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Fuzzy statistical methods in reliability and quality control

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Abstract

Statistical Quality Control (SQC) and Reliability and Safety (RS) are two important fields where both theory of probability and theory of fuzzy sets are used. In the paper we give a short overview of basic problems from these fields that have been solved using simultaneously both these theories. We also present problems which are still open, and whose solution should definitely increase the applicability of fuzzy sets in both areas.

1. Introduction

Statistical Quality Control (SQC) is probably the most popular application of statistical methods. It was introduced more than eighty years ago, and since that time it has been used by thousands of practitioners. One of its branches, acceptance sampling, has been so successful that for some statisticians it is the most convincing example of the applicability of the "classical" approach to probability and statistics based on observed frequencies of random events. Theory of Reliability and Safety (RS) does not have so long history. However, its successful applications are known for at least last fifty years. Thus, both SQC and RS are firmly established methodologies with many practical applications.

It is frequently observed in the case of application-oriented methodologies, like SQC and RS, that practitioners raise questions while facing problems with the practical application of some even basic concepts. Many of these problems are caused by unnecessary – in view of practitioners – precision required for the description of requirements and statistical data. Solutions to those problems that are offered by theoreticians are frequently viewed upon as impractical, and thus ignored in practice. Some twenty years ago it appeared to specialists in SQC and RS that the theory of Fuzzy Sets (FS) proposed by Lotfi A. Zadeh provides useful tools for dealing with many practical problems related to the lack of precision in statistical data and quality requirements. In the paper we are going to present the way, how fuzzy sets have been incorporated in theory and practice of SQC and RS.

In the second section of the paper we present the basic practical problems of SQC and RS that triggered interest of specialists from these fields to fuzzy sets. We present some solutions proposed by the pioneers of the application of fuzzy sets in both areas. In the third section of the paper we present the state of the art of current research activities in the applications of fuzzy sets in SQC and RS. Finally, in the last section of the paper, we discuss some important challenges that face both theoreticians of fuzzy probability and statistics and practitioners of SQC and RS. Overcoming these difficulties seems to be a prerequisite for the future practical successes of the fuzzy methodologies in quality control, reliability and safety, and related areas, like risk analysis.

2. Application of Fuzzy Sets in Statistical Quality Control and Reliability

Basic ideas of SQC have been developed in parallel with the ideas of statistical testing. Thus, some basic concepts of SQC, like, e.g., producer's and consumer's risks, have their clear statistical interpretation, and the theory of statistical tests has been used in designing of SQC procedures. However, in the 1950's some specialists in SQC noticed that economic consequences (a wide variety of costs) of the applied procedures should be also taken into account. Unfortunately, these consequences are never precisely known, so crisp "economic-oriented" models of SQC procedures have not been used in practice. However, there exists an obvious, but not well defined, relationship between economic consequences of the usage of SQC procedures and such concepts as allowable risks. Thus, the lack of precision in the estimation of involved costs leads to an obvious conclusion that the requirements for the statistical characteristics of SQC procedures could be defined in a more "soft" way, First attempts to "soften" classical SQC procedures were made in the area of acceptance sampling. In the case of the simplest and the most frequently used acceptance sampling procedure inspected items are classified as either conforming or nonconforming. A random sample of n items is taken from a lot (or a process), and the number of observed nonconforming items d is recorded. If this number is not greater than a certain acceptance number c, the whole lot is accepted. Otherwise, it is rejected with different consequences of this action. Thus, any single acceptance sampling plan by attributes is described by a pair of integers (n,c). In order to find the values of n and c, we usually specify four parameters: producer's quality level θ_1, consumer's quality level θ_2, producer's risk α, and consumer's risk β. Then we look for the sample size n, and the acceptance number c such that the following inequalities hold:

$$\begin{cases} P(\theta_1) \geq 1-\alpha \\ P(\theta_2) \leq \beta \end{cases} \tag{1}$$

Ohta and Ichihashi [1] were first authors who considered "softening" of (1) in a special case, when the requirements are stated in a form of equalities. A generalization of (1) with fuzzy inequalities was discussed by Kanagawa and Ohta [2]. In the most general case a fuzzy equivalent of (1) can be expressed as

$$\begin{cases} P(\approx \theta_1) >\approx 1-\alpha \\ P(\approx \theta_2) <\approx \beta \end{cases} \tag{2}$$

where $P(\approx \theta)$ denotes the probability that a lot of relaxed (fuzzy) quality θ will be accepted, $>\approx$ stands for "approximately greater or equal", and $<\approx$ stands for "approximately less or equal". Solution to (2) was considered by Tamaki, Kanagawa and Ohta [3], who solved a certain fuzzy mathematical programming problem with modal (possibility or necessity) constraints. Another solution to (2) was proposed by Grzegorzewski [4], who applied a methodology of fuzzy hypothesis testing introduced by Arnold [5].

Fuzzy acceptance sampling procedures mentioned above have been proposed for working with precise statistical data. However, in many practical cases it is difficult to classify inspected items as "conforming" or "nonconforming". We face this problem rather frequently when quality data comes from users who express their assessments in an informal way using such expressions like "almost good', "quite good", "not so bad", etc. First attempts to cope with the statistical analysis of such quality data can be found in Hryniewicz [6], who assumed that the quality of each inspected item is described by a family of fuzzy subsets of a set {0,1], with the following membership function $\mu_0 | 0 + \mu_1 | 1, 0 \le \mu_0, \mu_1 \le 1, \max\{\mu_0, \mu_1\} = 1$. When an inspected item "in general, fulfils quality requirements", the result of quality assessment is expressed as a fuzzy set with the membership function $1 | 0 + \mu_1 | 1$. Fully conforming items are described by crisp sets with the membership function $1 | 0 + 0 | 1$. On the other hand, if an inspected item "in general, does not fulfill quality requirements", the result of quality assessment is expressed as a fuzzy set with the membership function $\mu_0 | 0 + 1 | 1$, and fully nonconforming items are described by crisp sets with the membership function $0 | 0 + 1 | 1$.

Assume now, that in n_1 cases the quality of inspected items is characterized by a fuzzy set described by the membership function $\mu_{0,i} | 0 + 1 | 1, i = 1,\ldots,n_1$, and in the remaining $n_2 = n - n_1$ cases by a fuzzy set described by the membership function $1 | 0 + \mu_{1,i} | 1, i = 1,\ldots,n_2$. Without loss of generality we can assume that $0 \le \mu_{0,1} \le \ldots \le \mu_{0,n_1} \le 1$, and $1 \ge \mu_{1,1} \ge \ldots \ge \mu_{1,n_2} \ge 0$. Hence, the fuzzy total number of nonconforming items in a sample is given by [6]:

$$\tilde{d} = \mu_{0,1} | 0 + \mu_{0,2} | 1 + \ldots + 1 | n_1 + + \mu_{1,1} | (n_1 + 1) + \ldots + \mu_{1,n_2} | (n_1 + n_2). \tag{3}$$

This number has to be compared with a fuzzy acceptance number $\tilde{c}$ which can be found using either one of the previously mentioned methods or using a method proposed by Hryniewicz [6]. It is a well known fact that such unique method for the comparison of two fuzzy numbers does not exist. However, extensive simulations described in Hryniewicz [6] have revealed that the *Necessity of Strict Dominance (NSD)* index introduced by Dubois and Prade [7] seems to be the most useful for this purpose.

Another important field of SQC is *Statistical Process Control (SPC)*, whose main tools are so called control charts. Control charts are widely used in production practice where both quality requirements and quality data are usually precisely defined. Therefore, applications of fuzzy sets in SPC are not so obvious as in the case of acceptance sampling. Nevertheless, first attempts to propose fuzzy control charts appeared in the late 1980s in papers by Wang and Raz [8], Raz and Wang [9], and Kanagawa, Tamaki and Ohta [10].

The theory of fuzzy sets attracted specialist in RS in the 1980s. The main applications can be found in the area of the reliability and safety analysis of complex systems. The reason for this was simple: in case of complex systems we do not have enough precise information to build classical probabilistic models. For example, fault-trees - the most frequently used methodology for the analysis of reliability and safety of complex systems – require precise knowledge of all possible fault mechanisms and their probabilities. Thus, first applications of fuzzy sets in reliability were dedicated to that problem, as in the paper by Tanaka et al. [11]. Many interesting references to the papers on this problems can be found in the papers from books by Misra [12], and Onisawa and Kacprzyk [13].

Another reason for application of fuzzy sets in the area of reliability and safety is the so called "human factor". Reliability and safety of complex systems is strongly dependent on the behavior of people who control and maintain them. It is very difficult, if even possible, to asses the impact of human factors on reliability and safety in an objective way. Therefore, all analyses should take into account subjective information provided by people expressing their knowledge using natural language. Interesting results related to this problem can be found in papers by Onisawa [14] and [15].

Fuzzy sets are also useful for the analysis of reliability data, especially those coming from exploitation. In many practical situations it is very difficult to obtain precise data, and in extreme cases all data come from users whose reports are expressed in a vague way. First papers on this problem were published in the late 1980's. We have to note here the paper by Kanagawa and Ohta [22]. Some references to other papers can be found in the paper by Viertl and Gurker [17].

The vagueness of reliability data coming from the users has many different sources. In Hryniewicz [16] these sources have been divided into three groups:

- vagueness caused by subjective and imprecise perception of failures by a user,
- vagueness caused by imprecise records of reliability data,
- vagueness caused by imprecise records of the rate of usage.

First source of vagueness is typical for so called non-catastrophic failures. The tested item may be considered as failed, or – strictly speaking - as nonconforming, when at least one value of its parameters falls beyond specification limits. In practice, however, a user does not have possibility to measure all parameters, and is not able to define precisely the moment of a failure. For example, if there exists a requirement for an admissible level of noise it usually may be verified by a user only subjectively. The user can usually indicate only a moment when he noticed that the level of noise had increased, and the moment when he (or she) considered it as obviously excessive. Thus, it might be assumed that the first moment describes the time when the tested item (say, a car) may be considered as failed, and the second moment indicates the time of a sure failure. As the result, we obtain imprecise information about the real lifetime. Second source of vagueness is typical to retrospective data. Users do not record precisely the moments of failures, especially when they are not sure if they observed a real failure. So when they are asked about failures which were observed some time ago, they sometimes provide imprecise information. A lifetime of an individual is the actual length of life of that individual measured from some particular starting point. However, it may happen that the user cannot specify this starting point precisely but only in a vague way. In such situation the lifetime of the item under study is also vague. Third source of vagueness is related to the fact that users, who report their data in days (weeks, months), use the tested items with different intensity. In such a case, users are asked about the intensity of usage (for example, in hours per day), and their responses are, from obvious reasons, very often imprecise. Another example of the vagueness of that type is encountered in accelerated life tests, as it is described in the paper by Viertl and Gurker [17].

The lack of precision of reliability field data comes from all these sources and in many cases cannot be even identified. Precise probability models can be seldom applied only for clearly identified sources of vagueness, and they are very often impractical, because of many parameters which are either unknown or difficult to estimate. Therefore, we have to admit, that we often deal with really vague data expressed by imprecise words, and it is the only source of information which can be used for the verification of hypotheses about the mean lifetime of tested item

3. Current problems of fuzzy SQC and fuzzy reliability

In the previous section we have presented the main problems of SQC and RS where fuzzy sets have found many applications. The results published in 1980s and 1990s let us state that the basic theory of *Fuzzy Statistical Quality Control (FSQC)*, and *Fuzzy Reliability and Safety (FRS)* has been already established. Therefore, during the last ten years the efforts of specialists in these fields have been focused rather on solving particular problems than on more general issues.

In FSQC new results have been published in the areas of acceptance sampling and statistical process control. Many of these achievements have been possible thanks to the fundamental results in the theory of fuzzy statistical tests published in many books and papers, beginning with the seminal book by Kruse and Meyer [18]. The overview of these fundamental results can be found in the work of Kruse, Gebhardt and Gil [19].

In the area of statistical process control new results have been proposed in the papers by Grzegorzewski [20], and Grzegorzewski and Hryniewicz [21]. One of the charts proposed in these papers is based on the concepts of fuzzy statistical confidence intervals and the *NSD* index. Control lines *LCL* and *UCL* are calculated as critical values of certain fuzzy statistical tests. The inspection with the chart begins with setting significance level δ and necessity index ζ. Then, a fuzzy sample of n items is observed, and the interval I corresponding to $(1-\zeta)$th cut of the arithmetical mean $\overline{X}$ is plotted on the chart. If the whole interval lies outside the control lines we claim that the process is out of control. If this interval intersects one of the control lines, a warning signal is generated.

Interesting new application of fuzzy control charts has been recently proposed by Cheng [23]. He assumed that instead of usual measurements of quality characteristics aggregated fuzzy quality measures provided by experts are displayed on a control chart. Another interesting combination of classical SQC procedure and fuzzy technique, namely neural fuzzy technology, can be found in the paper by Chang and Aw [24].

In acceptance sampling new applications of FSQC are rather seldom. We have to note, for example, a fuzzy version of an acceptance sampling plan by variables proposed by Grzegorzewski [25]. General results from the theory of fuzzy statistical tests have been used in this paper for the construction of fuzzy sampling plans when the quality characteristic of interest is described by a fuzzy normal distribution.

The scope of current research in FRS is much broader than that of FSQC. The most important, from a point of view of a specialist in fuzzy sets, results have been obtained in possibilistic analysis of reliability. This approach was introduced by Cai et al. [26]. In fuzzy generalizations of classical theory of probability it is usually assumed that the behavior of a system is probabilistic, states of a system are binary, but the information about their characteristics is imprecise, and described by fuzzy sets. In a possibilistic approach the description of a system is more general: internal states of a system may be imprecisely defined. This assumption allows to describe complex multistate systems. Moreover, this approach is very useful for the description of software reliability. More information about this approach can be found in the paper by Cai [27]. Fuzzy description of complex reliability systems allows to solve complex problems of the design of systems. An example of such research is given in the paper by Zhao and Liu [28] on the optimization of redundancy of a different type using various algorithms of soft computing.

Structure reliability, or stress-strength models, is another important field of reliability theory. The results from this field are especially useful in safety and risk analysis. Interesting fuzzy generalizations of useful reliability models can be found in papers by Savchuk [29], and Jiang and Chen [30].

In the statistical analysis of imprecise reliability data the most interesting results have been obtained in the area of Bayesian statistics. The authors have focused their research on solving specific problems. Papers by Hryniewicz [31] and Wu [32] present some examples of the research activity of that type.

Even first look at the results presented in papers devoted to the problem of statistical analysis of fuzzy data gives impression that even in the simplest case of the estimation of the mean life time for the exponential distribution the analysis of fuzzy data may be too complicated for practitioners. In case of other life time distributions which are widely used in practice, such as the Weibull distribution or the gamma distribution, or in the case of slightly more complicated reliability characteristics like the reliability function *R(t)*, the calculation of the membership function becomes very difficult. Further complications will be encountered if we have to evaluate the reliability of a system using fuzzy data obtained for its components. In such cases practitioners need simple approximate solutions. An example of such approximation can be found in the paper by Hryniewicz [36] who has considered the problem of the estimation of reliability of a coherent system $R_s(t)$ consisted of independent components having exponentially distributed life times when available life times for components are imprecisely reported. He assumes that both observed life times and censoring times of individual components are described by trapezoidal fuzzy numbers. In such a case it is possible only to find the membership function for the probability of failure

$$R(t) = 1 - e^{-(t/\theta)}, t > 0, \tag{4}$$

but the obtained formulae are too complicated for the further usage in the calculation of the reliability of complex systems. Therefore, Hryniewicz [36] proposes to approximate fuzzy total time on test by shadowed sets introduced by Pedrycz [37] who proposes to approximate a fuzzy number by a set defined by four parameters: a_1, a_2, a_3, a_4. The interpretation of the shadowed set is the following: for values of the fuzzy number that are smaller than a_1 and greater than a_4 the value of the membership function is reduced to zero, in the interval (a_2,a_3) this value is elevated to 1, and in the remaining intervals, i.e. (a_1,a_2) and (a_3,a_4) the value of the membership function is not defined. It is easy to see that all arithmetic operations on so defined shadowed sets are simple operations on intervals, and their result is also a shadowed set. Thus, calculations of imprecise reliability of a system described by a given structure function is quite straightforward.

Finally, we have to mention the result that could be applied both in SQC (inspection interval for control charts) and reliability (inspection intervals for monitoring processes). Hryniewicz [33] has shown why in the case of imprecise input information optimal inspection intervals are usually determined using additional preference measures than strict optimization techniques.

3. Challenges for the future

The short overview of the applications of fuzzy sets in the areas of SQC and RS shows that there is a solid ground for the implementation of fuzzy sets methodology in practice, as it is the case for the theory of probability and mathematical statistics. There are, however, some serious problems of theoretical and practical nature that have to be overcome if we want to see real applications. In this section of the paper we present our personal and subjective view on challenges that have to be faced by specialists working in the area of fuzzy sets and their applications.

Problems of Statistical Quality Control, Reliability and Safety, and other related areas, like Risk Analysis, have both random (probabilistic) and imprecise (fuzzy, possibilistic) nature. Therefore, serious efforts have to be undertaken in order to clarify mutual relationships between these methodologies. Paper by Dubois and Prade [34] presents an interesting overview of this problem, and the recent results of de Cooman [35] should be regarded as an important step on the way to solve it. Unfortunately, there is still a lot to do if we want to have a general theory covering both randomness and fuzziness.

Another challenge for the fuzzy sets community is connected with operational rules that have to be used by practitioners. Fuzzification of existing results in SQC and RS usually leads to prohibitively complex computations. Therefore, there is a need for commonly agreed simplifications and approximations such as those proposed by Hryniewicz [36] mentioned in the previous section. Such simplifications and approximations should be proposed having in mind a certain ultimate view: to provide practitioners with some *standards* for dealing with imprecise concepts (like, e.g., fuzzy reliability states) and data (like, e.g., expert opinions).

To the end of this paper, we would like to cite a statement from the paper by Cai [27]: " .., the area of fuzzy methodology in system failure engineering is still staying in a speculative research period and is premature. From a speculative research period to an engineering practice period, from premature to mature, a lot of work has to be done". After nearly ten years, this statement still remains, unfortunately, true.

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