the failure rate $\widetilde{D}_\lambda$ be presented in a form of the interval [5*10^{-4} hrs^{-1} , 6,667*10^{-4} hrs^{-1}], and the value of δ be equal to 3 (it means that the prior distribution of λ is asymmetric, with large values of the hazard rate more probable that the small ones).

By merging both types of information in (24) we arrive at the imprecise Bayes estimator of the hazard rate $\widetilde{\lambda}_B$ in a form of the rectangular fuzzy number with the support given by the interval [6,237*10^{-4} hrs^{-1} , 6,984*10^{-4} hrs^{-1}]. Note, that the interval for $\widetilde{\lambda}_B$ is wider that the interval for $\widetilde{\lambda}$. However, its variance – as it is usually the case for Bayes estimators – should be much smaller.

The defuzzified value of $\widetilde{\lambda}_B$ in this particular example is extremely easy to calculate. It is just a weighted sum of the lower and upper limit of the interval for $\widetilde{\lambda}_B$.

In the example given above we have assumed the simplest representation of fuzziness, namely the intervals. In practical situations imprecise information may be described by other membership functions, such as triangular or trapezoidal. For example, triangular membership functions are very useful for the formal description of a vague information like "the time to failure was about x hours". In such a case, the form of the membership function of $\widetilde{\lambda}_B$ may be not so simple. However, even in such cases it still easy to calculate the α-cuts of $\widetilde{\lambda}_B$.

5. Summary

In the paper we have described the methodology of the Bayes estimation of the failure rate when the life-time data are given in a vague form. This situation may happen when times to failure are expressed in a common language. We model such a vague information using fuzzy sets. Moreover, we assume that the prior knowledge about the estimated failure rate is modelled by the gamma prior distribution with imprecisely defined parameters. We combine the imprecise information from both sources in a form of the fuzzy Bayes estimator of the failure rate. When the exact value of the fuzzy estimator of the failure rate is needed we may adopt a simple defuzzification method presented at the end of the paper.

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