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Reliability sampling

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Reliability Sampling

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Abstract

Reliability sampling is a part of acceptance sampling dedicated to the control of these quality characteristics of a product which are related to its functioning in time. Therefore, in reliability sampling we are interested in such quality characteristics of a product as: mean time to failure (MTTF), mean time between failures (MTBF), probability of good performance during a specified mission time, etc. The aim of the reliability sampling procedures, known as sampling plans, is to provide means for testing requirements related to these characteristics. In the article we present reliability sampling plans by attributes and variables which are useful in reliability practice. For sampling plans by variables, we present procedures valid in cases of the exponential and the Weibull distributions of lifetimes. We consider different types of sampling plans with different types of censoring. Special attention is devoted to procedures described in international standards related to reliability.

1. INTRODUCTION

Reliability Sampling is a part of acceptance sampling (*see* eqr134) that is dedicated to the control of these quality characteristics of a product which are related to its lifetime. In contrast to classical acceptance sampling, in reliability sampling procedures, such like lifetime tests, quality of sampled items is evaluated using tests that last in time. Other distinction between classical acceptance sampling and reliability sampling stems from a fact that the latter is rather used for the control of process parameters than for the control of finite lots or batches of products. Therefore, popular methods of sampling by attributes are seldom used in reliability sampling. On the other hand, methods of sampling by variables, whose applicability for the acceptance sampling of finite lots is questionable, are very useful in the context of reliability sampling.

Statistical procedures of reliability sampling can be roughly divided into two general groups:

- Sampling procedures used for the control of reliability characteristics defined in terms of probabilities (probability of survival of a pre-specified time, probability of correctly performing a certain mission, probability of passing a special test, etc.) , and
- Sampling procedures used for the control of reliability characteristics defined in terms of time (mean time to failure, mean time between failure, hazard rate, etc.).

Let p be reliability characteristic (parameter) such as, e.g. probability of failure or constant failure rate. In reliability sampling we define two characteristic values of p: its acceptable value p_0, and its disqualifying value p_1. Reliability sampling plans are the methods for the verification of a statistical null hypothesis $H_0 : p \leq p_0$ against an alternative one $H_1 : p \geq p_1$. Let α be the probability of wrong rejection of H_0 (producer's risk), β the probability of wrong acceptance of H_1 (consumer's risk), and $L(p)$ the probability of acceptance the null hypothesis when the true value of the considered characteristic is p. Then, the reliability sampling plan is designed in such a way that the following two requirements, $L(p_0) \geq 1-\alpha$, and $L(p_1) \leq \beta$, are fulfilled. One may note that the requirements defined by both these equations are exactly the same as in the case of classical acceptance sampling plans. It is generally true, but the difference between classical and reliability acceptance sampling plans lies rather in the way the statistical data are collected. We discuss these differences in details in the next section of this article.

2. RELIABILITY SAMPLING BY ATTRIBUTES

In statistical sampling by attributes we only note if a sampled item conforms or does not conform to certain requirements. In the context of reliability, conformance is usually understood as the correct functioning during a certain period of time, correct performing of a certain action or positive passing a special trial (e.g. an environmental trial). In any case, a sampled item is always classified as either conforming or nonconforming. Therefore, all well known sampling plans by attributes, like those described in the international standard ISO 2859-1 [1], may be applied. More useful sampling plans by attributes can be found in the international standard IEC 61123 [2]. The basic reliability sampling plans presented in [2] are the curtailed sequential sampling plans by attributes (*see* eqr136). In contrast to the original sequential sampling procedures a sample size curtailment in introduced in the plans given in [2]. If the cumulative number of tested items reaches the curtailment value n_t without taking a final decision, inspection terminates and the decision is then made using the curtailment values of the acceptance number of observed failures. Parameters of the plans given in [2] are calculated using simple approximate formulas proposed by Wald [3] for non-curtailed sampling plans. Thus, the exact statistical characteristics of the curtailed sequential sampling plans given in this standard are different from the designed ones. Curtailed sequential sampling plans by attributes based on exactly computed statistical properties can be found in the latest version of the international standard ISO 8422 [4]. In addition to curtailed sequential sampling plans, IEC 61123 [2] gives also reliability sampling plans where the number of trials (sample size) or the number of observed failures are predetermined.

3. RELIABILITY SAMPLING FOR LIFETIME DISTRIBUTIONS

3.1 General description

When reliability requirements are related to lifetimes (survival times) of tested items, reliability sampling procedures are different from those of classical sampling by variables. In classical sampling by variables the values of the quality characteristic of interest are observed for all items in the sample. In reliability tests, due to unavoidable limitations imposed on the possible time of the reliability test, not all lifetimes of tested items are observed. This feature of lifetime tests is known as censoring. Censoring arises in various ways. In case of the Type II censoring the test is terminated after the occurrence of the rth failure. Thus, the number of observed failures is fixed, but the length of the lifetime test is random. In such a case, the lifetimes of the remaining n-r tested items are known only to exceed the time of the rth failure. In case of the Type I censoring, the test is terminated after a predetermined time t_B. For this type of censoring the test time is fixed, but the number of observed failures is random. There exist also more complex censoring schemes. For example, in case of progressive censoring, still non-failed items are withdrawn from the test at times of observed failures. The presence of censoring makes the design of reliability sampling plans very complicated. Only for exponentially distributed lifetimes, sampling plans are relatively easily designed. Moreover, the efforts of many researchers resulted in some practical procedures for the cases of the Weibull and the log-normal distributions of lifetimes. For other probability distributions of lifetimes reliability sampling procedures are practically non-existent.

3.2 Reliability sampling in case of the exponential distribution

The exponential distribution, defined by the density function

$$f(t) = \frac{1}{\theta} exp\left(-\frac{t}{\theta}\right) = \lambda\, exp(-\lambda t),\, t > 0\,,$$

is the most popular statistical model that has been used for the analysis of lifetimes. The reason of its popularity stems from the fact that it is completely defined by only one parameter: the expected value θ, or its reciprocal – the constant hazard rate λ. This allows reliability analysts to estimate basic reliability characteristics when only few reliability data are available.

In the statistical analysis of lifetime data described by the exponential distribution we use the fact that all information from a sample is subsumed by two statistics: the observed number of failures r, and the total time on test (total observed lifetimes) T. From a practical point of view this feature is very important: the same information comes from lengthy lifetime tests of few items and shorter tests of many items.

The most popular reliability sampling procedure is a sequential one, originally proposed by Epstein and Sobel [6]. Its curtailed version is described in the international standard IEC 61124 [7]. For this test the observed number of failures r is plotted against the cumulative (total) time on test. If the cumulative time on test T, for the observed number of failures r, is greater than or equal to an acceptance critical value T_a, the test is terminated, and the specified reliability requirement is regarded as being complied with. If, on the other hand, this time is smaller than or equal to a rejection critical value T_r, the test is terminated, and the specified reliability requirement is regarded as not being complied with. If time T, for the observed number of failures r, lies between these two critical values, no decision can be taken, and the test is continued. When the number of observed failures exceeds the curtailment value, the test is terminated, and the specified reliability requirement is regarded as not being complied with. The international standard [7] provides also fixed time/failure terminated reliability sampling plans. For these sampling plans decisions rules of accepting or rejecting compliance are made when the test time for termination has been reached, or the acceptable number of failures has been exceeded.

3.3 Reliability sampling in case of the Weibull and other lifetime distributions

The Weibull distribution is described by the following density function

$$\frac{\beta}{a}\left(\frac{t}{a}\right)^{\beta-1}\exp\left[-\left(\frac{t}{a}\right)^{\beta}\right] \quad t>0,$$

where $\beta>0$ is the shape parameter, and $a>0$ is the scale parameter of the distribution. When the value of β is known, we can transform the Weibull distribution to the exponential distribution using a simple transformation $Z=T^{\beta}$. In such a case, we can use simple reliability sampling plans designed for the exponential distribution. However, when the value of the shape parameter is not known, the design of reliability sampling plans is not so simple. First of all, the most popular reliability sampling plans, namely sequential sampling plans, have not been proposed yet. However, several authors proposed reliability sampling plans censored by the number of observed failures for the transformed lifetime variable $X=lnT$ which is distributed according to the extreme value probability distribution. First such plans were proposed by Fertig and Mann [8] who used Best Linear Unbiased Estimators (BLUE's) for the estimation of the parameters of the distribution. The sampling plan (sample size n, the censoring number of failures r, and the critical value of the test statistics) was based on the approximate probability distribution of the test statistics – the standardized logarithm of the p-th quantile of the Weibull distribution. Another reliability sampling plan was proposed by Schneider [9] who used the Maximum Likelihood Estimators (MLE's) of the parameters of the distribution. The decision criterion was typical for the sampling plans by variables. The result of the reliability test is positive if $\hat{\mu} - k\hat{\sigma} \geq ln\, L$, where $\hat{\mu}$ and $\hat{\sigma}$ are the estimators of the parameters $\mu = ln\, a$ and $\sigma = 1/\beta$, respectively, k is the acceptability constant, and L is the lower specification limit for the lifetime. The parameters of the sampling plan are in this case computed from the corrected asymptotic distribution of the MLE's. Similar sampling plans have been also proposed for the log-normal and the gamma probability distributions.

3.4 Reliability sampling plans for complex censoring schemes

In real reliability sampling experiments certain items are removed from the test at its different stages. For instance, they are removed in order to investigate physical degradation processes that lead to future failures. When a prespecified number of tested items are withdrawn from the test at the moments of failures of some other items, the censoring scheme is known as progressive censoring of Type II. The general methodology for the design of such reliability sampling plans was proposed by Balasooriya and coauthors. The case of the exponential distribution is considered in [10], and the case of the Weibull distribution is described in [11]. The results presented in these papers, and in other recently published papers on similar topics, are rather far from being applicable in practice,

4. BAYES RELIABILITY SAMPLING PLANS

The reliability sampling plans described in the previous sections of this article are based on purely statistical requirements. It is also assumed that all pertinent information come from tested samples. One might think about reliability sampling procedures that are based also on the other information related e.g. to sampling costs, losses induced by wrong decisions etc. Moreover, usually there exists certain prior information about the reliability of tested items that is especially useful in case of highly reliable objects. Statistical procedures that utilize all such information are named Bayesian procedures. In the area of reliability sampling an interesting general framework for the design of optimal sampling plans (in case of the exponential distribution) was proposed in the paper of Dunsmore and Wright [12]. Bayesian life test plans for the Weibull distribution with given shape parameter are also given in the paper by Zhang and Meeker [13]. In these both instances, however, particular sampling plans are to be determined by a user.

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