Model laws for granular media and powders with a special view to silo models (*)

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A CONTINUATION of earlier work on silo structures carried out at the Structural Research Laboratory is described. On the basis of very general basic equations, two model laws are given, one corresponding to the individual particles of the particulate material being similar in the same ratio as the remaining geometry, and the other corresponding to the use of the same particles in model and prototype. Account is taken of both time-dependent material properties in the particulate medium and the effect of the pore medium on the stress and strain field. The model laws contain the requirement to an increased, homogeneous gravitational field, and an account is given of the errors that arise if, instead, a gravitational field produced in a centrifuge is used. Finally, the question of the degree to which the model conditions can be satisfied for various groups materials is discussed. The results can be widely applied to other problems relevent to particulate media.

W pracy opisano wyniki stanowiące kontynuację wcześniejszych badań konstrukcji silosów, prowadzonych w Laboratorium Mechaniki Konstrukcji. Na podstawie ogólnej analizy przedstawionych równań zaproponowano dwa modele ośrodka. W pierwszym z nich poszczególne cząstki materiału rozdrobnionego są zmniejszone w tym samym stosunku co wymiary geometryczne silosa. W drugim natomiast zastosowano te same cząstki w modelu i prototypie. Uwzględniono zarówno zależne od czasu własności ośrodka rozdrobnionego, jak też wpływ porowatości na wielkości naprężeń i odkształceń. Rozpatrzono oddzielnie przypadek występowania niejednorodnych pól grawitacyjnych oraz pól pochodzących od sił odśrodkowych. Przedstawiono dyskusję zagadnienia, w jakim stopniu wprowadzona idealizacja odpowiada rzeczywistemu zachowaniu się różnych grup materiałów. Otrzymane wyniki mogą mieć zastosowanie w innych problemach dla ośrodków rozdrobnionych.

В работе описаны результаты составляющие продолжение более ранних исследований конструкций силосов, проводимых в Лаборатории Механики Конструкций. На основе общего анализа представленных уравнений предложены две модели среды. В первой из них отдельные частицы размельченного материала уменьшены в этом же самом отношении, что геометрические размеры силоса. Во второй модели применены эти же самые частицы в модели и в прототипе. Учтены так зависящие от времени свойства размельченной среды, как и эффект пористости и его влияние на величины напряжений и деформаций. Рассмотрен отдельно случай выступания неоднородных гравитационных полей и полей происходящих от центробежных сил. Представлено обсуждение вопроса, в какой степени введенная идеализация отвечает действительному поведению разных групп материалов. Полученные результаты могут иметь применение в других проблемах для размельченных сред.

Notation

Tensor notation, with i and j as indices, is used together with the following:

Symbol	Meaning
B	width of silo,
c	coefficient of resistance,
F	area,

F_α $F_\alpha = 0$; constitutive equations ($\alpha = 1, 2, 3, 4 \ldots$),

g gravitational field,

g_J gravitational field of the earth,

H height of silo,

k material parameter,

K_z scale ratio $\dfrac{z_{\text{prototype}}}{z_{\text{model}}}$,

l measure of particle size,

m porosity, pore volume/total volume,

n g/g_J,

p air pressure: $p = \dfrac{1}{3}(\sigma_{11}^L + \sigma_{22}^L + \sigma_{33}^L)$,

P force (air resistance),

q constant in Darcy's law,

q_c, q_θ mass forces,

Q reaction force,

R half-length of rotor,

R_t Reynold's number,

t time,

u angle,

v velocity of falling grain,

Δv difference between velocity of grains and mean velocity of surrounding pore medium in jet,

V volume,

x_t coordinates,

x arbitrary length,

α coefficient of compressibility,

δ_{tJ} Kronecker's delta,

ε strain,

μ viscosity,

ν dynamic viscosity,

ϱ density,

σ stress,

φ inclination of model in relation to horizontal.

Index

L pore medium,

s particulate medium,

1. Introduction

THE constitutive equations for particulate media are frequently poorly elucidated, as a result of which theoretical results are encumbered with considerable uncertainty. Despite the fact that tests—and particularly model tests—are thus an obvious tool for the analysis of structures in connection with these media, the procedure is rarely used, compared with the use made of model tests for the investigation of steel and concrete structures. The reason for this seems to be difficulties in satisfying the criteria for the model laws. These difficulties are due especially to the fact that the physical properties of

particulate materials are not so well known and that the dead load of particulate materials can seldom be neglected.

Consequently, model tests with particulate media have largely been restricted to tests of a qualitative nature, especially in cases in which the scale ratio has been large.

The studies reported here must be regarded as a continuation of the work that has been going on at the Structural Research Laboratory for several years. Interest has been particularly focussed on the pressure conditions in silo structures [1], [2], [3], [4] and [5]. The experimental parts of this work were all carried out in full scale, but the possibility of continuing the investigations in model was discussed in [5], and it was concluded that model tests in an artificial gravitational field offered the most promising possibility.

2. Model laws

In order to define the criteria, the model laws are based on well-known basic equations which, under given assumptions, describe the phenomena to be investigated.

Correct results depend on the theoretical description being correct for both prototype and model, on the model criteria being satisfied, and on the boundary conditions and initial conditions being in accordance with the model law. If these conditions are not satisfied, scale errors will occur. It can, to a certain extent, be said that the simpler the theory on which the model law is based, the fewer the model criteria that must be satified. On the other hand the theory may be based on assumptions that are not fulfilled, with scale errors as a result. As the magnitude of the scale errors is unknown, efforts must be made to make the theoretical basis as broad as possible. This means that special cases may occur in which satisfactory investigations can be carried out in accordance with a less rigorous model law (see discussion in [5]), possibly after it has been proved that the two model laws lead to the same results.

The investigation covers only stress and strain fields as a function of time. The temperature is thus assumed to be constant.

The pores in the particulate medium are assumed to be filled with a fluid medium which has a considerable influence on the stress and strain fields.

As it will be seen later, the question of the necessity of regarding the particulate medium as a continuum is related to the question of similitude in the individual particles as well, and in this respect there appear to be only two possibilities of practical and theoretical interest. One of these is complete similitude, with the same scale ratio for the particle geometry as for the silo geometry. A model law can well be constructed for this case while the conception of the medium as a particulate medium is retained. The other possibility consists in using the same particles in model and prototype. This proves possible provided the particulate medium can be considered as a continuum, which implies that the particle size is of no significance.

The description is otherwise based on the processes that, together, constitute the work sequence in a silo, viz. fall, impact, rest and discharge. The reason for including fall and impact is that these processes may affect the lodgement which is an important parameter in the stress-strain relationship during rest and discharge. Some equations are omitted

Table 1.

Process		Equations	Model requirements	Remarks
Fall and impact	(1)	$\varrho_s V \dfrac{d^2 x_i}{dt^2} = \varrho_s V g_i + P_i + Q_i$	$K_{\rho_s} K_x^3 K_x \dfrac{1}{K_t^2} = K_{\rho_s} K_x^3 K_g = K_P = K_Q$	Equations of movement for individual particles. P_i are surface forces from air. Q_i are forces between adjacent grains
	(2)	$\dfrac{d\varrho_L}{dt} + \varrho_L \dfrac{\partial v_i}{\partial x_i} = 0$		Equations of continuity
	(3)	$\dfrac{\partial \sigma_{ij}}{\partial x_j} + \varrho_L g_i = \varrho_L \dfrac{d^2 x_i}{dt^2}$	$K_\sigma \dfrac{1}{K_x} = K_{\rho_L} K_g = K_{\rho_L} K_x \dfrac{1}{K_t^2}$	Equations of movement
	(4)	$\sigma_{ij} = -p\delta_{ij} + \mu(v_{i,j} + v_{j,i}) + \alpha \delta_{ij} v_{k,k}$	$K_\sigma = K_p = K_\mu \dfrac{1}{K_t} = K_\alpha \dfrac{1}{K_t}$	Physical conditions
	(5)	$P_i = \int\limits_F \sigma_{ij} dF_j$	$K_P = K_\sigma K_x^2$	δ_{ij} is determined by the air flow
	(6)	$\dfrac{d\varrho_s}{dt} + \varrho_s \dfrac{\partial v_i}{\partial x_i} = 0$		
	(7)	$\dfrac{\partial \sigma_{ij}}{\partial x_j} + \varrho_s g_i = \varrho_s \dfrac{d^2 x_i}{dt^2}$	$K_\sigma \dfrac{1}{K_x} = K_{\rho_s} K_g = K_{\varrho_s} K_x \dfrac{1}{K_t^2}$	Stresses inside particles during fall (impact). k_β is material parameters.
	(8)	$F_\alpha(\sigma_{ij}, \varepsilon_{ij}, k_\beta) = 0 \ (\alpha, \beta = 1, 2 \ldots)$	$K_\sigma = K_\varepsilon = K_k = 1$	
	(9)	$Q_i = \int\limits_F \sigma_{ij} dF_j$	$K_Q = K_\sigma K_x^2$	σ_{ij} is determined by stresses inside the particles (zero for areas outside the contact zone).
Rest and discharge	(10)	$\dfrac{d\varrho_s}{dt} + \varrho_s \dfrac{\partial v_i}{\partial x_i} = 0$		Equations of continuity
	(11)	$\dfrac{d\sigma_{ij}}{\partial x_j} + \varrho_s g_i = \varrho_s \dfrac{dx_i}{dt^2}$	$K_\sigma \dfrac{1}{K_x} = K_{\rho_s} K_g = K_{\rho_s} K_x \dfrac{1}{K_t^2}$	Equation of movement
	(12)	$F_\alpha(k_\beta, \sigma_{ij}, \varepsilon_{ij}, t) = 0$	$1 = K_k = K_\varepsilon = K_t = K_\sigma$	Physical conditions, incl. conditions for mutual sliding
	(13)	$\dfrac{d\varrho_L}{dt} + \varrho_L \dfrac{\partial v_i}{\partial x_i} = 0$		
	(14)	$\dfrac{\partial \sigma_{ij}}{\partial x_j} + \varrho_L g_i = \varrho_L \dfrac{d^2 x_i}{dt^2}$	$K_\sigma \dfrac{1}{K_x} = K_{\rho_L} K_g = K_{\rho_L} K_x \dfrac{1}{K_t^2}$	Description of pore flow
	(15)	$\sigma_{ij} = -p\delta_{ij} + \mu(v_{i,j} + v_{j,i}) + \alpha \delta_{ij} v_{k,k}$	$K_\sigma = K_p = K_\mu \dfrac{1}{K_t} = K_\alpha \dfrac{1}{K_t}$	

Right-hand remarks (spanning):
- Fall and impact, eqs (2)–(5): Description of air flow around falling particles.
- Rest and discharge, eqs (10)–(15): Stress-strain description in individual particles.

[550]

[551]

because it can directly be seen that they do not contribute to the model laws. This applies to the compatibility conditions and — because the silo geometry is assumed to be similar — to the geometrical boundary conditions, together with equations that results in the fact that the stress scale has to be identical in both media, e.g. certain boundary conditions in stresses.

2.1. Complete similitude—basic equations

The basic equations are shown in Table 1.

The model conditions must be chosen in such a way that the model requirements (Tabl. 1) are satisfied. There is some degree of freedom in this choice, but certain conditions can lead to impracticable model laws. With a view to viability, the following model conditions are chosen:

(16) Complete geometrical similitude in the ratio K_x.

It will presumably be extremely difficult to ensure that two media consist of similar particles with identical physical conditions, but in investigations (see [6]) in which the same model and the same particles are placed in variable gravitational fields to elucidate scale effects, the prototypes correspond, according to the model law, to silos of different size with particles similar in the same ratio.

(17) The model is placed in an artificial gravitational field $K_g = \dfrac{1}{K_x}$.

(18) Same solid medium: $K_k = K_{\rho_s} = 1$.

If the fluid medium satisfies: $K_{\rho_L} = 1$, $K_\mu = K_\alpha = K_x$, all model requirements will be satisfied with the following transformation laws: $K_\sigma = 1$, $K_\varepsilon = 1$, $K_t = K_x$ (forces of inertia), $K_t = 1$ (time-dependence in constitutive equations) and $K_t = K_x$ (pore flow). However, to all intents and purposes it would be impossible to find such a fluid medium for the model, and this affects both the rate of fall and the pore flow. The simplest solution, from the point of view of testing technique, would be to use the same pore medium, viz.

(19) $$K_{\rho_L} = K_\mu = K_\alpha = 1.$$

If the forces of inertia in the pore flow can be assumed to have no effect and if α and μ in the requirement to rate of fall, Eq. (4), can be neglected, the other requirements in this case can be satisfied by the following transformation laws:

(20) $\qquad K_\sigma = 1$,

(21) $\qquad K_\varepsilon = 1$;

(22) $\qquad K_t = K_x \qquad$ (forces of inertia in the particles),

(23) $\qquad K_t = 1 \qquad$ (time-dependence in const. rel.),

(24) $\qquad K_t = 1 \qquad$ (pore flow).

The rate of fall can be evaluated by taking into account the fact that the effect of μ on the stress field is of minor importance after the commencement of turbulence. If, in addition, it is assumed that the compressibility of the pore medium is negligible, the rate of fall will be correctly modelled.

The model law for a silo test is thus constituted by the model conditions (16) and (19) and by the transformation laws (20) to (24). It will be seen that Eq. (22) is incompatible

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with Eqs. (23) and (24), which means that forces of inertia must not have any influence while the other two timedependent properties have an influence. This will be clarified later.

2.2. Continuum considerations—basic equations

When the particles are not similar in the same ratio as the remaining geometry, the model law cannot be based on the general equations given in 2.1 on account of the geometrical boundary conditions. To arrive at a suitable model law it is necessary to make a number of other assumptions:

Regarding the fall it is assumed that all particles in a cross-section of the jet have the same velocity and that the air within the cross-section of the jet is partially carried along with the particles, such that the difference between the velocity of the particles and that of the air is Δv. It is assumed that the particles do not touch each other during the fall. The effect of the air is described on the basis of the theory for flowing bodies, with a supplement for buoyancy, Eq. (26).

The relative velocity of the air, $\Delta v/v$, is imagined to depend primarily on the ratio between the pore volume and the total volume, described by the porosity m. Furthermore, it is assumed that Reynold's number, $R_1 = \Delta v l/v$, where l is a measure of the particle size, plays a role, especially at the end of the fall, when the particle density is at its minimum, Eq. (27).

This description (see Tabl. 2) cannot be expected to be as good as the corresponding description in Sect. 2.1, but it is not the particle velocity itself that is of interest in silo models, only its influence on the properties of the medium via lodgement, so that scale errors on the velocity do not necessarily mean a serious scale error on the properties of the medium.

Impact is treated in relation to a subsequent continuum-mechanical consideration of the resting medium which is assumed to be affected, via the impact, by an edge stress σ. An impulse consideration within the impact area of the jet is used, neglecting the fact that the air does not have quite the same velocity as the particles, Eq. (28).

After lodgement it is assumed that the medium can be regarded as a porous continuum. It is further assumed that when failure occurs in the medium, the fluid medium follows the movement of the surrounding particles corresponding to the occurrence of forces of inertia of the order of magnitude $\varrho(dv_i/dt)$.

The pore flow is described on the basis of Darcy's law, one assumption of which is that forces of inertia are negligible. The constant q in Darcy's law depends on the geometry of the particles and on the physical properties of the pore medium. In order to clarify these relationships a homogeneous flow is considered in accordance with the particle scale l. This flow can be described partly by Darcy's law and partly by the more general equation from Sect. 2.1, where the forces of inertia are neglected. This yields:

Darcy: $$v_i = -q\left(\frac{dp}{dl_i} - \varrho_L g_i\right),$$

General: $$\frac{dp}{dl_i} - \varrho_L g_i = \mu(v_{i,jj} + v_{j,ij}) + \alpha v_{k,ki},$$

Table 2.

Process		Equations	Model requirements	Remarks
Fall	(25)	$\varrho_s V_s \dfrac{dx_i}{dt^2} = \varrho_s V_s g_i + P_i$	$K_{\varrho_s} K_l^3 K_x \dfrac{1}{K_t^2} = K_{\varrho_s} K_l^3 K_g = K_P$	For the fall. P_i are surface forces from air
	(26)	$P_1 = -c(R_1)\dfrac{1}{2}\varrho_L(\Delta v)^2 F - V_s \varrho_L g_1$ $\text{and } (P_2, P_3) = (0, 0)$	$K_P = K_c K_\varrho K_{\Delta v}^2 K_l^2 = K_l^3 K_{\varrho_L} K_g$	The effect of the air (x_1—axis in direction of the fall) (individual particles) $R_1 = \dfrac{\Delta vl}{v}$ and F is a characteristic area of
	(27)	$\Delta v_{\text{fin}} = v_{\text{fin}} f(m, R_1)$	$K_{\Delta v} = K_v; \ K_m = K_{R_1} = 1$	the particles, e.g. $F = l^2$, where l is a measure of the particle size. v_{fin} is the final velocity.
Impact	(28)	$\sigma = \varrho v_{\text{fin}}^2$	$K_\sigma = K_\varrho K_v^2$	where $\varrho(x)$ is the specific weight of the jet when this is regarded as a continuum and v is the final velocity
Rest and discharge	(29)	$\varrho = m\varrho_L + (1-m)\varrho_s$	$K_m = 1; \ K_{\varrho_L} = K_{\varrho_s} = K_\varrho$	Density of continuum
	(30)	$\dfrac{d\varrho}{dt} + \varrho\dfrac{\partial v_i}{\partial x_i} = 0$		Equations of continuity
	(31)	$\dfrac{\partial \sigma_{ij}}{\partial x_j} + \varrho g_i = \varrho\dfrac{dv_i}{dt}$	$K_\sigma\dfrac{1}{K_x} = K_\varrho K_g = K_\varrho\dfrac{K_v}{K_t}$	Equations of movement
	(32)	$F_\alpha(k_\beta, \sigma_{ij}, \varepsilon_{ij}, t) = 0$	$K_k = K_\sigma = K_\varepsilon = K_t = 1$	Physical conditions
	(33)	$\sigma_{ij} = p\delta_{ij} + \sigma_{ij}^{\text{eff}}$	$K_\sigma = K_p$	p is pressure in the pore medium
	(34)	$v_i = -q\left(\dfrac{dp}{dx_i} - \varrho_L g_i\right)$	$K_v = K_q K_p\dfrac{1}{K_x} = K_q K_{\varrho_L} K_g$	Darcy's law
	(35)		$K_q = \dfrac{K_l^2}{K_\mu} = \dfrac{K_l^2}{K_\alpha}$	

stress-strain description in continuum (spanning rows: Equations of continuity, Equations of movement, Physical conditions)

each of which leads to its own model requirements. Correspondence requires that $K_q =$
$= \dfrac{K_l^2}{K_\mu} = \dfrac{K_l^2}{K_\alpha}$, which is included in Tabl. 2, Eq. (35).

The model conditions are now chosen taking account of the requirements shown in Tabl. 2 although all of these cannot be satified, tor which reason further assumptions are necessary. It is assumed that the pore medium's contribution to the density is infinitesimal, that the buoyancy can be neglected and that Reynold's number, R_1, has no significant effect on Δv and on the shape factor c. The corresponding requirements in Tabl. 2 disappear, and a model law can be obtained by choosing the following model conditions:

(36)   The model silo is similar in the ratio K_x.

(37)   Same particles, i.e. $K_k = K_l = K_{\rho_s} = 1$.

This implies the restriction that the particle diameter must be small in relation to the silo geometry in general, especially in relation to the outlet diameter, in the model.

(38)   The test is performed in an artificial gravitational field of the size $K_g = \dfrac{1}{K_x}$.

(39)   The pore medium satisfies the conditions $K_{\rho_L} = \dfrac{1}{K_x}$, $K_\mu = K_\alpha = 1$.

   As far as gases are \concerned such a pore medium can be obtained performing the test under increased pressure.

(40) $K_c = 1$.

We then get the following transformation laws:

(41) $K_\sigma = 1$,

(42) $K_\varepsilon = 1$,

(43) $K_t = K_x$ (forces of inertia),

(44) $K_t = 1$ (time-dependence in const. equations),

(45) $K_t = K_x^2$ (pore flow).

The model conditions, Eqs. (36) to (40), and the transformation laws, Eqs. (41) to (45), constitute the model law, which is valid on the assumption that the description in Tabl. 2 is correct and under the above-mentioned special assumptions which can be expected to be satisfied with reasonable accuracy when the pore medium is air. It should be noted that Eqs. (43), (44) and (45) are incompatible, for which reason model tests cannot be performed if they all have an influence at the same time. This is elucidated in Sect. 3.2.

The model conditions include the requirement to altered pore medium. This is dependent on the scale ratio, which this is inconvenient so we seek to replace Eq. (39) by the following:

(46)   Same pore medium: $K_{\rho_L} = K_\mu = K_\alpha = 1$.

It will be seen that buoyancy is now taken into account correctly, both during fall and at rest, but that the air resistance to the falling particles will only be correct (see Eq. (26)) if Eq. (40) is substituted by

(47)
$$K_c = \frac{1}{K_x}.$$

This requirement cannot be satisfied because c is a function of Reynold's number, R_1, which, with $K_v = 1$, will follow $K_{R_1} = 1$ and thereby give $K_c = 1$. The air resistance will thus be too low in the model, corresponding to too great a rate of fall, which is desirable if concentrated forces in the impact govern the lodgement. This is so because a single particle in the model represents a group of particles in the prototype, which thus has a possibility of internal movement, which is not found in the model. If single particles are to have the same possibility of movement, the impact forces must be made similar in the same ratio as the contact pressures, Q, which are $K_Q = K_g$ for same particle size and density. As the same particles with the same velocity result in the same impact forces, i.e. $K_Q = 1$, it will be seen that the final velocity must be increased in the model to achieve the same possibility of movement for single particles. The stress scale can be maintained by reducing the number of falling particles. Considering the large number of particles hitting the surface, it must be expected that the statistical equalization of the individual impact forces will be so pronounced at a depth of a few particle diameters that the requirement to the stress scale will be decisive for the lodgement.

3. Possibilities for performance of tests in accordance with the model laws

The model conditions included the requirement to a homogeneous gravitational field in the ratio $K_g = 1/K_x$. It was mentioned that this might be achieved by means of a centrifuge, but this introduces an axisymmetrical component to the gravitational field, linearly increasing with the distance from the axis of rotation. The uncertainties which this results in are discussed in Sect. 3.1. It appeared from the model laws that forces of inertia, time-dependent constitutive relations and pore flows lead to different time scales, so that model tests cannot be performed if more than one of these scales applies simultaneously. This matter is discussed in Sect. 3.2.

3.1. Centrifugal field

Let as consider such big gravitational fields that the contribution of the earth can be neglected. The errors dealt with are greatest in these circumstances. The deviation of the gravitational from a homogeneous, parallel field can be expressed as follows in the area of the model (see Fig. 1):

$$(48) \qquad \frac{g_{max}}{g_{min}} \cong 1 \pm \frac{H}{2R},$$

$$(49) \qquad \tan u \cong \frac{B}{2R}.$$

Until the matter has been further investigated, we must simply decide which deviations can be accepted on the basis of an estimate. However, it is commonly assumed that H/R and B/R should be kept below 0.1 [7] and [8]. For silo models, the height of which is normal several times the width, problems arise in connection with the former expression.

Another unintended effect of the gravitational field of the centrifuge is Corioli's forces. These forces can to a certain extent be calculated. They are mass forces directed perpendicular to the gravitional field. For a unit volume with a velocity (see Fig. 2), their magnitude is given by the following expression [5]:

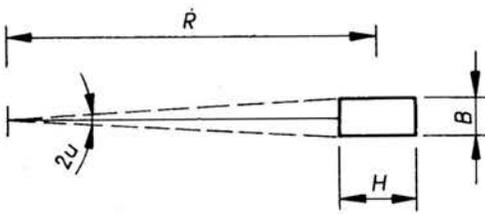$$(50) \qquad\qquad q_c = 2\varrho v_1 \dot{\Theta}.$$
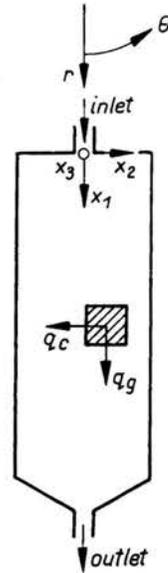

FIG. 1.

FIG. 2.

A comparison with the force of gravity, the magnitude of which is $q_g = \varrho n g_J$, yields the following, in that $\dot{\Theta} = \sqrt{n g_J / R}$:

$$(51) \qquad\qquad \frac{q_c}{q_g} = \frac{2v_1}{\sqrt{n g_J R}}.$$

For a given model v_1 is by and large proportional to $\sqrt{n}$ and, correspondingly, the relative significance of the Coriolis forces is independent of the size of the gravitational field, while their significance decreases with increasing radius of centrifuge. v_1 is greatest immediately above the inlet opening, but the Coriolis forces will already have fallen by a factor of approximately 10 at a dept of a couple of times the outlet diameter, where a considerable greater proportion of the particles in the cross-section are in motion. The order of magnitude of the forces can be illustrated by the fact that in a model containing sand and placed in a centrifuge with a radius of 150 cm, the Coriolis force directly above the outlet opening will be about 10 per cent of the force of gravity.

It is sometimes difficult to predict the significance of these Coriolis forces, but as they

are directed in such a way as to result in lack of symmetry in the rupture pattern, control on this point can form the basis for an evaluation.

3.2. Time scale

The discussion is based on the classification of silo media shown in Table 3. Table 4 shows the time scales as found in Sect. 2.

Table 3. Classification of silo media.

Group	Coarse-grained	Fine-grained	Const. rel. not infl. by time	Time dep. Const. rel.
I	+		+	
II		+	+	
III	+			+
IV		+		+

+) Coarse-grained media shall be taken to mean media in which the pore pressures are so low that they can be neglected.

Table 4. Time scales.

Time scale for	Single particle conside-ration (similar particles)	Continuum (same particles)
Forces of inertia[+]	$K_t = K_x$	$K_t = K_x$
Time dep. const. rel.	$K_t = 1$	$K_t = 1$
Pore flow	$K_t = 1$	$K_t = K_x^2$

+) For discharge.

Group I

This group includes, for example, coarse-grained sand. Only the time scale for forces of inertia is significant, and then only in the discharge period.

For both types of models tests can therefore be performed taking account of the following time scale:

(52)   Filling and rest: No requirements

Emptying: $K_t = K_x$

Group II

This group includes many types of clay and, for example, also cement, the air content of which has proved to have a considerable effect on the pressure conditions in silos [9].

It will be seen from Tabl. 3 and 4 that there is an inconsistency in the emptying situation. Furthermore, there is a boundary problem in connection with the pore flow, because filling must take place in accordance with the time scale for the pore flow.

Assuming that the boundary condition for impact stresses can be neglected, the time

scale for filling cari be altered to accord with that for the pore flow. This is done by altering the inlet geometry.

If the pore flow is still accompanied by appreciable rearrangement of pressure at the beginning of the emptying process, or if rupture in the medium causes pore medium transport with appurtenant alterations in pressure while forces of inertia have an influence in the particulate medium, model tests will not be possible. If this is not the case, the time scale for emptying can be used for forces of inertia.

Tests can thus be performed in accordance with the following:

(53)

$$\text{Filling:} \quad \begin{array}{ll} (K_t = 1) & \text{similar particles,} \\ (K_t = K_x^2) & \text{same particles.} \end{array}$$

$$\text{Rest:} \quad \begin{array}{ll} K_t = 1 & \text{similar particles,} \\ K_t = K_x^2 & \text{same particles.} \end{array}$$

$$\text{Emptying:} \quad (K_t = K_x).$$

The parentheses indicate that the extra assumptions mentioned are necessary for performance of the tests.

Group III

This group includes, for example, certain coarse-grained plastics. As filling takes place in accordance with $K_t = K_x$, there are inconsistencies during both filling and emptying.

If filling in both model and prototype takes place so rapidly that no significant change in the material properties occurs, tests can be carried out in accordance with the requirements to impact. If, on the other hand, the filling proceeds very slowly, so that it can be assumed that the material properties are mainly controlled (governed) by the time-dependent factors and, in relation hereto, only to a slight degree by the lodgement resulting from the impact, filling can be performed taking account of the time scale for time-dependent properties. In the intermediate cases considerable scale errors must be anticipated.

If emptying takes place so rapidly that the material properties are not altered, the emptying can be carried out in accordance with the time scale for forces of inertia. This criterion is often satisfied, especially during the first part of the emptying process, because changes in pressure are so small and because it is a long time since the particles were loaded, so that the properties no longer change so rapidly. In the opposite event considerable errors must be anticipated.

There are thus two possibilities for model tests:

Rapid filling:

(54)

$$\begin{array}{ll} \text{Filling:} & (K_t = K_x), \\ \text{Rest:} & K_t = 1, \\ \text{Emptying:} & (K_t = K_x). \end{array}$$

Slow filling:

(55)

$$\begin{array}{ll} \text{Filling:} & (K_t = 1), \\ \text{Rest:} & K_t = 1, \\ \text{Emptying:} & (K_t = K_x). \end{array}$$

Again, the parentheses indicate that special assumptions mentioned in this section are necessary.

Group IV

This group includes, for example, some fine-grained plastics. It will be seen from Tabl. 3 and 4 that for this group model tests will only be possible in very special cases. However, before a medium is placed in this group it should be investigated whether it can, with an acceptable error, be placed in one of the other groups.

Model tests can only be performed with similar particles and only if filling can take place in accordance with $K_t = 1$ and if neither alterations in pore pressure nor alternations in material properties are of significance during emptying. In such a case, the tests must be performed in accordance with the following:

$$\text{Filling:} \quad (K_t = 1).$$
(56) $$\text{Rest:} \quad K_t = 1.$$
$$\text{Emptying:} \quad (K_t = K_x).$$

Here, too, the parentheses indicate that special assumptions have been made in this section.

Conclusion

It can be concluded that an artificial gravitational field does not normally suffice to prevent scale errors in model tests. The analysis provides a certain basis for evaluating whether model tests may be a possibility in a given case by defining the necessary assumptions. The basis for the analysis is so broad that other models than silo models are also covered or can easily be covered by adding other model requirements.

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