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The interface of statistical product inspection and statistical process control

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The Interface of Statistical Product Inspection and Statistical Process Control

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Abstract

Acceptance control charts, originally proposed by Freund, are the tools of statistical process control (SPC) that combine features of classical control charts and sampling plans used in acceptance sampling of products and services. In contrast to classical Shewhart control charts, their design is based on the requirements typical for acceptance sampling plans by variables. In the article, we present the design principles of the acceptance control charts. The similarity between acceptance control charts and acceptance sampling systems show that the latter might be used for the purpose of SPC.

1 INTRODUCTION

Acceptance sampling and statistical process control (SPC) are the most frequently used methodologies of statistical quality control (SQC). In acceptance sampling the result of evaluation of a random sample taken from a group of products (a lot) or services is used for making decisions about these products or services. Therefore, acceptance sampling is aimed at product or service disposition decisions, based on statistical testing of stated quality requirements. As opposed to classical acceptance sampling, control charts, the best known tools of SPC, are used to find whether monitored processes are in a "state of statistical control". In other words, control charts are primarily used as a management tool for the determination if a controlled process is stable. For this application of control charts any quality requirements need not be taken into account.

In acceptance control charts, introduced by Freund [1], [2], features of acceptance sampling and control charts, are combined.; They are used not only to monitor the stability of a process, but also to verify if product or service quality requirements are satisfied. These requirements may be satisfied even in the presence of small shifts of the process level around its target values. For example, if uniform large batches of raw material are used during a production process, the differences between the process levels related to between-batches variability may be tolerable. Similarly, small infrequent and irregular shifts of the process levels that cannot be eliminated because of economic or engineering (possibility of "over-control" of the process) considerations may be tolerated if their impact on the quality of the product is negligible. Thus, in real applications quality requirements should be taken into consideration on equal footing with striving to eliminate all possible sources of process variation.

2 ACCEPTANCE CONTOL CHARTS. DESIGN AND OPERATION

The design principles, and the operation of acceptance control charts have been described in the international standard ISO 7966 [3]. In this standard it is assumed that there exist two characteristic process levels. The acceptable process level (*APL*) represents a process that should be accepted almost always, with probability 1-α. On the other hand, the rejectable process level (*RPL*) represents a process that should almost never be accepted. When the process operates at the *RPL* level, or worse, the probability of its acceptance should be not greater than β. Thus, any process centered between the target value and *APL* will have a risk smaller than α of not being accepted. On the other hand, the risk of acceptance of processes located further away from the target value than *RPL* is smaller than β. The process levels lying in a "indifference zone" between the *APL* and *RPL* yield products of borderline quality. From a point of view of quality control this "indifference zone" should be as narrow as possible. Unfortunately, the narrower this zone, the larger will the sample size have to be. Therefore, some economic considerations have to be present in the design process of acceptance control charts.

In order to operate an acceptance control chart it is necessary to define two additional parameters: the sample size n, and the action criterion or acceptance control limit (*ACL*). If the sample average value of the quality characteristic $\overline{X}$, plotted on an acceptance control chart, falls beyond *ACL* (i.e. above the upper acceptance control limit ACL_U or below the lower acceptance limit ACL_L, in case of double specification limits) the process shall be considered non-acceptable. The location of the *APL* and *RPL* levels, and the *ACL* limits is presented on Figure 1.

< Figure 1 near here>

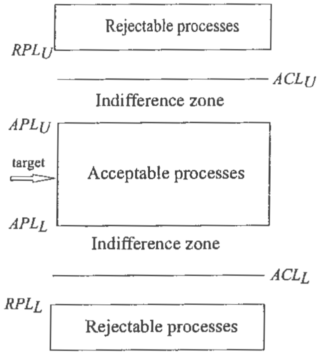

Figure 1: Basic elements of the acceptance control chart

When the *APL* and α, and *RPL* and β are defined, the upper and lower acceptance control limits are calculated as

$$ACL_U = APL_U + \left(\frac{y_{1-\alpha}}{y_{1-\alpha} + y_{1-\beta}}\right)(RPL_U - APL_U)$$

and

$$ACL_L = APL_L - \left(\frac{y_{1-\alpha}}{y_{1-\alpha} + y_{1-\beta}}\right)(APL_L - RPL_L),$$

respectively. The values of $y_{1-\alpha}$ and $y_{1-\beta}$ are the quantiles of the standard normal distribution of order 1-α and 1-β, respectively. For the calculation of the sample size *n* it is necessary to know the value of the standard deviation σ_w within the sample (rational sub-group). The value of *n* is obtained by rounding-up to the nearest integer the value calculated from the following expression

$$n = \left[\frac{(y_{1-\alpha} + y_{1-\beta})\sigma_w}{(RPL - APL)}\right]^2.$$

Let p_0 be the acceptable fraction of nonconforming items, at the output of the process, and p_1 be the rejectable fraction of nonconforming items. If these two values are known from economic considerations, and the values of the lower (*LSL*) and upper (*USL*) limits are given, we can calculate the values of *APL* and *RPL* from the following formulae:

$$APL_U = USL - y_{1-p_0}\sigma_w$$

$$APL_L = LSL + y_{1-p_0}\sigma_w$$

$$RPL_U = USL - y_{1-p_1}\sigma_w$$

$$RPL_L = LSL + y_{1-p_1}\sigma_w .$$

The formulae given above are valid when the quality characteristic is normally distributed. In cases of other probability distributions the quantiles of the standardized normal distribution should be replaced with the respective quantiles of the standardized versions of those distributions.

3 EXAMPLE

Bottles of nominal value of 1000 cm^3 are filled with sunflower oil. It has been verified that the amount of oil in a bottle is described by the normal distribution. The filling process is considered acceptable if less than 1% of the bottles does not meet the requirement described by the lower specification limit *LSL*=995 cm^3 and the upper specification limit *USL*=1005 cm^3. The process is considered non-acceptable if the percentage of the bottles that does not meet these requirements exceeds 5%. The standard deviation σ_w, estimated from historical data, is equal to 1,5 cm^3. The risks α and β are set at the same value equal to 0.05.

In order to design the appropriate acceptance control chart we have to do the following computations

Step 1.

For p_0=0.01 we have $y_{0,99} = 2.326$, and $y_{0,96} = 1.751$.

Step 2.

$$APL_U = 1005 - 2.326 * 1.5 = 1001.51$$

$$APL_L = 995 + 2.326 * 1.5 = 998.49$$

$$RPL_U = 1005 - 1.751 * 1.5 = 1002.37$$

$$RPL_L = 995 - 1.751 * 1.5 = 997.63.$$

Step 3.

The acceptance control limits are calculated as follows

$$ACL_U = 1001.51 + \left(\frac{1.645}{1.645 + 1.645}\right)(1002.37 - 1001.51) = 1001.94$$

$$ACL_L = 998.49 + \left(\frac{1.645}{1.645 + 1.645}\right)(998.49 - 997.63) = 998.06$$

Step 4.

The initial calculation of the sample size gives

$$n = \left[\frac{(1.645 + 1.645) * 1.5}{1002{,}37 - 1001{,}51}\right]^2 = 3{,}128$$

Thus, the required sample size is n=4.

4 DISCUSSION

The acceptance control charts presented in this article are practically the same as acceptance sampling plans by variables. As the matter of fact, popular acceptance sampling plans by variables, presented in the international standards from the series

ISO 3951 (see, for example, ISO 3951-1 [4]) should be rather used to accept (or reject) process levels expressed in terms of fraction nonconforming and labeled by Acceptance Quality Limits (AQL's), than to accept (or reject) individual lots of products. Accordingly, individual sampling plans from these standards could be used for the purpose of statistical process control. For example, the economically optimal control chart, based on the sampling plans taken from [4] (for unknown values of the standard deviation σ) is proposed by Hryniewicz [5].

References

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