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L. Klukowski

Instytut Badań Systemowych
Polska Akademia Nauk

Systems Research Institute
Polish Academy of Sciences

POLSKA AKADEMIA NAUK

Instytut Badań Systemowych

ul. Newelska 6

01-447 Warszawa

tel.: (+48) (22) 3810100

fax: (+48) (22) 3810105

Kierownik Pracowni zgłaszający pracę:
Prof. dr hab. inż. Zbigniew Nahorski

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OPTIMIZATION OF PUBLIC DEBT MANAGEMENT IN CASE OF STOCHASTIC BUDGETARY CONSTRAINTS

Leszek Klukowski*

* Systems Research Institute Polish Academy of Sciences

6 Newelska Str., 01-447 Warsaw, e-mail:

Leszek.Klukowski@ibspan.waw.pl

The paper presents stochastic approach to strategic optimization of public debt management in Poland - aimed at minimization of two criterions: servicing costs of the debt and costs resulting from stochastic budgetary constraints. The main results comprise: formulation of the problem, determination of necessary components (parameters, forecasts, etc.) and the method of problem solution. The results show complexity of the problem and gains from its implementation (budgetary savings). The paper is based on a research made in Polish Ministry of Finance (see Klukowski, Kuba 2002b).

1. Introduction

Decision problems, which appear in optimization of public debt management, are typically of stochastic nature. The main stochastic components are: forecasts of interest rates and constraints of budgetary requirements. Risk resulting from interest rates is discussed broadly in the literature (see e.g. Cleassens S., Kreuser J., Seigel L, Wets R., J-B. 1995, *Danish Government Borrowing and Debt* 2000). Random character of budgetary requirements is of similar importance, because changes of their level together with non-linear form of criterion function and constraints can influence optimal solutions in unexpected way. The range of methods, which take into account stochastic form of the constraints is quite extensive. However, some empirical limitations, e.g. computation time, mathematical complexity, knowledge of necessary functions, parameters, etc., constraint feasible set in this area. The approach used in the paper combines mathematical simplicity and main features of actual problem. It exploits the idea of goal programming with stochastic constraints expressing budgetary requirements. The constraints indicate surplus or deficit, which form some costs. They are incorporated into criterion function together with servicing costs of the debt. The random constraints generate additional decision variables and increase the size of the problem - proportionally to sizes of sets of values of the random variables.

The aim of this paper is to present complete solution of stochastic problem based on empirical data, i.e.: formulation of the task, an algorithm for its solution and empirical results.

The paper consists of five sections. The main results – formulation of optimization problem, determination of its components (i.e. functions, parameters, forecasts) and empirical results (an example of optimal solution) are presented in Sections 2 – 4. Last section summarizes the results.

2. Formulation of optimisation task

The problem examined in the paper can be stated as follows:

To determine the optimal portfolio of treasury securities (bonds) issued,

- aimed at minimizing criterion function comprising: servicing costs of securities and costs of deficit/surplus resulting from stochastic constraints of budgetary requirements - in three years period,
- under constraints on: risk level and other features of the debt.

The optimization task for the problem can be formulated as an extension of the deterministic approach (see Klukowski, Kuba 2002a), i.e.

without costs of deficit and surplus, resulting from stochastic budgetary requirements. The deterministic task, formulated for the set of bonds issued in Poland (in 2001 year), can be written as follows (with budgetary constraints only):

$$\min_{x_{it}} \left\{ \sum_{t=1}^{3} \sum_{i=1}^{\kappa} x_{it}(M - d^{(it)}(x_{it}))\varphi^{(it)}(x_{it}) \right\}, \tag{1}$$

$$\sum_{i=1}^{\kappa} x_{i1}(M - d^{(i1)}(x_{i1})) = \mathrm{A}_1, \tag{2}$$

$$\sum_{i=1}^{\kappa} x_{i2}(M - d^{(i2)}(x_{i2})) = \mathrm{A}_2, \tag{3}$$

$$\sum_{i=1}^{\kappa} x_{i3}(M - d^{(i3)}(x_{i3})) - M\, x_{1,1} = \mathrm{A}_3, \tag{4}$$

$$x_{it}^{\min} \le x_{it} \le x_{it}^{\max} \quad (i=1, \ldots, \kappa;\ t=1, 2, 3), \tag{5}$$

where:

x_{it} (i=1, ...,κ; t=1, 2, 3) – sale of i-th bond in year t – decision variable,

κ - number of bonds issued;

$d^{(it)}(x_{it})$ – average discount of i-th bond corresponding to sale level x_{it};

$\varphi^{(it)}(x_{it})$ – compound rate of return (CRR) of i-th bond, corresponding to sale level x_{it} (8 – years investment horizon);

A_t – budgetary requirement (in capital constraint), in year t;

M – nominal value of one bond (1000 of Polish zlotys).

The set of bonds issued in 2001 year comprises three fixed rates bonds (two-years - x_{1t}, five-years - x_{2t}, ten-years - x_{3t}) and one ten-years variable rate bond x_{4t}. The constraint (4) includes the term $M\,x_{11}$, which reflects an amount of redemption of two-years bond, issued in the first year of the period; it increases budgetary requirements in the third year. The investment horizon (8 years) in compound rate of return (see Klukowski, Kuba 2001) has been determined as a median in redemption schedule; it is clear that the median exceeds optimization period (three years).

Stochastic level of budgetary requirements indicates replacement of the vector $\mathbf{A}' = [A_1, A_2, A_3]$ (symbol $\mathbf{A}'$ - means transposed vector) with the vector of random variables $\mathbf{\Lambda}' = [\Lambda_1, \Lambda_2, \Lambda_3]$. The distribution functions of the variables Λ_t (t=1, 2, 3) can be written in the form:

$$P(\Lambda_t = A_{tr}) = p_{tr} \qquad (r = 1, \ldots, s_t;\ s_t \geq 1), \qquad \sum_{r=1}^{s_t} p_{tr} = 1, \qquad (6)$$

where:

A_{tr} (t=1, 2, 3; r=1, …, s_t) – an element of the value set of the random variable Λ_t; at least one value s_t $(1 \leq t \leq 3)$ satisfies $s_t \geq 2$.

The random variables Λ_t, incorporated into the constraints (2) – (4), indicate possibility of discrepancy in capital constraint, i.e. realization of the random value A_{tr} can be not the same as a deterministic value A_t.

The case, when $\sum_{i=1}^{K} x_{it}(M - d^{(it)}(x_{it}))$ is lower than actual capital requirement means deficit, opposite case – surplus. These situations can generate some costs; deficit – necessity of extra borrowing under higher rates, surplus -- necessity of deposits with rates lower, than profitability of bonds issued. For simplicity, the costs of deficit and surplus are assumed constant (for any level and structure of bonds issued in year t). Moreover, it is assumed that:

$$\gamma_t,\ \eta_t \geq 0, \tag{7}$$

$$\gamma_t + \eta_t > 0, \tag{8}$$

where:

γ_t $(t = 1, 2, 3)$ - cost of deficit,

η_t $(t = 1, 2, 3)$ - cost of surplus.

The variables expressing deficit y_{tr} and surplus z_{tr}, included into a set of decision variables, are defined as follows $(t = 1, 2, 3; r = 1, ..., s_t)$:

$$y_{tr} = \max\{A_{tr} - \sum_{i=1}^{K} x_{it}(M - d^{(it)}(x_{it})),\ 0\}, \tag{9}$$

$$z_{tr} = \max\{\sum_{i=1}^{K} x_{it}(M - d^{(it)}(x_{it})) - A_{tr},\ 0\}. \tag{10}$$

Cost resulting from the deficit y_{tr} is equal to $\lambda_t y_{tr}$, while the cost resulting from the surplus – equal to $\eta_t z_{tr}$. Each of the values y_{tr} or z_{tr} appears with the probability p_{tr} and therefore expected value of the cost of incorrect capital level equals $\sum_{t=1}^{3}\sum_{r=1}^{s_t} p_{tr}(\gamma_t y_{tr} + \eta_t z_{tr})$. This expression is added to the criterion function (1) as the second criterion. It is clear that the term expressing costs of deficit and surplus have to be compatible with the term expressing servicing costs of the debt. Therefore the costs of deficit and surplus have to be precisely determined - also with possibility of different values for individual levels of budgetary requirements.

The random level of budgetary requirements implies modifications of feasible set of the task (1) – (5); the differences: $y_{tr} - z_{tr}$ are added to left hand sides of the inequalities (2) – (4). It is also rational to include the costs resulting from the deficit and surplus into constraints for servicing costs of the debt. Taking into account the modifications, the task (1) – (5) assumes the form:

$$\sum_{t=1}^{3}\sum_{i=1}^{K} x_{it}(M\text{-}d^{(it)}(x_{it}))\varphi^{(it)}(x_{it}) + \sum_{t=1}^{3}\sum_{r=1}^{s} p_r(\gamma_t y_{tr} + \eta_t z_{tr}) \rightarrow \min, \qquad (11)$$

$$\sum_{i=1}^{K} x_{i1}(M - d^{(i1)}(x_{i1})) + y_{1r} - z_{1r} = A_{1r} \qquad (r=1, ..., s_1), \qquad (12)$$

$$\sum_{i=1}^{\kappa} x_{i2}(M-d^{(i2)}(x_{i2}))+y_{2r}-z_{2r}=A_{2r} \qquad (r=1, ..., s_2), \qquad (13)$$

$$\sum_{i=1}^{\kappa} x_{i3}(M-d^{(i3)}(x_{i3}))+y_{3r}-z_{3r}-M\cdot x_{1,1}=A_{3r} \quad (r=1, ..., s_3), \qquad (14)$$

$$x_{it}^{\min} \le x_{it} \le x_{it}^{\max} \qquad (i=1, ..., \kappa, t=1, 2, 3), \qquad (15)$$

(y_{tr}, z_{tr} - defined in (9), (10)).

The stochastic task generates additionally $2\prod_{t=1}^{3} s_t$ variables and constraints. Thus, the complexity of stochastic task increases in comparison to the deterministic one.

3. Determination of components of stochastic task

The parameters and functions necessary to formulate numerical form of the problem (11) – (15) comprise:

a) the probability functions of the random variables Λ_{t}.;
b) the rates γ_t, η_t;
c) the functions $d^{(it)}(x_{it})$ and $\varphi^{(it)}(x_{it})$;
d) feasible sets (intervals) for decision variables x_{it}.

The functions $d^{(it)}(x_{it})$, $\varphi^{(it)}(x_{it})$ and the feasible intervals appear also in deterministic form of the problem.

3.1. Parameters of stochastic constraints

The parameters of stochastic constraints together with the cost of deficit and surplus are of crucial importance for empirical results. They are typically determined on the basis of experts opinions or with the use of statistical methods (probability functions, forecasts). The parameters, costs and functions have been determined in the following way. The rates of deficit - on the basis of compound rate of return of bonds from previous years, the rates of the surplus – on the basis of credit and deposit spread. The values of these parameters are presented in Table 1.

The probability functions of budgetary requirements have been determined on the basis of budget realizations from previous years. The number of possible levels of the requirements has been assumed three in each year - minimal, medium and maximal - with the same probability of each level in consecutive years (see Table 2). Such number allows avoiding large size of optimization problem (number of variables and constraints). The number of levels of the requirements can be increased, if necessary.

Table 1. Rates of shortage and surplus

	2002	2003	2004
Rate of shortage	0,1011	0,1004	0,0952
Rate of surplus	0,0101	0,0100	0,0095

Table 2. Variants of budgetary requirements in years 2002 – 2004 and their probability functions.

Year	Variant I (r=1)	Variant II (r=2)	Variant III (r=3)
2002	61 719 000 000	63 719 000 000	59 719 000 000
2003	60 596 000 000	62 696 000 000	58 496 000 000
2004	56 554 000 000	58 854 000 000	54 254 000 000
Probab. function	0,5	0,3	0,2

3.2. Forecasting of CRR functions

The compound rate of return of treasury bonds (symbol $\varphi^{(it)}(x_{it})$ (i=1, …, 4) in the formula (11)) assume nonlinear form, with parameters determined by auctions' results (see Klukowski 2003). Prediction of the functions is not easy problem; the method used in the paper rests on two basic assumptions:

- there exists a typical shape (pattern) of the function of each type of bond;
- the form of any forecasted function $\varphi^{(it)}(x_{it})$ (i=1, ..., 4; t=1, 2, 3) can be expressed as the product of the pattern and the forecast of interest rate in the year t=1, 2, 3.

Thus, the forecast of each function has been obtained in the following way: • to predict interest rates for the years t=1, 2, 3; • to determine the pattern of compound rate of return of each bond, • to determine the product of rate and product of each bond, with adjustment to expected demand level (for details see Klukowski, Kuba 2002b).

The patterns of compound rate of return have been determined on the basis of data from previous years, with the use of two methods of classification: the first one – based on statistical pairwise algorithm (see Klukowski 1990) and the second - based on Kohonen neuronal network (SPSS Neuronal Connecting® 2.2 has been used). Empirical results of both approaches are similar.

It is clear that components of the stochastic task, based on estimates, forecasts and experts' opinions, include imprecise variables. Such variables require careful analytical research, because can influence significantly optimal solution. However, application of such data does not weaken practicability of the optimization approach. The optimal solution provides a broad set of information for decision maker, especially resulting from properties of criterion function and constraints. The results of optimization can be applied in other decision models, e.g. based on game theory (see Klukowski 2003).

It should be stressed that optimal solution of stochastic task is not comparable with deterministic one, because of difference in assumptions; deterministic solution does not take into account costs of surplus and deficit and is solved for one level of budgetary requirements.

4. Empirical results

The example presented in the section is based on actual functions and empirical data.

Each component $x_{it}(M\text{-}d^{(it)}(x_{it}))\varphi^{(it)}(x_{it})$ (i=1, ..., 4) of the criterion function is non-linear and non-convex (for 8 years investment horizon), but convergent to convex piecewise linear function under weak conditions (Klukowski 2003, Chapter 4). Empirical experience shows that polynomial approximation obtained with the use of least squares method provides convex form of the approximated components and appropriate precision. The functions, expressing capital of bonds, i.e. $x_{it}(M\text{-}d^{(it)}(x_{it}))$, are piecewise linear concave functions. They can be also approximated in the same way. An alternative approach is to approximate the components of criterion function with the use of piecewise linear function, without approximation of capital constraints. However, it increases considerably the number of decision variables of the task, which includes typically non-linear constraints, and hinders solving the problem. Therefore, polynomial approximation, indicating moderate number of variables, has been applied. The parameters of approximated criterion function (polynomial form) are presented in Table 3.

Table 3. Parameters of polynomial approximations of criterion function for Polish treasury bonds (2002 year).

Power of polynomial	Type of bond			
	2-years (x_{1i})	5-years (x_{2i})	10-years (fixed rate) (x_{3i})	10-years (variable rate) (x_{4i})
0 (constant)	1474,20	-6023,09	692,43	-273959,70
1	79,20	91,52	77,88	100,46
2	1,17	2,01E-08	2,69E-08	×
3	-2,06E-15	-2,89E-15	-5,81E-15	×
4	2,31E-22	2,69E-22	1,57E-21	×

5	-1,53E-29	-1,51E-29	-3,43E-28	×
6	6,32E-37	5,31E-37	5,19E-35	×
7	-1,67E-44	-1,16E-44	-4,93E-42	×
8	2,85E-52	2,54E-52	2,77E-49	×
9	-3,01E-60	-1,13E-60	-8,39E-64	×
10	1,79E-68	3,53E-69	1,05E-64	×
11	-4,61E-77	×	×	×

The approximated form of the task can be written as follows:

• the criterion function:

$$\sum_{t=1}^{3}\sum_{i=1}^{4}\sum_{k=0}^{m_{it}} a_{itk}\, x_{it}^{k} + \sum_{t=1}^{3}\sum_{r=1}^{s} p_r(\gamma_t\, y_{tr} + \eta_t\, z_{tr}) \rightarrow \min,$$

where:

x_{it}^{k} - variable x_{it} in *k*-th power;

a_{itk} - polynomial coefficient of the variable x_{it} in *k*-th power;

m_{it} - the degree of polynomial for the variable x_{it};

• the constraints:

- intervals for decision variables:

$x_{it}^{\min} \leq x_{it} \leq x_{it}^{\max}$ (i =1, ..., 4; t=1, ..., 3),

values $x_{it}^{\min}$ and $x_{it}^{\max}$ (in thousands) in the table below;

	x_{1t}	x_{2t}	x_{3t}	x_{4t}
$x_{it}^{\min}$ $(t = 1,2,3)$	20000	30000	5000	1100
$x_{it}^{\max}$ $(t = 1,2,3)$	35000	50000	12000	2000

- budgetary requirements for individual values of surplus and shortage (i.e. y_{tr} and z_{tr}):

$$\sum_{i=1}^{4}\sum_{k=0}^{n_{i1}} b_{i,1,k}x_{i,1}^{k} + y_{1,1} - z_{1,1} = 61\ 719\ 000\ 000,$$

$$\sum_{i=1}^{4}\sum_{k=0}^{n_{i1}} b_{i,1,k}x_{i,1}^{k} + y_{1,2} - z_{1,2} = 63\ 719\ 000\ 000,$$

$$\sum_{i=1}^{4}\sum_{k=0}^{n_{i1}} b_{i,1,k}x_{i,1}^{k} + y_{1,3} - z_{1,3} = 59\ 719\ 000\ 000,$$

$$\sum_{i=1}^{4}\sum_{k=0}^{n_{i2}} b_{i,2,k}x_{i,2}^{k} + y_{2,1} - z_{2,1} = 60\ 596\ 000\ 000,$$

$$\sum_{i=1}^{4}\sum_{k=0}^{n_{i2}} b_{i,2,k}x_{i,2}^{k} + y_{2,2} - z_{2,2} = 62\ 696\ 000\ 000,$$

$$\sum_{i=1}^{4}\sum_{k=0}^{n_{i2}} b_{i,2,k}x_{i,2}^{k} + y_{2,3} - z_{2,3} = 58\ 496\ 000\ 000,$$

$$\sum_{i=1}^{4}\sum_{k=0}^{n_{i3}} b_{i,3,k}x_{i,3}^{k} + y_{3,1} - z_{3,1} - Mx_{1,1} = 69\ 054\ 000\ 000,$$

$$\sum_{i=1}^{4}\sum_{k=0}^{n_{i3}} b_{i,3,k}x_{i,3}^{k} + y_{3,2} - z_{3,2} - Mx_{1,1} = 71\ 354\ 000\ 000,$$

$$\sum_{i=1}^{4}\sum_{k=0}^{n_{i3}} b_{i,3,k}x_{i,3}^{k} + y_{3,3} - z_{3,3} - Mx_{1,1} = 66\ 754\ 000\ 000,$$

where:

b_{itk} – coefficients of polynomial (similar as a_{itk} in the criterion function),

n_{it} – a power of the polynomial for the variable x_{it} (from the range 9 – 11 for individual variables);

- servicing costs (in the years 2003 – 2006):

$$85\,x_{2,1} + 60\,x_{3,1} + 112{,}5\,x_{4,1} + 0{,}5(0{,}1011\,y_{1,1} + 0{,}0101\,z_{1,1}) + 0{,}3(0{,}1011\,y_{1,2} + 0{,}0101\,z_{1,2}) + 0{,}2(0{,}1011\,y_{1,3} + 0{,}0101\,z_{1,3}) \le 21\ 000\ 000\ 000,$$

$$1000\,x_{1,1} + \sum_{k=0}^{1} b_{1,1,k}\,x_{1,1}^{k} + 85\,x_{2,1} + 60\,x_{3,1} + 104{,}1\,x_{4,1} + 85\,x_{2,2} + 60\,x_{3,2} + 104{,}1\,x_{4,2} + 0{,}5(0{,}1004\,y_{2,1} + 0{,}01\,z_{2,1}) + 0{,}3(0{,}1004\,y_{2,2} + 0{,}01\,z_{2,2}) + 0{,}2(0{,}1004\,y_{2,3} + 0{,}01\,z_{2,3}) \le 27\ 000\ 000\ 000,$$

$$1000\,x_{1,2} - \sum_{k=0}^{1} b_{1,2,k}\,x_{1,2}^{k} + 85\,x_{2,1} + 60\,x_{3,1} + 98{,}3\,x_{4,1} + 85\,x_{4,2} + 60\,x_{3,2} + 98{,}3\,x_{4,2} + 85\,x_{2,3} + 60\,x_{3,3} + 98{,}3\,x_{4,3} + 0{,}5(0{,}0952\,y_{3,1} + 0{,}0095\,z_{3,1}) + 0{,}3(0{,}0952\,y_{3,2} + 0{,}0095\,z_{3,2}) + 0{,}2(0{,}0952\,y_{3,3} + 0{,}0095\,z_{3,3}) \le 31\ 000\ 000\ 000,$$

$$1000\,x_{1,3} - \sum_{k=0}^{1} b_{1,3,k}\,x_{1,3}^{k} + 85\,x_{2,1} + 60\,x_{3,1} + 91{,}6\,x_{4,1} + 85\,x_{2,2} + 60\,x_{3,2} + 91{,}6\,x_{4,2} + 85\,x_{2,3} + 60\,x_{3,3} + 91{,}6\,x_{4,3} \le 31\ 000\ 000\ 000,$$

- a share of fixed-rate bonds in total sale of bonds in each year:

$$0,75 \leq \sum_{i=1}^{3} x_{it} / \sum_{i=1}^{4} x_{it} \leq 0,985 \qquad (t=1, 2, 3),$$

- a share of variable-rate bonds in total sale of bonds in each year:

$$0,015 \leq x_{4t} / \sum_{i=1}^{4} x_{it} \leq 0,25 \qquad (t=1, 2, 3),$$

- average maturity of bonds issued in each year:

$$3,5 \leq (2\, x_{1,t} + 5\, x_{2,t} + 10(x_{3,t} + x_{4,t})) / \sum_{i=1}^{4} x_{it} \leq 5,4 \qquad (t=1, 2, 3),$$

- average duration of fixed rate-bonds issued in each year:

$$3,0 \leq (2\, x_{1,t} + 4,2\, x_{2,t} + 7,5\, x_{3,t}) / \sum_{i=1}^{3} x_{it} \leq 4,3 \qquad (t=1, 2, 3),$$

- constraint of the expression including semivariance and semicovariance matrix (see Klukowski 2003, chapt. 6):

$$[z_{1,t}, z_{2,t}, z_{3,t}, z_{4,t}]\mathbf{Q}[z_{1,t}, z_{2,t}, z_{3,t}, z_{4,t}]' \leq 0,005 \qquad (t=1, 2, 3).$$

Numerical solution of the stochastic task has been obtained with the use of *solver* procedure from Excel system. The value of the criterion function corresponding to the optimal solution equals: 18 673 631 500; the optimal values of variables are presented in Table 4 (sale of bonds) and Table 5 (shortage and surplus). Servicing costs of the debt assume the values (in the period from 2003 to 2006 respectively): 18 862 224 981;

22 116 427 533; 22 354 043 699; 22 247 884 741. The values of remaining constraints are presented in Table 6.

5. Summary and conclusions

The paper presents an application of multiple criteria optimization approach in the area of public debt management, under assumption about stochastic constraints of budgetary requirements.

The "quality" of debt management with the use of optimisation tools exceeds in meaningful way "traditional" approach. Especially, it provides budgetary savings, increases transparency of decision process, reduces employment costs and hastens decisions. Moreover, empirical experience shows that computation time (with the use of *solver* procedure from Excel worksheet) is acceptable for assumed size of the task (number of variables and constraints). It seems possible to solve more complex tasks - without simplifications made – e.g. aggregation of bonds in one year period. However, up to now, the optimisation approach has been not applied in Poland.

Table 4. Optimal solution of the stochastic task (sale of bonds).

Type of the bond	Absolute values in the year			Relative values (%) in the year		
	2002	2003	2004	2002	2003	2004
2-year bond (x_{1t})	20000	35000	35000	27,1	46,4	38,3
5-years bond (x_{2t})	45820	31328	43957	62,2	41,5	48,0
10-years (fixed) bond (x_{3t})	6770	8030	10520	9,2	10,6	11,5
10-years (variable) bond (x_{4t})	1105	1139	2000	1,5	1,5	2,2

Table 5. Values of shortage (y_{it}) and surplus in the optimal solution (z_{it})

Probability	2002		2003		2004	
	shortage	surplus	shortage	surplus	Shortage	surplus
0,5	0	0	0	2100	0	2300
0,3	2000	0	0	0	0	0
0,2	0	2000	0	4200	0	4600

Table 6. Values of remaining constraints in optimal solution

	Year 2002	Year 2003	Year 2004
Share of fixed-rate bonds	0,985	0,985	0,978
Share of variable-rate bonds	0,015	0,015	0,022

Average maturity	4,72	4,22	4,54
Duration	3,90	3,52	3,734
Risk (quadric of semivariance and semicovariance matrix)	0,0039	0,0047	0,0042

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