279/2006

Raport Badawczy

Research Report

RB/14/2006

Statistics with fuzzy data in statistical quality control

O. Hryniewicz

Instytut Badań Systemowych
Polska Akademia Nauk

Systems Research Institute
Polish Academy of Sciences

POLSKA AKADEMIA NAUK

Instytut Badań Systemowych

ul. Newelska 6

01-447 Warszawa

tel.: (+48) (22) 8373578

fax: (+48) (22) 8372772

Kierownik Pracowni zgłaszający pracę:
prof. dr hab. Olgierd Hryniewicz

Warszawa 2006

Statistics with fuzzy data in statistical quality control

Olgierd Hryniewicz

Systems Research Institute, Newelska 6, 01-447 Warsaw, Poland
hryniewi@ibspan.waw.pl

Abstract

Statistical Quality Control (SQC) is an important field where both theory of probability and theory of fuzzy sets may be used. In the paper we give a short overview of basic problems of SQC that have been solved using both these theories simultaneously. Some new results on the applications of fuzzy sets in SQC are presented in details. We also present problems which are still open, and whose solution should definitely increase the applicability of fuzzy sets in quality control.

Key words: *fuzzy sets, statistical quality control, acceptance sampling, statistical process control, control charts*

1. Introduction

Statistical Quality Control (SQC) is probably the most popular application of statistical methods. It was introduced more than eighty years ago, and since that time it has been used by thousands of practitioners. One of its methodologies, acceptance sampling, has been so successful, that for some statisticians it is the most convincing example of the applicability of the "frequentist" approach to probability and statistics. Thus, SQC is a firmly established methodology with many practical applications.

It is frequently observed in the case of application-oriented methodologies, like SQC, that practitioners raise questions while facing problems with the practical application of basic concepts. Many of these problems are caused by unnecessary – in view of practitioners – precision, required for the description of quality requirements and statistical data. Solutions for those problems that are offered by theoreticians are frequently viewed upon as impractical, and thus ignored in practice. Some twenty years ago it appeared to specialists in SQC that the theory of fuzzy sets proposed by Lotfi A. Zadeh provides useful tools for dealing with many practical problems related to the lack of precision in statistical data and imprecisely defined quality requirements. In the paper we are going to present the way, how fuzzy sets have been incorporated in theory and practice of SQC.

In the second section of the paper we present the basic practical problems of SQC that triggered interest of specialists working in quality control to fuzzy sets. In the first subsection of this section we present an overview of some solutions proposed by the pioneers of the application of fuzzy sets in this area. In the second subsection we present an original methodology for designing fuzzy acceptance sampling plans for attributes which may be used for the analysis of fuzzy data. In the third section of the paper we present the state of the art of current research activities in the applications of fuzzy sets in SQC. Finally, in the last section of the paper, we discuss some important challenges that face both theoreticians of fuzzy probability and statistics and practitioners of SQC. Overcoming these difficulties seems to be a prerequisite for the future practical successes of the fuzzy methodologies in quality control, and related areas like reliability and safety analysis.

2. Application of Fuzzy Sets in Statistical Quality Control

2.1 Fuzzy methodology in Statistical Quality Control – an overview

Basic ideas of SQC have been developed in parallel with the ideas of statistical testing. Thus, some basic concepts of SQC, like, e.g., producer's and consumer's risks, have their clear statistical interpretation, and the theory of statistical tests has been used in designing of SQC procedures. However, in the 1950's some specialists in SQC noticed that economic consequences (a wide variety of costs) of the applied procedures should be taken into account. For example, there exists an obvious relationship between economic consequences of the usage of SQC procedures and such concept as allowable risk. Unfortunately, these consequences are never precisely known, so crisp "economic-oriented" models of SQC procedures have not been used in practice. Thus, the lack of precision in the estimation of involved costs leads to an obvious conclusion that the requirements for the statistical characteristics of SQC procedures should be defined in a "softer" way.

First attempts to "soften" classical SQC procedures were made in the area of acceptance sampling. In the case of the simplest and the most frequently used procedure of acceptance sampling by attributes, inspected items are classified as either conforming or nonconforming. A random sample of n items is taken from a lot (or a process), and the number of observed nonconforming items d is recorded. If this number is not greater than a certain acceptance number c, the whole lot is accepted. Otherwise, it is rejected with different consequences of this action. Thus, any single acceptance sampling plan by attributes is described by a pair of integers (n,c). In order to find the values of n and c, we usually specify four parameters: producer's quality level p_0, consumer's quality level p_1, producer's risk α, and consumer's risk β. The values of p_0 and p_1 are usually expressed in terms of fractions (or percentages) of nonconforming items in a lot or a process. The value of p_0 is assumed to be acceptable, and the value of p_1 is assumed to be non-acceptable. The value of α represents the probability of wrongly rejection of inspected lots of good quality (i.e. when the actual fraction nonconforming in the lot is equal to p_0), and the value of β represents the probability of wrongly acceptance of inspected lots of bad quality (i.e. when the actual fraction nonconforming in the lot is equal to

p_1). The required sample size n, and the acceptance number c, we find from the following inequalities:

$$\begin{cases} P(p_0) \geq 1 - \alpha \\ P(p_1) \leq \beta \end{cases} \qquad (1)$$

where $P(p)$ is the probability of acceptance when the actual quality level is equal to p.

Ohta and Ichihashi [19] were first authors who considered "softening" of (1) in a special case, when the requirements are stated in a form of equalities. A generalization of (1) with fuzzy inequalities was discussed by Kanagawa and Ohta [16]. In the most general case a fuzzy equivalent of (1) can be expressed as

$$\begin{cases} P(\widetilde{p}_0) \geq 1 - \widetilde{\alpha} \\ P(\widetilde{p}_1) \leq \widetilde{\beta} \end{cases} \qquad (2)$$

where $P(\widetilde{p})$ denotes the probability that a lot of imprecisely defined (fuzzy) quality $\widetilde{p}$ will be accepted. Solution to (2) was considered by Tamaki et al. [23], who solved a certain fuzzy mathematical programming problem with modal (possibility or necessity) constraints. Another solution to (2) was proposed by Grzegorzewski [9], who applied a methodology of fuzzy hypothesis testing proposed by Arnold [1].

Another important field of SQC is *Statistical Process Control (SPC)*. The main tools of SPC are control charts introduced by W. Shewhart in the 1920's. Control charts are widely used in production practice where both quality requirements and quality data are usually precisely defined. Therefore, applications of fuzzy sets in SPC are not so obvious as in the case of acceptance sampling. Nevertheless, first attempts to propose fuzzy control charts appeared in the late 1980's in papers by Wang and Raz [26], and Raz and Wang [21]. In these papers it has been assumed that the set of possible fuzzy values of observations is finite, and the membership functions of such observations are given in advance. The fuzzy sample mean is then represented by a certain crisp value. This value is plotted on a chart whose control lines are calculated using defuzzified values of previous observations. Kanagawa et al. [17] also assumed that the fuzzy values of observed statistics are represented by certain crisp values, and proposed a more general, but very complicated from a practical point of view, construction of a control chart. Control charts proposed in [17], [21], and [26] are relatively easy to use. On the other hand, their assumptions are not very realistic, and due to the usage of defuzzified values of sample statistics some important information may be lost.

2.2 Fuzzy acceptance sampling plans for fuzzy data

Fuzzy acceptance sampling procedures mentioned in the previous section have been proposed for working with precise statistical data. However, in many practical cases it is difficult to classify inspected items as "conforming" or "nonconforming". We face this problem rather frequently when quality data come from users who express their assessments in an informal way, using such expressions like “almost good’, “quite good”, “not so bad”, etc. First attempts to cope with the statistical analysis of such quality data can be found in Hryniewicz [13], who assumed that the quality of each inspected item is described by a family of fuzzy subsets of a set {0,1], with the following membership function

$$\mu_0 \mid 0 + \mu_1 \mid 1,\ 0 \le \mu_0, \mu_1 \le 1,\ \max\{\mu_0, \mu_1\} = 1, \tag{3}$$

where the notation $\mu|k$ stands for *<membership grade>*|*<value>*. Thus, equation (3) defines a possibility distribution on a binary set representing the result of the test.

When an inspected item “in general, fulfils quality requirements”, the result of quality assessment is expressed as a fuzzy set with the membership function $1 \mid 0 + \mu_1 \mid 1$. Fully conforming items are described by crisp sets. In this case the membership function is given by $1 \mid 0 + 0 \mid 1$. On the other hand, if an inspected item “in general, does not fulfill quality requirements”, the result of quality assessment is expressed as a fuzzy set with the membership function $\mu_0 \mid 0 + 1 \mid 1$, and fully nonconforming items are described by crisp sets with the membership function $0 \mid 0 + 1 \mid 1$.

Let us now consider the mathematical model for the process that generates such fuzzy data. Let X be a random variable describing quality of an inspected item. If the observed values of X are fuzzy we can describe them using the notion of δ-cuts. Let $\delta \in (0,1]$ be the level of the δ-cut. For this level there exist three possible observed values of X: $A_1^\delta = \{0\}$, $A_2^\delta = \{0,1\}$, and $A_3^\delta = \{1\}$, where $\{0\}$ means that the inspected item is considered as conforming, $\{1\}$ means that the inspected item is considered as nonconforming, and $\{0,1\}$ means that (at the given δ level) it is not possible to distinguish if the inspected item is conforming or nonconforming. Let us introduce now the following probabilities: $p_{00}^\delta = P(X = A_1^\delta)$, $p_{01}^\delta = P(X = A_2^\delta)$, and $p_{11}^\delta = P(X = A_3^\delta)$, where $p_{00}^\delta + p_{01}^\delta + p_{11}^\delta = 1$. Hence, at the given δ level, the result of inspection X is described by a three-point probability distribution. This distribution induces a Shafer belief function on the binary frame representing the result of the test of an individual item.

If a sample of n items is inspected the result of inspection is described by a random vector $(X_1, X_2, \ldots, X_n)$.

Let

$$I_{A_j^\delta}(X) = \begin{cases} 1 & X = A_j^\delta \\ 0 & \text{otherwise} \end{cases}, j = 1,2,3 \tag{4}$$

be the indicator function describing the result of the inspection of one item, and

$$K_j^\delta = \sum_{i=1}^{n} I_{A_j^\delta}(X_i), j = 1,2,3. \tag{5}$$

Then, at the given δ level, the result of the inspection of the whole sample is described by a trinomial distribution

$$P\left(K_1^\delta = k_1, K_2^\delta = k_2, K_3^\delta = k_3\right) = \frac{n!}{k_1!k_2!k_3!}\left(p_{00}^\delta\right)^{k_1}\left(p_{01}^\delta\right)^{k_2}\left(p_{11}^\delta\right)^{k_3} \qquad (6)$$

In quality control we are interested in the evaluation of the fraction of nonconforming items that are either surely observed $\left(X = A_3^\delta\right)$ or only possibly observed $\left(X = A_2^\delta\right)$. Therefore, the probability of observing a nonconforming item is, at the given δ level, represented as an interval $p_1^\delta = \left[p_{11}^\delta, p_{11}^\delta + p_{01}^\delta\right]$. The set of such intervals (for all $\delta \in (0,1]$) defines the *fuzzy* probability of finding a nonconforming item that describes the previously defined fuzzy observations. When results of a test are crisp, the total number of nonconforming items in a sample is distributed according to a binomial distribution. Thus, in the considered fuzzy case it is described by a fuzzy random variable distributed according to a fuzzy binomial distribution.

Assume now, that in the sample of n items in n_1 cases the quality of inspected items is characterized by fuzzy sets described by the membership function $\mu_{0,i} \mid 0 + 1 \mid 1, i = 1, \ldots, n_1$ and in the remaining $n_2 = n - n_1$ cases by a fuzzy set described by the membership function $1 \mid 0 + \mu_{1,i} \mid 1, i = 1, \ldots, n_2$. Without loss of generality we can assume that $0 \le \mu_{0,1} \le \ldots \le \mu_{0,n_1} \le 1$, and $1 \ge \mu_{1,1} \ge \ldots \ge \mu_{1,n_2} \ge 0$. Hence, the fuzzy total number of nonconforming items in this sample, calculated using Zadeh's extension principle, is given by Hryniewicz [13]:

$$\widetilde{d} = \begin{array}{c} \mu_{0,1} \mid 0 + \mu_{0,2} \mid 1 + \ldots + 1 \mid n_1 + \\ + \mu_{1,1} \mid (n_1 + 1) + \ldots + \mu_{1,n_2} \mid (n_1 + n_2) \end{array} \quad (7)$$

This number has to be compared with a fuzzy acceptance number $\widetilde{c}$ which can be found using a methodology for the design of fuzzy acceptance sampling plans suggested by Hryniewicz [13], and described in details below.

Let us assume that the parameters p_0, p_1, α, and β that describe an acceptance sampling plan are imprecisely defined by triangular fuzzy numbers $\widetilde{p}_0 = (p_{0L}, p_0^0, p_{0U})$, $\widetilde{p}_1 = (p_{1L}, p_1^0, p_{1U})$, $\widetilde{\alpha} = (\alpha_L, \alpha_0, \alpha_U)$, and $\widetilde{\beta} = (\beta_L, \beta_0, \beta_U)$, respectively. Using the results of Hald [12] we can calculate the sample size n as the solution (rounded to the nearest integer) of the following equation

$$n\left(p_1^0 - p_0^0\right) - \sqrt{n}\left(y_{1-\beta_0}\sqrt{p_1^0 q_1^0} - y_{1-\alpha_0}\sqrt{p_0^0 q_0^0}\right) + \left(k_1 - k_0\right) = 0, \quad (8)$$

where

$$q_0^0 = 1 - p_0^0,\ q_1^0 = 1 - p_1^0,\ u_0 = y_{1-\alpha_0},\ u_1 = y_{1-\beta_0} \quad (9)$$

and

$$k_i = -0{,}5 + \left(q_i^0 - p_i^0\right)\left(u_i^2 - 1\right)/6,\ i = 0{,}1. \quad (10)$$

By y_γ we denote the quantile of order γ in the standard normal distribution.

When all parameters used for the design of the sampling plan (p_0, p_1, α, β) are crisp, we can use an approximate formula proposed by Hald [12] for the calculation of the acceptance number c. In the case of fuzzy values of these parameters we can use Zadeh's extension principle to fuzzify the value of c. The fuzzy acceptance number $\widetilde{c}$ can be described by its δ-*cuts*. For the given δ level we can calculate the interval $\left(c_L^\delta, c_U^\delta\right)$ with limiting values calculated from the following expressions:

$$c_L^\delta = \inf_{\substack{p_0 \in \left(p_{0L}^\delta, p_{0U}^\delta\right), p_1 \in \left(p_{1L}^\delta, p_{1U}^\delta\right) \\ \alpha \in \left(\alpha_L^\delta, \alpha_U^\delta\right), \beta \in \left(\beta_L^\delta, \beta_U^\delta\right)}} \left[n\left(p_0 + p_1\right) + \sqrt{n}\left(y_{1-\alpha}\sqrt{p_0 q_0} - y_{1-\beta}\sqrt{p_1 q_1}\right) + \right.$$
$$\left. + \left(k_0 + k_1\right)\right]/2 \tag{11}$$

$$c_U^\delta = \sup_{\substack{p_0 \in \left(p_{0L}^\delta, p_{0U}^\delta\right), p_1 \in \left(p_{1L}^\delta, p_{1U}^\delta\right) \\ \alpha \in \left(\alpha_L^\delta, \alpha_U^\delta\right), \beta \in \left(\beta_L^\delta, \beta_U^\delta\right)}} \left[n\left(p_0 + p_1\right) + \sqrt{n}\left(y_{1-\alpha}\sqrt{p_0 q_0} - y_{1-\beta}\sqrt{p_1 q_1}\right) + \right.$$
$$\left. + \left(k_0 + k_1\right)\right]/2 \tag{12}$$

All the symbols in the formulae (11) and (12) have the same interpretation as in (8).

Let $c_1, c_2, \ldots, c_m$ be set of all integers that belong to the support of the fuzzy set $\widetilde{c}_0$ defined by the δ–cuts $\left(c_L^\delta, c_U^\delta\right)$, and let us denote its membership function by $\mu_0(c)$. Now, for each $c_i, i = 1, \ldots, m$ let us define the interval $A_i = [c_i - 0,5, c_i + 0,5)$. Next, for each $c_i, i = 1,\ldots,m$ let us calculate the following values of a membership function

$$\mu(c_i) = \sup_{c \in A_i} \mu_0(c),\ i = 1,\ldots,m. \tag{13}$$

Hence, the fuzzy equivalent $\widetilde{c}$ of an acceptance number c may be defined by the following fuzzy set:

$$\widetilde{c} = \mu(c_1) | c_1 + \cdots + \mu(c_i) | c_i + \cdots + \mu(c_m) | c_m. \tag{14}$$

The decision to reject the sampled lot or process has to be taken if the inequality $\widetilde{d} \succ \widetilde{c}$ holds. It is a well known fact that a unique method for the comparison of two fuzzy numbers does not exist. Therefore, in practice we have to use one of the methods which have been proposed for that purpose.

Let $\mu(x)$ be the membership function of the fuzzy set $\widetilde{x}$, and $\nu(y)$ be the membership function of the fuzzy set $\widetilde{y}$. The *Possibility of Strict Dominance (PSD)* index, introduced by Dubois and Prade [6], that measures the possibility that $\widetilde{x}$ strictly dominates $\widetilde{y}$, is calculated from the following formula

$$PSD(\widetilde{x} \succ \widetilde{y}) = \sup_{x} \left\{ \inf_{y: x \le y} \left[\min(\mu(x), 1 - \nu(y)) \right] \right\}. \tag{15}$$

Positive, but smaller than 1, values of this index indicate certain weak evidence that $\tilde{x}$ strictly dominates $\tilde{y}$. When this evidence is stronger we can measure it using the *Necessity of Strict Dominance (NSD)* index, introduced by Dubois and Prade [6], and defined as follows

$$NSD(\tilde{x} \succ \tilde{y}) = 1 - \sup_{x,y:x \le y} [min(\mu(x), \nu(y))]. \tag{16}$$

Extensive simulations described in [13] have revealed that the *NSD* index seems to be the most useful for the purpose of the comparison of $\tilde{d}$ and $\tilde{c}$. Thus, the decision of rejection should be taken if $NSD(\tilde{d} \succ \tilde{c}) \ge \eta_0$, where $0 \le \eta_0 \le 1$ is a certain required value of *NSD.*

Let us illustrate the usage of the proposed method with the following example. The management of a firm producing household appliances has decided to verify if the quality of certain product fulfills expectations of users. It has been decided that a sample of *n* randomly chosen users should be asked about their opinion about this product. The design of the sampling plan was initially based on experience coming from production tests. The level of acceptable quality, p_0, measured in percent of negative opinions, was set to 5%, and the level of non-acceptable quality, p_1, was set to 15%. The probability of wrong rejection, when the product meets stated quality requirements (producer's risk), α, was set to 3%. The probability of wrong acceptance, when the quality of the product is on the non-acceptable level (consumer's risk), β, was set to 5%. In further discussions it has been decided that design parameters typical for production tests are too stringent. Thus, their relaxed versions have been proposed in a form of fuzzy triangular numbers: (0.05, 0.05, 0.08) for $\tilde{p}_0$, (0.03, 0.03, 0,06) for $\tilde{\alpha}$, (0.15, 0.15, 0.20) for $\tilde{p}_1$, and (0.05, 0.05, 0.10) for $\tilde{\beta}$.

The sample size n, calculated from (8), is now equal to 103. The acceptance criterion, calculated from the formulae (11) – (14) is given as the following fuzzy set

$$\tilde{c} = 1.0|9 + 0.93|10 + 0.70|11 + 0.47|12 + 0.25|13 + 0.0|14. \quad (17)$$

In the questionnaire sent to the users, five possible answers have been available:

a) "I am fully satisfied" (represented by the fuzzy set 0|1 + 1|0),

b) "I am rather satisfied" (represented by the fuzzy set 0|1 + 1|0.5),

c) "I do not know" (represented by the fuzzy set 0|1 + 1|1),

d) "I am rather not satisfied" (represented by the fuzzy set 0|0.5 + 1|1), and

e) "I am definitely not satisfied" (represented by the fuzzy set 0|0 + 1|).

From among n=103 persons, 90 have chosen option a), 3 have chosen option b), 5 have chosen option c), 3 have chosen option d), and only 2 have chosen option e). Thus the fuzzy set that represents the result of the investigation is given as follows:

$$\tilde{d} = 0|0 + 0|1 + 0.5|2 + 0.5|3 + 0.5|4 + 1|5 + 1|6 + 1|7 + 1|8 + 1|9 + 1|10 + 0.5|11 + 0.5|12 + 0.5|13 + 0|14. \qquad (18)$$

Now, let us verify if the result of this investigation supports the claim that the quality of the considered products is worse than required. In order to do so, we have to evaluate the relation $\tilde{d} \succ \tilde{c}$, where $\tilde{d}$ is given by (18), and $\tilde{c}$ is given by (17). The *NSD* index, calculated from (16) is equal to 0. Thus, there is no strong evidence that this relation is true. The *PSD* index, calculated from (15), is in the considered case equal to 0.5. It means that there exists certain weak evidence that the quality requirements are not fulfilled. This is hardly unexpected, if we look at the results of the investigation. Despite the fact that only 2 opinions were definitely negative, altogether 13 opinions were not absolutely positive. This number is obviously greater than the acceptance number calculated for the most stringent quality requirements (c=9). On the other hand, it is not greater than the acceptance number for the most relaxed quality requirements (c=13). Thus, the results of the investigation cannot be regarded as a decisive one.

3. Current Problems of Fuzzy SQC

In the previous section we have presented the main problems of SQC where fuzzy sets have found many applications. The results published in 1980s and 1990s let us state that the basic theory of fuzzy SQC has been already established. Therefore, during the last ten years the efforts of specialists in these fields have been focused rather on solving particular problems than on more general issues.

3.1 Applications in Statistical Process Control

In the area of statistical process control new results have been proposed in the papers by Grzegorzewski [8], Grzegorzewski and Hryniewicz [11], Taleb and Limam [22], and Tannock [24]. One of the charts proposed in [8] and [11] is based on the concepts of fuzzy statistical confidence intervals and the necessity of strict dominance (*NSD*) index defined by (16). Control (decision) lines of the chart are calculated as critical values of certain fuzzy statistical tests. The inspection with the chart begins with setting significance level α and necessity index η. Then, a fuzzy sample of n items is observed, and the interval corresponding to (1-η)th cut of the arithmetical mean $\overline{X}$ is plotted on the chart. If the whole interval lies outside the control lines we claim that the process is out of control. If this interval intersects one of the control lines, a warning signal is generated.

Nearly all papers devoted to fuzzy control charts deal with fuzzy versions of a classical Shewhart control chart. A cumulative sum (CUSUM) control chart for fuzzy quality data has been proposed recently by Wang [25]. In this paper, Wang proposes to represent original fuzzy data by appropriately chosen "representative values", and then to plot these values on a classical CUSUM control chart.

Interesting new application of fuzzy control charts has been recently proposed by Cheng [4]. He assumed that instead of usual measurements of quality characteristics aggregated fuzzy quality measures provided by experts are displayed on a control chart. Another interesting combination of classical SQC procedure and fuzzy technique, namely neural fuzzy technology, can be found in the paper by Chang and Aw [3].

3.2 Applications in acceptance sampling

In acceptance sampling new applications of fuzzy SQC are rather seldom. Interesting, from a purely theoretical point of view, results were obtained by Krätschmer [18], who proposed a mathematically sound basis for the sampling inspection by attributes in fuzzy environment. Unfortunately, no new practical SQC procedures have been proposed using that general model.

Much more practical procedure, namely a fuzzy version of an acceptance sampling plan by variables, has been proposed by Grzegorzewski [10]. General results from the theory of fuzzy statistical tests have been used in this paper for the construction of fuzzy sampling plans when the quality characteristic of interest is described by a fuzzy normal distribution.

Finally, we have to mention the result that could be applied not only in SQC (inspection interval for control charts) but also in the theory and practice of reliability (inspection intervals for monitoring processes). Hryniewicz [14] has shown why in the case of imprecise input information optimal inspection intervals are usually determined using additional preference measures than strict optimization techniques.

4. Challenges for the Future

The short overview of the applications of fuzzy sets in the area of SQC shows that there is a solid ground for the implementation of fuzzy sets methodology in practice, as it is the case for the theory of probability and mathematical statistics. There are, however, some serious problems of theoretical and practical nature that have to be overcome if we want to see real applications. In this section of the paper we present our personal and subjective view on challenges that have to be faced by specialists working in the area of fuzzy sets and their applications.

Problems of Statistical Quality Control have both random (probabilistic) and imprecise (fuzzy, possibilistic) nature. Therefore, serious efforts have to be undertaken in order to clarify mutual relationships between these methodologies. The paper by Dubois and Prade [7] presents an interesting overview of this problem, and the recent results of de Cooman [5] should be regarded as an important step on the way to solve it. Unfortunately, there is still a lot to do if we want to have a general theory covering both randomness and fuzziness.

Another challenge for the fuzzy sets community is connected with operational rules that have to be used by practitioners. Fuzzification of existing results in SQC usually leads to prohibitively complex computations. Therefore, there is a need for commonly agreed simplifications and approximations. An example of such work can be found in Hryniewicz [15], where complex calculations of system's fuzzy reliability have been dramatically simplified using the concept of shadowed sets introduced by Pedrycz [20]. Such simplifications and approximations should be proposed having in mind a certain ultimate view: to provide practitioners with some *standards* for dealing with imprecise concepts. By *standards* I mean standardized procedures for simple description of fuzzy events. For example, if we decide that trapezoidal fuzzy numbers should be used for the description of fuzzy results of measurements, we have to give *operational* methods for the evaluation of the parameters defining the membership function of such numbers.

To the end of this paper, we would like to cite a statement from the paper by Cai [2]: " .., the area of fuzzy methodology in system failure engineering is still staying in a speculative research period and is premature. From a speculative research period to an engineering practice period, from premature to mature, a lot of work has to be done". After nearly ten years, this statement still remains, unfortunately, true. Not only in the area of system failure engineering, but in statistical quality control, as well.

References

1. Arnold BF (1996) An Approach to Fuzzy Hypothesis Testing. Metrika 44:119-126.
2. Cai KY (1996) System failure engineering and fuzzy methodology. An introductory overview. Fuzzy Sets and Systems 83:113-133.
3. Chang SI, Aw CA (1996) A Neural Fuzzy Control Chart for Detecting and Classifying Process Mean Shifts. International Journal of Production Research 34:2265-2278.
4. Cheng CB (2005) Fuzzy process control: construction of control charts with fuzzy numbers. Fuzzy Sets and Systems 154:287-303.
5. de Cooman G (2005) A behavioral model for vague probability assessments. Fuzzy Sets and Systems 154:305-358.
6. Dubois D, Prade H (1983) Ranking Fuzzy Numbers in the Setting of Possibility Theory. Information. Science 30, 1983:183-224.
7. Dubois D, Prade H (2002) Quantitative Possibility Theory and its Probabilistic Connections. In: Grzegorzewski P, Hryniewicz O, Gil MA (eds) Soft Methods in Probability, Statistics and Data Analysis. Physica-Verlag, Heidelberg, pp 3-26.
8. Grzegorzewski P (1997) Control Charts for Fuzzy Data. Proc. of the 5th European Congress EUFIT'97, Aachen, pp 1326-1330.
9. Grzegorzewski P (2001) Acceptance Sampling Plans by Attributes with Fuzzy Risks and Quality Levels. In: Wilrich PTh, Lenz HJ (eds) Frontiers in Statistical Quality Control 6. Physica-Verlag, Heidelberg, pp 36-46.
10. Grzegorzewski P (2002) Acceptance Sampling Plans by Variables for Vague Data. In: Grzegorzewski P, Hryniewicz O, Gil MA (eds) Soft Methods in Probability, Statistics and Data Analysis. Physica-Verlag, Heidelberg, pp 197-206.
11. Grzegorzewski P, Hryniewicz O (2000) Soft methods in statistical quality control. Control and Cybernetics 29:119-140.
12. Hald A (1981) Statistical theory of sample inspection by attributes. Academic Press, New York.
13. Hryniewicz O (1994) Statistical Decisions with Imprecise Data and Requirements. In: Kulikowski R, Szkatula K, Kacprzyk J (eds) Systems Analysis and Decisions Support in Economics and Technology. Omnitech Press, Warszawa, pp 135-143.
14. Hryniewicz O (2003) User-preferred solutions of fuzzy optimization problems—an application in choosing user-preferred inspection intervals. Fuzzy Sets and Systems 137:101-111.

15. Hryniewicz O (2005) Evaluation of reliability using shadowed sets and fuzzy lifetime data. In: Kolowrocki K (ed) Advances in Safety and Reliability. A.A.Balkema, Amsterdam, pp 881-886.
16. Kanagawa A, Ohta H (1990) A Design for Single Sampling Attribute Plan based on Fuzzy Sets Theory. Fuzzy Sets and Systems 37:173-181.
17. Kanagawa A, Tamaki F, Ohta H (1993) Control Charts for Process Average and Variability Based on Linguistic Data. International Journal of Production. Research 31:913-922.
18. Krätschmer V (2005) Sampling inspections by attributes which are based on soft quality standards. Proceedings of EUSFLAT'2005, Barcelona, September 2005, 611-614.
19. Ohta H, Ichihashi H (1988) Determination of Single-Sampling Attribute Plans Based on Membership Functions. International Journal of Production Research 26:1477-1485.
20. Pedrycz W (1998) Shadowed sets: Representing and processing fuzzy sets. IEEE Transactions on Systems, Man and Cybernetics – Part B: Cybernetics 28:103 – 109.
21. Raz T, Wang JH (1990) Probabilistic and Membership Approaches in Construction of Control Charts for Linguistic Data. Production Planning & Control Vol. 1:147-157.
22. Taleb H, Limam M (2002) On fuzzy and probabilistic control charts. International Journal of Production. Research 40:2849-2863.
23. Tamaki F, Kanagawa A, Ohta H (1991) A Fuzzy Design of Sampling Inspection Plans by Attributes. Japanese Journal of Fuzzy Theory and Systems 3:315-327.
24. Tannock TDT (2003) A fuzzy control charting methods for individuals. International Journal of Production. Research 41:1017-1032.
25. Wang D (2006) A CUSUM Control Chart for Fuzzy Quality Data. In: Lawry J, Miranda E, Bugarin A, Li S, Gil MA, Grzegorzewski P, Hryniewicz O (eds) Soft Methods for Integrated Uncertainty Modelling. Springer-Verlag, Heidelberg, pp 357-364.
26. Wang JH, Raz T (1990) On the Construction of Control Charts Using Linguistic Variables. International Journal of Production Research 28:477-487.