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ABSTRACT: In the paper we consider the problem of the evaluation of systems reliability using statistical data coming from the reliability tests of its elements when the life times of the elements are described by the exponential distribution. We assume that these lifetime data may be reported imprecisely, and that this lack of precision may be described by fuzzy sets. As the direct application of the fuzzy sets methodology leads, in the considered case, to very complicated and time consuming calculations, we propose simple approximations of fuzzy numbers by the shadowed sets introduced by Pedrycz (1998). The proposed methodology may be simply extended to the case of general lifetime probability distributions.

1 INTRODUCTION

Statistical methods for the analysis of lifetime data have been intensively developed for the last fifty years. Numerous papers and textbooks dealing with different statistical aspects of the analysis of reliability data have been published. In the books written by Bain & Englehardt (1991), Meeker & Escobar (1998), Lawless (2003), and other authors one can find practically all methods that could be used for the analysis of such data. However, practically all well known statistical procedures have been developed for the analysis of precisely measured lifetime data. Only in few books and papers (e.g. in Lawless (2003)) one can find information about the statistical analysis of interval data with precisely defined time intervals covering unknown lifetimes.

Precisely reported lifetimes are common when the data come from specially designed life tests. In such a case a failure should be precisely defined, and all tested items should be continuously monitored. However, in real situation these test requirements might not be fulfilled. In the extreme case, the reliability data come from users whose reports are expressed in a vague way. The vagueness of reliability data coming from the users has many different sources. In Hryniewicz (1995) these sources have been divided into three groups:

- vagueness caused by subjective and imprecise perception of failures by a user,
- vagueness caused by imprecise records of reliability data,
- vagueness caused by imprecise records of the rate of usage.

First source of vagueness is typical for so called non-catastrophic failures. The tested item may be considered as failed, or – strictly speaking - as nonconforming, when at least one value of its parameters falls beyond specification limits. In practice, however, a user does not have a possibility to measure all parameters, and is not able to define precisely the moment of a failure. For example, if there exists a requirement for an admissible level of noise it usually may be verified by a user only subjectively. The user can usually indicate only a moment when he noticed that the level of noise had increased, and the moment when he (or she) considered it as obviously excessive. Thus, it might be assumed that the first moment describes the time when the tested item (say, a car) may be considered as failed, and the second moment indicates the time of a sure failure. As the result, we obtain imprecise information about the real lifetime. Moreover, this type of vagueness causes situations in which even at the end of a test (i.e. at a censoring time) a user is not sure whether the tested item has failed or not. In such a case, we have not only imprecise values of lifetimes, but we have imprecise information about the number of observed failures as well. This type of imprecise reliability data was considered in Hryniewicz (1995) and in Grzegorzewski & Hryniewicz (2002).

Second source of vagueness is typical to retrospective data. Users do not record precisely the moments of failures, especially when they are not sure if they observed a real failure (see above). So when they are asked about failures which were observed some time ago, they sometimes provide imprecise information. A lifetime of an individual is the actual length of life of that individual measured from some particular starting point. However, it may happen that the user cannot specify this starting point precisely but only in a vague way. In such situation the lifetime of the item under study is also vague.

Third source of vagueness is related to the fact that users, who report their data in days (weeks, months), use the tested items with different intensity. Two reports about items that failed after 20 weeks of work may have completely different meaning for the measurement of *MTTF* expressed in hours of continuous work, when the intensity of usage significantly differs in both these cases. In practice, the users are asked about the intensity of usage (for example, in hours per day), and their responses are, from obvious reasons, very often imprecise.

The lack of precision of reliability field data comes from all these sources and in many cases cannot be even identified. Precise probability models can be seldom applied only for clearly identified sources of vagueness, and they are very often impractical, because of many parameters which are either unknown or difficult to estimate. Therefore, we have to admit, that we often deal with really vague data expressed by imprecise words, and it is the only source of information which can be used for the verification of hypotheses about the mean lifetime of tested items.

In the second section of the paper we introduce the concept of the fuzzy modeling of imprecise lifetime data. The obtained results, however, may be too difficult to implement by practitioners. Therefore, there is a need to propose a much simpler approximate formulae. In the third section of the paper we present such results using the concept of shadowed sets introduced by Pedrycz (1998).

2 FUZZY MODELING OF IMPRECISE LIFE-DATA

One of the most frequently used reliability characteristics is the mean time to failure *MTTF*. It can be easily estimated from sample (W_1, W_2, ..., W_n) of the times to failure (life-times) when all the lifetimes are known. However, in the majority of practical cases reliability tests are terminated before the failure of all items. Thus, the lifetime data are usually censored using fixed censoring times $Z_i>0$, $i = 1,\ldots,n$, where n is the sample size. In such a case we observe lifetimes W_i only if $W_i \leq Z_i$. Thus, the lifetime data consist of pairs (T_1,Y_1), ..., (T_n, Y_n), where

$$T_i = \min(W_i, Z_i), \tag{1}$$

$$Y_i = \begin{cases} 1 & \text{if} \quad T_i = W_i \\ 0 & \text{if} \quad T_i = Z_i \end{cases} \tag{2}$$

Further analysis of censored lifetime data depends upon the assumed probability distribution. The simplest, and frequently used in many areas of lifetime analysis, is the exponential model. In this model the lifetime W is described by the probability density function

$$f(w) = \begin{cases} \dfrac{1}{\theta}\exp(-w/\theta) & \text{if} \quad w > 0 \\ 0 & \text{if} \quad w \leq 0 \end{cases} \tag{3}$$

where $\theta>0$ is the mean lifetime.

The exponential model is rather restrictive (constant hazard rate), but due to its simplicity is frequently used by practitioners. Thus in this paper we assume that the exponential distribution is used for the mathematical description of lifetimes.

Let

$$T = \sum_{i=1}^{n} T_i = \sum_{i \in O} W_i + \sum_{i \in C} Z_i \tag{4}$$

be the total survival time (total time on test), where O and C denote the sets of items whose exact lifetimes are observed and censored, respectively. Moreover, let

$$r = \sum_{i=1}^{n} Y_i \tag{5}$$

be the number of observed failures. In the considered exponential model (r,T) is a minimally sufficient statistics for θ, and the maximum likelihood estimator of the mean life time is given by

$$\hat{\theta} = \frac{T}{r}. \tag{6}$$

Now suppose that the lifetimes (times to failure) and censoring times are not necessarily precisely reported. In such a case it is possible to model this kind of vagueness by fuzzy numbers. This approach has been recently used by several authors such like Viertl & Gurker (1995), Grzegorzewski & Hryniewicz (1999, 2002), and Hryniewicz (1995). Grzegorzewski & Hryniewicz (1999) proposed the generalization of the exponential model using the concept of fuzzy numbers and fuzzy random variable.

The fuzzy subset A of the real line $\boldsymbol{R}$, with the membership function $\mu_A : \boldsymbol{R} \to [0,1]$ is a fuzzy number iff

a) A is normal, i.e. there exists an element x_0 such that $\mu(x_0) = 1$;

b) A is fuzzy convex, i.e. $\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \mu_A(x_1) \wedge \mu_A(x_2)$, $\forall x_1, x_2 \in \boldsymbol{R}, \forall \lambda \in [0,1]$;

c) μ_A is upper semicontinous;

d) supp A is bounded.

From the definition given above one can easily find that for any fuzzy number A there exist four real numbers a_1, a_2, a_{13}, a_4 and two functions: nondecreasing function $\eta_A : \mathbf{R} \to [0,1]$, and nonincreasing function $\xi_A : \mathbf{R} \to [0,1]$, such that the membership function μ_A is given by

$$\mu_A(x) = \begin{cases} 0 & \text{if} \quad x < a_1 \\ \eta_A(x) & \text{if} \quad a_1 \leq x < a_2 \\ 1 & \text{if} \quad a_2 \leq x < a_3 \\ \xi_A(x) & \text{if} \quad a_3 \leq x < a_4 \\ 0 & \text{if} \quad a_4 < x \end{cases} \tag{7}$$

Functions η_A and ξ_A are called the left side and the right side of a fuzzy number A, respectively.

Fuzzy numbers may be also described by a set of α-cuts. The α-cut of a fuzzy number A is a nonfuzzy set defined as

$$A_\alpha = \{x \in R : \mu_A(x) \geq \alpha\}. \tag{8}$$

A family $\{A_\alpha : \alpha \in (0,1]\}$ is a set representation of the fuzzy number A. From the so called resolution identity we get

$$\mu_A(x) = \sup\{\alpha I_{A_\alpha}(x) : \alpha \in (0,1]\}, \tag{9}$$

where $I_{A_\alpha}(x)$ denotes the characteristic function of A_α. Every α-cut of a fuzzy number is a closed interval. Hence we have $A_\alpha = \lfloor A_\alpha^L, A_\alpha^U \rfloor$, where

$$\begin{aligned} A_\alpha^L &= \inf\{x \in R : \mu_A(x) \geq \alpha\}, \\ A_\alpha^U &= \sup\{x \in R : \mu_A(x) \geq \alpha\}. \end{aligned} \tag{10}$$

Membership functions of fuzzy numbers that are defined as functions of other fuzzy numbers may be calculated using the following extension principle introduced by Zadeh (1975), and described in Dubois & Prade (1980) as follows:

Let X be a Cartesian product of universe $X = X_1 \times X_2 \times \cdots \times X_r$, and $A_1, \ldots, A_r$ be r fuzzy sets in $X_1, \ldots, X_r$, respectively. Let f be a mapping from $X = X_1 \times X_2 \times \cdots \times X_r$ to a universe Y such that $y = f(x_1, \ldots, x_r)$. The extension principle allows us to induce from r fuzzy sets A_i a fuzzy set B on Y through f such that

$$\mu_B(y) = \sup_{x_1,\ldots,x_r : y = f(x_1,\ldots,x_r)} \min\lfloor \mu_{A_1}(x_1), \ldots, \mu_{A_r}(x_r) \rfloor \tag{11}$$

$$\mu_B(y) = 0 \text{ if } f^{-1}(y) = \varnothing \tag{12}$$

Fuzzy numbers may be treated like a generalization of real numbers. Similarly, real-valued random variables may be generalized by fuzzy random variables. There exist many definitions of a fuzzy random variable. The following definition is similar to that given in Kruse & Meyer (1987).

Suppose that a random experiment is described by a probability space (Ω, A, P), where Ω is the set of all possible outcomes of the experiment, A is a σ-algebra of subsets of Ω (the set of all possible events) and P is a probability measure. A mapping $X : \Omega \rightarrow FN$, where FN is a space of all fuzzy numbers, is called a fuzzy random variable if it satisfies the following properties:

a) $$\{X_\alpha(\omega) : \alpha \in [0,1]\}$$

is a set representation of $X(\omega)$ for all $\in \Omega$,

b) for each $\alpha \in [0,1]$ both

$$X_\alpha^L = X_\alpha^L(\omega) = \inf X_\alpha(\omega)$$

and

$$X_{\alpha}^{U} = X_{\alpha}^{U}(\omega) = \sup X_{\alpha}(\omega)$$

are usual real-valued random variables in (Ω, A, P).

Thus, a fuzzy random variable X is considered as a perception of an unknown (unobservable) usual random variable V, called the original of X. If only vague data are available we can calculate the grade of acceptability that a fixed random variable V is the original of the fuzzy random variable X (see Kruse & Meyer (1987)).

Fuzzy random variables are described by probability distributions that a similar to usual probability distributions except for their parameters which are described by fuzzy numbers. Fuzzy parameters of a fuzzy random variable X may be considered as fuzzy perceptions of unknown parameters of its original V. Thus, in the case of imprecise fuzzy lifetime data, important reliability characteristics like the mean time to failure are also fuzzy.

Let us consider the case of imprecise lifetime data. In the exponential model time to failures and censoring times equally contribute, see (4), to the total time on test T. Thus, each of these times may be regarded as one lifetime datum, denoted by T_i, i=1,…,n. Suppose now that times T_i may be described by fuzzy numbers. In order to simplify calculations we assume that for each i=1,…,n they are described by the trapezoidal fuzzy numbers defined by the following membership function:

$$\mu_{T_i}(t) = \begin{cases} 0 & \text{if} \quad t < t_{1,i} \\ (t - t_{1,i})/(t_{2,i} - t_{1,i}) & \text{if} \quad t_{1,i} \le t < t_{2,i} \\ 1 & \text{if} \quad t_{2,i} \le t < t_{3,i} \\ (t_{4,i} - t)/(t_{4,i} - t_{3,i}) & \text{if} \quad t_{3,i} \le t < t_{4,i} \\ 0 & \text{if} \quad t_{4,i} \le t \end{cases} \tag{13}$$

It is easy to note that precise lifetime data can be considered as a special case of (13) when the following equality holds: $t_{1,i} = t_{2,i} = t_{3,i} = t_{4,i} = t_i$. When the following equalities hold: $t_{1,i} = t_{2,i} = t_{l,i}$ and $t_{3,i} = t_{4,i} = t_{u,i}$, then the expression (13) represents well known interval data. On the other hand, if we have $t_{2,i} = t_{3,i} = t_{p,i}$ than (13) represents a typical imprecise information "t_i is about $t_{i,p}$".

Now, let us consider the fuzzy equivalent of the total time on test when individual fuzzy data are described by (13). From the extension principle one can easily find that the fuzzy total time on test $\widetilde{T}$ is defined by the following membership function:

$$\mu_T(t) = \begin{cases} 0 & \text{if} \quad t < T_1 \\ (t - T_1)/(T_2 - T_1) & \text{if} \quad T_1 \le t < T_2 \\ 1 & \text{if} \quad T_2 \le t < T_3 \\ (T_4 - t)/(T_4 - T_3) & \text{if} \quad T_3 \le t < T_4 \\ 0 & \text{if} \quad T_4 \le t \end{cases} \tag{14}$$

where

$$T_1 = \sum_{i=1}^{n} t_{1,i} \, , \tag{15}$$

$$T_2 = \sum_{i=1}^{n} t_{2,i} \, , \tag{16}$$

$$T_3 = \sum_{i=1}^{n} t_{3,i} \, , \tag{17}$$

and

$$T_4 = \sum_{i=1}^{n} t_{4,i} \, . \tag{18}$$

Now, let us suppose that the number of failures r is precisely defined. It means that all failures may occur at imprecisely reported times, but their existence is sure. [The case of imprecise (fuzzy) failure counting is considered in Grzegorzewski and Hryniewicz (2002)]. In such a case the estimated mean time to failure $\tilde{\theta}$ is described by the membership function similar to that given by (14) – (18). The only difference is in the scale of the membership function, as everything in (14) – (18) has to be divided by r.

The situation becomes more complicated if we are interested in the evaluation of other reliability characteristics, as e.g. probability of failure

$$P(t) = 1 - e^{-t/\theta}, t > 0 \, . \tag{19}$$

When the total time on test is imprecise, and described by the membership function (14) we may apply the extension principle to find the fuzzy version of the estimated probability of failure. By straightforward calculations we can find that in this case the membership function is given by the following expression:

$$\mu_P(p)=\begin{cases} 0 & \text{if} \quad p<p_1 \\ \dfrac{T_4+\dfrac{rt}{\ln(1-p)}}{T_4-T_3} & \text{if} \quad p_1\le p<p_2 \\ 1 & \text{if} \quad p_2\le p<p_3 \\ \dfrac{-T_1-\dfrac{rt}{\ln(1-p)}}{T_2-T_1} & \text{if} \quad p_3\le p<p_4 \\ 0 & \text{if} \quad p_4\le p \end{cases} \tag{20}$$

where

$$p_1=1-e^{-rt/T_4}, \tag{21}$$

$$p_2=1-e^{-rt/T_3}, \tag{22}$$

$$p_3=1-e^{-rt/T_2}, \tag{23}$$

and

$$p_4=1-e^{-rt/T_1}. \tag{24}$$

Situation is even more complicated if we want to evaluate the probability of failure of a system when probabilities of failures of its elements are estimated from imprecise fuzzy lifetime data. Consider a coherent system whose elements have lifetimes described by exponential probability distributions. In the most general case we can write

$$P_S(t)=h(P_1(t),P_2(t),\ldots,P_m(t)) \tag{25}$$

where probabilities $P_1(t),P_2(t),\ldots,P_m(t)$ have a form given by (19). It is a well known fact that for a coherent system the probability of failure $P_S(t)$ is an nondecreasing function of its arguments. We can use this property for the calculation of the membership function of $\widetilde{P}_S(t)$ when the membership functions of $\widetilde{P}_1(t),\widetilde{P}_2(t),\ldots,\widetilde{P}_m(t)$ have a form given by (20). To calculate this membership function let us consider the α-cut representation of $\mu_P(p)$. For a given value of $\alpha\in[0,1]$ we can define the following α-cuts:

$$\widetilde{P}_{\alpha,j}(t)=\left[P_{\alpha,j}^L,P_{\alpha,j}^U\right],\ j=1,\ldots,m \tag{26}$$

where

$$P_{\alpha,j}^{L}=1-\exp\left(-\frac{r_j t}{T_{4,j}-\alpha\left(T_{4,j}-T_{3,j}\right)}\right),\ j=1,\ldots,m \tag{27}$$

$$P_{\alpha,j}^{U}=1-\exp\left(-\frac{r_j t}{T_{1,j}-\alpha\left(T_{2,j}-T_{1,j}\right)}\right),\ j=1,\ldots,m \tag{28}$$

Now we can find the α-cut representation of the membership function of $\widetilde{P}_S(t)$ using the following α-cuts:

$$\widetilde{P}_{\alpha,S}(t)=\left[P_{\alpha,S}^{L},P_{\alpha,S}^{U}\right] \tag{29}$$

where

$$P_{\alpha,S}^{L}=h\left(P_{\alpha,1}^{L},P_{\alpha,2}^{L},\cdots,P_{\alpha,m}^{L}\right) \tag{30}$$

$$P_{\alpha,S}^{U}=h\left(P_{\alpha,1}^{U},P_{\alpha,2}^{U},\cdots,P_{\alpha,m}^{U}\right). \tag{31}$$

However, all these calculations may be prohibitively complicated for systems with a complicated structure function $h(P_1(t),P_2(t),\ldots,P_m(t))$. Therefore, there is a need to simplify necessary calculations. In the next section we propose a method for such simplification that is based on the concept of shadowed sets.

3 DESCRIPTION OF VAGUE LIFEDATA USING SHADOWED SETS

The theory of fuzzy sets provides a precise methodology for dealing with vagueness. However, in many practical cases these precision seems to be excessive. See, for example, the description of imprecise life data and its consequences. Very simple, and intuitively well understood, fuzzy description of imprecise total time on test (14) is transformed to a definitely much complicated description of the imprecise probability of failure (20), and further to an incomprehensible for practitioners fuzzy description of the probability of failure of a coherent system with a general reliability structure. Therefore, there is a practical need to simplify this description by using some approximations of fuzzy sets. One of such approximations has been proposed by Pedrycz (1998) who introduced the notion of shadowed sets.

Let us introduce the concept of a shadowed set in a rather informal way. Suppose that a vague lifetime is described by a fuzzy number. If the value of the membership function for a given value of lifetime is small we can say that such a lifetime is rather not plausible. On the other hand, if such a value is rather high (close to 1) we can regard that lifetime as very plausible. The concept of shadowed sets consists in elevating to 1 all the values of the membership functions that are high enough, and in reducing to 0 all the values of the membership functions that are small enough. The membership function for the region where it is not possible neither to elevate it to 1 nor to reduce it to 0 becomes undefined. Thus, a fuzzy number A can be approximated by a set of intervals: a central interval with the membership grade equal to one, two adjacent intervals with the undefined (shadowed) grade of membership, and the open side intervals, where the grade of membership is equal to 0.

Consider a fuzzy number A with the membership function given by (7). The elevation of its membership function to 1 should be done for any

$$\{x \in [a_2', a_2], a_2' = \eta_A^{-1}(\alpha_U)\}$$

and

$$\{x \in [a_3, a_3'], a_3' = \xi_A^{-1}(\alpha_U)\}$$

where $\alpha_U < 1$ is a given upper threshold. Similarly, the reduction of its membership function to 0 should be done for any

$$\{x \in [a_1, a_1'], a_1' = \eta^{-1}(\alpha_L)\}$$

and

$$\{x \in [a_4', a_4], a_4' = \xi^{-1}(\alpha_L)\},$$

where $\alpha_L < \alpha_U$ is a given lower threshold. Thus, each fuzzy number can be represented by a shadowed fuzzy number defined by the set of four real numbers $\{a_1', a_2', a_3', a_4'\}$ that have been defined above. Further transformations of shadowed fuzzy numbers can be done using interval arithmetic.

Pedrycz (1998) proposed a general methodology for finding such thresholds that $\alpha_L = \alpha$, $\alpha < 0{,}5$, and $\alpha_U = 1-\alpha$. The value of α can be find by taking

$$a_1' = \eta_A^{-1}(\alpha),$$

$$a_2' = \eta_A^{-1}(1-\alpha),$$

$$a_3' = \xi_A^{-1}(1-\alpha),$$

$$a_4' = \xi_A^{-1}(\alpha),$$

and solving the following equation

$$\int_{a_1}^{a_1'}\eta_A(x)dx+\int_{a_2'}^{a_2}(1-\eta_A(x))dx-\int_{a_1'}^{a_2'}\eta_A(x)dx+$$
$$\int_{a_4'}^{a_4}\xi_A(x)dx+\int_{a_3}^{a_3'}(1-\xi_A(x))dx-\int_{a_3'}^{a_4'}\xi_A(x)dx=0. \tag{32}$$

In the case of the fuzzy probability of failure defined by (20) solving (32) is extremely difficult as it involves numerical evaluation of special functions named integral logarithms. The situation may be simplified when we assume that the probabilities of failure are small. In such a case we can use the following approximation:

$$\mu_P(p)\approx\begin{cases}0 & \text{if} \quad p<p_1\\ \dfrac{T_4-\dfrac{rt}{p}}{T_4-T_3} & \text{if} \quad p_1\le p<p_2\\ 1 & \text{if} \quad p_2\le p<p_3\\ \dfrac{-T_1+\dfrac{rt}{p}}{T_2-T_1} & \text{if} \quad p_3\le p<p_4\\ 0 & \text{if} \quad p_4\le p\end{cases} \tag{33}$$

where p_1, p_2, p_3, and p_4 are given by (21) – (24), respectively. The solution of (32) when the fuzzy number is described by (33) is much easier, but it can be done only numerically. The amount of computation is comparable with that which is necessary when we use the concept of α-cuts, and thus there is a practical need to find a simpler approximate solution.

A very simple solution can be found, however, when we apply the methodology of the shadowed sets just on the level of the fuzzy total time on test. Pedrycz (1998) has shown that in the case of linear functions $\eta_A(x)$ and $\xi_A(x)$ the value of α that fulfills (32) is always equal to 0,414. Simple calculations lead to the representation of the fuzzy time on test $\widetilde{T}$ described by (14) by the set of four numbers $\{t_1,t_2,t_3,t_4\}$ defined by the following expressions:

$$t_1=T_1+0{,}414(T_2-T_1) \tag{34}$$

$$t_2=T_1+0{,}586(T_2-T_1) \tag{35}$$

$$t_3=T_4-0{,}586(T_4-T_3) \tag{36}$$

$$t_4=T_4-0{,}414(T_4-T_3) \tag{37}$$

Hence, we can represent the fuzzy probability of failure $\widetilde{P}(t)$ defined by (20) by its shadowed counterpart $\{p_1^s, p_2^s, p_3^s, p_4^s\}$, where

$$p_1^s = 1 - e^{-rt/t_4}, \tag{38}$$

$$p_2^s = 1 - e^{-rt/t_3}, \tag{39}$$

$$p_3^s = 1 - e^{-rt/t_2}, \tag{40}$$

and

$$p_4^s = 1 - e^{-rt/t_1}. \tag{41}$$

Now, the calculation of the fuzzy probability of system's failure $\widetilde{P}_S(t)$ becomes much simpler, and this characteristic is now given as a shadowed fuzzy number $\{P_{S,1}^s, P_{S,2}^s, P_{S,3}^s, P_{S,4}^s\}$, where

$$P_{S,1}^s(t) = h\left(p_{1,1}^s, p_{2,1}^s, \ldots, p_{m,1}^s\right), \tag{42}$$

$$P_{S,2}^s(t) = h\left(p_{1,2}^s, p_{2,2}^s, \ldots, p_{m,2}^s\right), \tag{43}$$

$$P_{S,3}^s(t) = h\left(p_{1,3}^s, p_{2,3}^s, \ldots, p_{m,3}^s\right), \tag{44}$$

$$P_{S,4}^s(t) = h\left(p_{1,4}^s, p_{2,4}^s, \ldots, p_{m,4}^s\right), \tag{45}$$

and $p_{j,1}^s, p_{j,2}^s, p_{j,3}^s, p_{j,4}^s$, $j = 1, \ldots, m$ are given by (38) – (41) for each element of the system.

4 CONCLUSIONS

In the paper we have proposed a new methodology for the evaluation of reliability for a coherent system when the reliability of all its elements is described by an exponential model. The novelty of our approach consists in the assumption that the observed time to failures of system's elements may be reported imprecisely. We have assumed that this imprecision can be described by fuzzy numbers. Unfortunately, the direct application of the fuzzy sets methodology to the considered problem requires significant computational effort. To alleviate these computational problems we propose simple approximations of fuzzy numbers by shadowed fuzzy sets introduced in Pedrycz (1998).

The proposed methodology may be used for a more general problem when times to failures are distributed according to other probability distributions. In such a case we have to approximate individual observed fuzzy times to failure by the respective shadowed fuzzy sets. Further calculations should be done using the rules of interval arithmetic and the respective formulae known from the mathematical statistics.

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