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Analysis of the usefulness of diffusion equations for the description of dislocation mobility and related phenomena

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DISLOCATION motion induced by the process of thermal activation and mechanical loading is considered. An analysis is given of possible reasons explaining that the motion of dislocation is more restricted as compared to point defects. The influence of material properties on the displacements (displacements in two-dimensional and three-dimensional spaces) is studied. A good correlation between the theory and experiments indicates the possibility of describing the process by the diffusion equations.

Rozpatrzono ruchy dyslokacji wywołane przez aktywację termiczną i przez obciążenia mechaniczne. Przeanalizowano przyczyny ograniczenia możliwości przemieszczeń dyslokacji w porównaniu z defektami punktowymi i wpływu na te przemieszczenia własności materiału (przemieszczeń w przestrzeni 2- lub 3-wymiarowej). Zgodność wyników badań doświadczalnych z wnioskami teoretycznymi wskazuje na możliwość opisu procesu przy pomocy równań dyfuzji.

Рассмотрены движения дислокаций, вызванные термической активацией и механической нагрузкой. Анализируются причины ограничения возможности перемещений дислокаций по сравнению с точечными дефектами и влияния на эти перемещения свойств материала (перемещений в двухмерном или трехмерном пространстве). Совпадение результатов экспериментальных исследований с теоретическими выводами указывает на возможность описания процесса при помощи уравнений диффузии.

1. Introduction

IN THE papers [1, 2, 3] and others devoted to problems of the fatigue of metals the author applied the diffusion equations to estimate the ability of dislocation displacements (mobility), their density changes, maximum density etc. The equation for the density of the mobile dislocations had the form

$$\frac{\partial\varrho_m}{\partial N} = D_{13}\Delta\varrho_m + a_2 kF\varrho_m + a_3\varrho_{im} - a_4F\varrho_m - 2(a_5+a_7+a_9+a_{11})\varrho_m^2 - (a_6+a_8+a_{10}+a_{12})\varrho_m\varrho_{im} \tag{1.1}$$

and for immobile dislocations was

$$\frac{\partial\varrho_{im}}{\partial N} = -a_3\varrho_{im} + a_4F\varrho_m + (2a_5+a_7+a_9)\varrho_m^2 + (a_6-a_{10})\varrho_m\varrho_{im}. \tag{1.2}$$

The equation for all dislocations after adding by sides Eqs. (1.1) and (1.2) is

$$\frac{\partial\varrho}{\partial N} = D\Delta\varrho + b\varrho - c\varrho^2, \tag{1.3}$$

where D, b, c, are the coefficients of diffusion, multiplication and annihilation of dislocations, respectively. These coefficients are dependent on the corresponding coefficients a_i appearing in Eqs. (1.1) and (1.2) and on $\beta(N)$, where

$$\beta(N) = \frac{\varrho_m}{\varrho} \cong 1 - \frac{\varrho}{\varrho_{\text{lim}}}. \tag{1.4}$$

Analysis of the approximate solution of Eq. (1.3) shows [1] that the change of $\partial\varrho/\partial N$ depends mainly on the value of the coefficient H

$$H = b\beta - \frac{5.76}{R^2} D\beta - c_1\beta^2, \tag{1.5}$$

where R is the radius of the cross-section of the specimen.

Depending on the value of the coefficients which appear in Eq. (1.5), the value of H may be positive, negative or equal to zero. If $H > 0$ the dislocation density rapidly increases to the limit state of saturation ϱ_{lim} (Fig. 1) and the greater ϱ_{lim}, the larger H is; if $H < 0$ the dislocation density rapidly decreases to the minimum limit values and the smaller ϱ_{lim}, the smaller H is; if $H = 0$ the dislocation density does not change.

It should be stressed that at the fixed values of the coefficients b, c and D the limit dislocation density, Eq. (1.3), does not depend on the initial density existing in metal before the start of action of the variable loading. From the numerical computations it results that if $b > 5.78D/R^2$, then the limit density depends solely on the coefficient ratio

$$\varrho_{\text{lim}}(N) = \frac{b}{c} \tag{1.6}$$

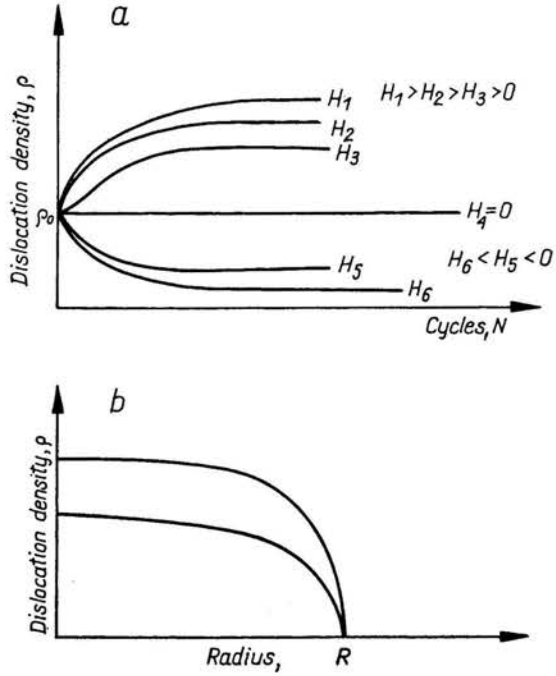

FIG. 1. Changes in dislocation density at different values of H in terms of a) number of cycles, N; b) along the radius of the cross-section of the specimen, R.

and

$$\varrho(\bar{r}) = \varrho_{\lim}(N)f(D/R). \tag{1.7}$$

If the initial density $\varrho_0 < b/c$, then the variable loading involves the growth of dislocation density to the value $\varrho_{\lim}$; if $\varrho_0 > b/c$ then under action of the variable loads the dislocation density decreases to $\varrho_{\lim}$.

2. The notion of diffusion of dislocation

The notion of diffusion used in this paper may raise some objections since in the literature it was really not encountered in this sense. In the literature the notion of diffusion of dislocation caused by the diffusive motion of point defects toward the dislocation or from the dislocation was generally used. The displacement of the edge of dislocation occurs then in a dislocation plane (in a half-plane being the dislocation wedge). But our considerations deal with the displacements of dislocations in their slip planes.

In the case of single dislocation such a displacement is described by its direction, velocity and distance passed. If, however, the great number of the dislocations moves in different directions, what occurs especially under variable stresses, the estimation of the summary effects of the objects moving chaotically in different directions may be described by means of the diffusion equations which were used by the author for describing the effects of the dislocation displacements. The dislocation density is sufficiently large so it is possible to say something about the average effects of the dislocation displacements, and at the same time it is sufficiently small so that the particular dislocation can preserve the individual features of the objects forming the sets.

The doubts mentioned above as to whether the application of the diffusion equations for describing the effects of dislocation displacements is sufficiently argumented may arise from the fact that:

one may have objections as to whether the condition of chaoticity is satisfied for dislocations;

in distinction to the classic processes of the diffusion of atoms or molecules in gases, fluids and solids, the only existence of the density gradient of dislocations does not lead to the corresponding dislocation displacements, causing the decrease of this gradient;

the decrease of the dislocation density and dislocation density gradient may in certain conditions occur only under variable external loads.

These problems will be discussed successively.

3. Chaoticity of elementary dislocation displacements

The displacements of dislocation in a determined direction is the resulting effect of overcoming by particular dislocation segments of the consecutive barriers of the potential coinciding with the net planes. The dislocation motion connected with its vibrations between two extreme positions coinciding with the neighbouring maxima of the potentials was first considered by SEEGER [4]. This problem was further developed by DONTH [5] and others.

At a fixed temperature the dislocation segments vibrate similarly as a string with a vibration frequency which, depending on the kind of crystal, degree of its purity etc., equals $\nu = 10^6 - 10^{11}$ sec^{-1}, [6], and the amplitude of the vibrations is of the order of the lattice parameter. These vibrations exhibit the features of the thermally-activated process and under thermal fluctuations their amplitude changes locally. The pair of transient kinks is formed on the dislocation lying in a plane of dense packing (Fig. 2) due to thermal fluctuations and first of all due to applied external load. These kinks in the case of overcoming the barriers of the potential and the attractive force between kinks (Fig. 2c) become permanent deflections and then move away along the dislocation line (Fig. 2d). The displacement of dislocation in its slip plane has then a two-dimensional character since it consists in overcoming the successive Peierls-Nabarro barriers and is connected with the displacement of kinks in a direction normal to the direction of dislocation motion.

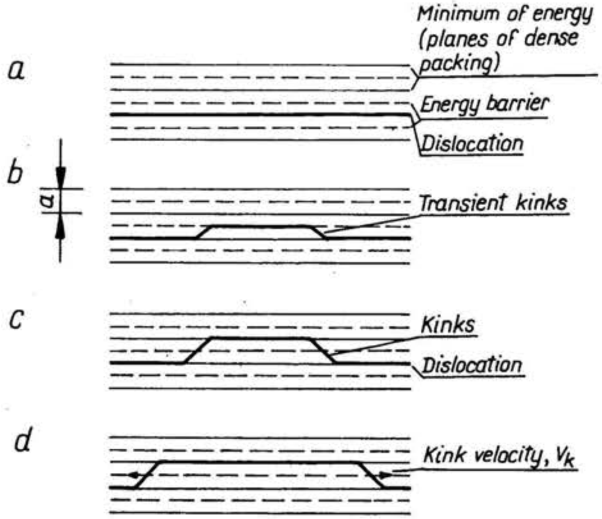

FIG. 2. The model of thermically-activated overcoming of Peierls-Nabarro barriers, [6].

In equilibrium conditions the kinks of dislocations, similarly as the atoms from which these kinks are built, under the influence of the thermal fluctuations vibrate in a chaotic way. The flux of the kinks assumes the form [15]

$$J_k = -D_k \frac{\partial c_k}{\partial x}. \tag{3.1}$$

The diffusion coefficient of kinks, D_k, is

$$D_k = \alpha a^2 \omega \approx a^2 \nu_0 \exp\left(-\frac{W_m}{kT}\right), \tag{3.2}$$

where c_k is the concentration of kinks, a denotes the length of the elementary jump, ω is the frequency of jumps in a given direction, the coefficients $\alpha \approx 1$, ν_0 denotes the vibration frequency of the kinks which is of the order 10^{12}–10^{13} sec^{-1}, W_m is the activation energy of the jump of kink.

Both possible directions of kink jumps are equally probable, hence no favoured direction of dislocation motion can arise. When acted upon by a constant shear stress τ, the motion of kinks is biased. The jumps to one side are aided by the force τbh, while those to the other side are retarded, so that a net drift velocity, V_k, in the direction of the stress results:

$$V_k = D_k \frac{\tau bh}{kT}, \tag{3.3}$$

where b is the Burgers vector and h is the height of the kink (Fig. 3).

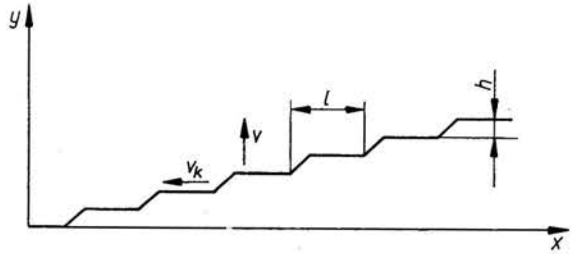

FIG. 3. Dislocations with kinks distributed at the same distances l [15].

If the distances between the kinks are the same and equal l, and the kinks move with the velocity V_k, then the whole dislocation moves in a direction perpendicular to its line with the velocity

$$V = D_k \frac{\tau bh^2}{lkT}. \tag{3.4}$$

According to recent concepts the dislocations in fcc metals manifest the ability to move even at a temperature close to 0°K. The resulting motion of dislocation is then a consequence of the motion of its particular segments which are being displaced thanks to the continuous randomly vibrating motions with the largest probability of overcoming the barriers of the potential in one plane (in a slip plane).

4. Dislocation density gradient and dislocation displacement

It has been confirmed above that the noticeable dislocation displacements can not be proved experimentally when $\tau = 0$ or $\operatorname{grad} T = 0$, but only when $\operatorname{grad} \varrho > 0$. The possibility of equalization of the dislocation densities between the micro-regions even with the significant values of the $\operatorname{grad} \varrho$ and high temperature is not confirmed, while for the atoms the equalization of the concentrations when $\tau = 0$, $\operatorname{grad} T = 0$ and $T = \text{const}$ but only when $\operatorname{grad} c > 0$ is one of the basic symptoms of the diffusion. Moreover, the self-diffusion phenomenon confirms the fact that for atoms the existence of $\operatorname{grad} c > 0$ is not necessary for the diffusion to develop with noticeable macroscopic effects.

4.1. Dislocation displacements at symmetric elementary displacements

In conditions of symmetric elementary displacements ($\tau = 0$, $\operatorname{grad} T = 0$, $T = \text{const}$ or even $\tau > 0$ but $\tau_m = 0$) the character of the displacement of atoms and point defects in solids differs essentially from that of the displacement of dislocations. Although the possibilities of displacements of atoms and dislocations exhibit in crystals directional anisotropy, the probability of the displacement of the atom or point defect in the three mutually orthogonal directions is, however, the same (in cubic lattice) or almost the same (in hexagonal lattice). Each of these objects being in a determined point may be displaced with the same probability to one of at least six similar neighbouring points. The diffusion processes of atoms or point defects are then symmetric processes occurring in the three-dimensional space.

The dislocations—even in a body with directional isotropy and particularly in real anisotropic crystals—may displace only in the slip plane. A given atom may be displaced with the same probability P_y forward and backward together with the dislocation line (Fig. 2) and with the same probability P_x to the left and to the right with the kink. The atoms of a dislocation half-plane lying at a distance of n parameters of the lattice away from the edge of this half-plane may be subjected to analogous displacements only in the plane parallel to the slip plane, at a distance of n parameters of lattice away from it. Similar reduction of the motions of atoms to two-dimensional space exists also in the displacement of the screw dislocations.

Since several differently oriented planes of the easy slips exist in the crystal, the three-dimensionality of the processes may be sometimes also taken into account; however, the elementary displacements of atoms causing the motion of the particular dislocations occur in two-dimensional space. Some deviations from this rule take place at the cross slips. But this will be discussed later in Sect. 5.

There is a full analogy between the character of the displacements of atoms or point defects in the crystal lattice of metals (especially cubic) and the model of displacements considered in the mathematical theory of the symmetric random walks [7]. In both cases the objects may rest only in the nodal points of the lattice composed of the elementary squares or cubics and may pass by jumps the discrete distances to the neighbouring points. Thanks to this analogy and to the fact that the theory of the random walks lies in the foundations of the diffusion theory, two basic theorems of the theory of random walks are applied in the analysis of our problem.

1. In the case of the symmetric random walks the probability that in the infinite increase of time the random walking point will come back to its initial position in 1- and 2-dimensional space equals unity, while in $3, 4, \ldots, n$-dimensional spaces this probability is less than unity [12, 13, 14].

2. Two randomly walking points in 1- and 2-dimensional space will meet with a probability equal to 1 infinitely multiple times, while in the $3, 4, \ldots, n$-dimensional spaces the probability that these points will never meet is positive (they rather go away from each other toward the boundary of the region) [12, 13, 14].

The essential conclusions result from these theorems.

1. The atoms or point defects, if they are only sufficiently mobile, displacing in the 3-dimensional space show a tendency to increase continously the distance from their initial positions. These motions account for the transport of the particular atoms for significant distances what in fact causes the decrease of grad c. This also contributes to the forming of the self-diffusion process occurring in the crystals built from the atoms of the single element, i.e., at $\operatorname{grad} c = 0$.

2. The dislocations, moving because of the displacements of atoms in the 2-dimensional space, show a tendency to return unceasingly to their initial positions. In the conditions when $T = \text{const}$, $\operatorname{grad} T = 0$ and $\tau = 0$, or even $\tau > 0$ but $\tau_m = 0$, the displacements of the kinks and the whole dislocations may have solely a local character and their summary effect equals zero. This is just the reason why the displacements of dislocations are not in a sufficient number required to cause noticeable equalization of the differences of their densities between different (neighbouring) microvolumes.

The self-diffusion phenomenon mentioned above confirms the fact that the existence of $\operatorname{grad} c > 0$ is not an indispensable condition for the diffusion processes. These processes are the result of the enormous number of the randomly occurring elementary displacements of atoms. Fick's formula in general reflects correctly the quantitative effects of the process governed by the law of large numbers, but the existence of $\operatorname{grad} c > 0$ is certainly not a motor mobilizing the mechanism of the directional diffusion. Besides, it is known from the experiments that the process of reverse diffusion may occur when the flux of the determined kind of atoms displaces to the regions of their greater concentrations. The nonhomogeneous distribution of the stresses, the differences of the chemical potential, etc. may be the reason for this phenomenon.

The discussed restriction of the possibility of displacement of the dislocations to the 2-dimensional space is certainly responsible for the fact that the differences alone of the dislocation densities in the particular micro-regions are not a sufficient factor for equalizing these densities. The stress fields produced by the dislocation and, in particular, by their tangles may act against such an equalization. In certain conditions the differentiation of the local dislocation densities may even increase since the dislocations should demonstrate, because of the limited mobility mentioned above, the tendency to gather in microregions of higher efficiency of their multiplication, i.e., in the vicinity of the boundaries of the subgrains and grains.

4.2. Dislocation displacements at asymmetric elementary displacements

In the equation of the flux of dislocations

$$J = V_{\text{drift}}\varrho = \frac{D}{kT}\tau bh\varrho, \tag{4.1}$$

in the equation for changes of dislocation density

$$\frac{\partial\varrho}{\partial N} = D\Delta\varrho \tag{4.2}$$

or in Eqs. (1.1) and (1.3) the dislocation diffusion coefficient D determines the number of dislocations passing in unit time through the unit width of the plane (in the 2-dimensional model) or through the unit surface (in the 3-dimensional model when more than one slip plane exist).

In the conditions of variable stresses when the direction of the dislocation displacement changes cyclically the flux J expresses a difference between the number of dislocations moving back and forward during one cycle of loading through the unit width or surface. In the case of the symmetry of dislocation displacements the flux J equals zero, i.e. the situation is similar to that considered in Sect. 4.1, since the noticeable effects of the displacements of dislocations in a given region will not appear. In the case of asymmetry of displacements the flux $J > 0$ and the directional stream of dislocations arise with the density given by Eq. (4.1).

Mean dislocation velocity v_m or drift velocity in Eq. (4.1) plays a significant role in the process of dislocation displacements and through v_m the coefficient D depends on different factors. The sign of dislocation velocity depends on the sign of stress; the velocity depends on the stress, temperature, kind of lattice, etc; the path of the free displacements depends on the kind of crystal, the degree of its imperfection, the density of the barriers, i.e. density of inclusions or density of immobile dislocations, etc...

The velocities of dislocations in different crystals, and even in the same crystal but in different ranges of stress and temperature, are strongly differentiated. In the quantitative analysis of the problem one may use the direct results of measuring or the empirical formulae. For example, one such relation has the form [8]

$$v_m = \left(\frac{\tau}{\tau_0}\right)^m, \tag{4.3}$$

where v_m denotes the mean dislocation velocity, τ is the applied stress, τ_0 is the stress causing the unit velocity and m denotes the experimental coefficient different for various crystals and temperature ranges.

The values of m for certain materials are as follows [8]: Si (600–900°C)—1.4; Ge (420–700°C)—1.4–1.9; Cr—ca 7; Mo—ca 8; W—5; LiF—14.5–25; Fe—3.25% Si—35; Cu—ca 200; Ag—ca 300.

The external surface should play a particular role in the conditions where, at the stresses acting on the crystal, the flux $D = 0$ (symmetry of dislocation displacements) and $D > 0$ (asymmetry of dislocation displacements). However, even in such loading conditions which lead to $D = 0$ in the core (when, for example, for symmetric stresses and a variable sign the mean stress equals zero), near the surface $D > 0$ will occur due to the additional interaction force of attracting the dislocation to the surface. For the screw dislocation parallel to the surface this force referred to unit length is

$$\frac{F}{L} = \frac{Gb^2}{4\pi l} \tag{4.4}$$

and for edge dislocation

$$\frac{F}{L} = \frac{Gb^2}{4\pi(1-\mu)l}, \tag{4.5}$$

where G and μ denote the elastic shear modulus and the Poisson coefficient respectively, and l is the distance from the dislocation to the surface.

The flux of dislocations appearing in a surface layer, directed to the surface and caused by the joint interaction of the external and internal forces, has an essential influence on the mechanism of fatigue. It is not excluded that the whole agglomerations of dislocations, gathered in the tangles or in boundaries of cells of dislocations, diffuse to the surface. Thus, it was confirmed experimentally that the surface layer can be free of the dislocations [9, 10].

5. Diffusive mechanism of cycling hardening and softening

The consequences of different possible mechanisms of dislocation displacements appear in the macroscopic effects (work-hardening or softening curves) as well as in microscopic effects (distributions of dislocations). FELTNER and LAIRD [11] have presented the work-hardening and softening curves for the polycrystal copper and alloy Cu—7,5% Al and have evaluated the densities and distributions of dislocations in these materials. They used specimens annealed and cold-worked (decrease in diameter 5%, 15%, 23% and more) and fatigued at temperature of 78°K or 300°K.

It should be pointed out that in such materials as Cu—7,5% Al or Fe—3% Si the dislocation displacements have a 2-dimensional character, i.e. the slips occur only in the determined planes (due to large anisotropy), but for example in Cu, Al or Fe the displacements have a 3-dimensional character since in these materials cross slips of the screw dislocations may occur easily [11].

The result of examinations of the hardening and softening and distribution of dislocations were as follows:

1. At a given test temperature, copper, which has a 3-dimensional or wavy glide slip character, has a unique cyclic stress-strain curve independent of history. The results in Fig. 4 show that cyclic stress-strain curves for Cu are coincident, regardless of the annealing treatment (resulting in different grain sizes), the amount of cold work, the cold working process, and the temperature of cold work. Other results on pure Al demonstrate the generality that saturation conditions produced by cyclic strain in materials which display a wavy slip character are independent of the history of the metal.

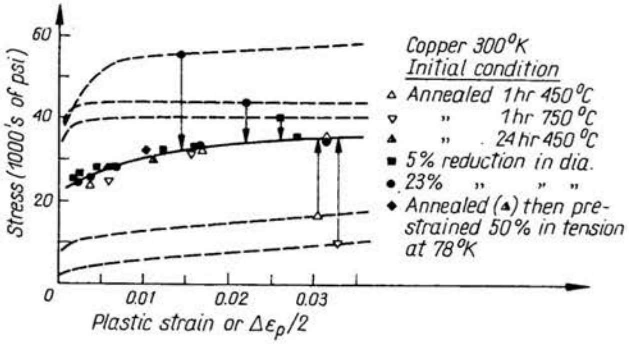

FIG. 4. Cyclic (——) and monotonic (- - - - -) stress-strain curves for copper at 300°K [11].

2. Cu—7.5% Al, which has a planar glide slip character, has a different cyclic stress-strain curve for each different initial condition of the material. The results in Fig. 5 show that cyclic stress-strain curves of Cu—7.5% Al are never coincident and depend strongly on annealing and cold working treatment.

3. In annealed copper subjected to fatigue the dislocation cells appeared (Fig. 6a, c), the diameters of which were larger at higher temperature and at lower amplitude of cyclic stress. The interiors of cells, in which after static deformation some amounts of spreading dislocations remained (Fig. 6b), were "cleaned" after cyclic stress from dislocations

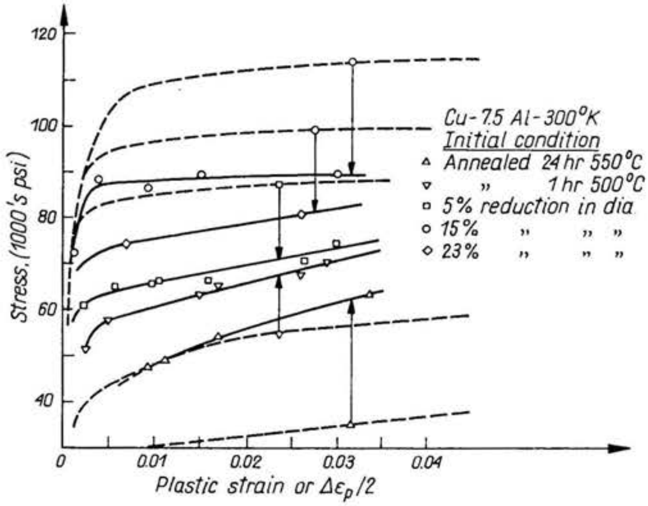

FIG. 5. Cyclic (——) and monotonic (- - - - -) stress-strain curves for Cu–7.5% Al at 300°K [11].

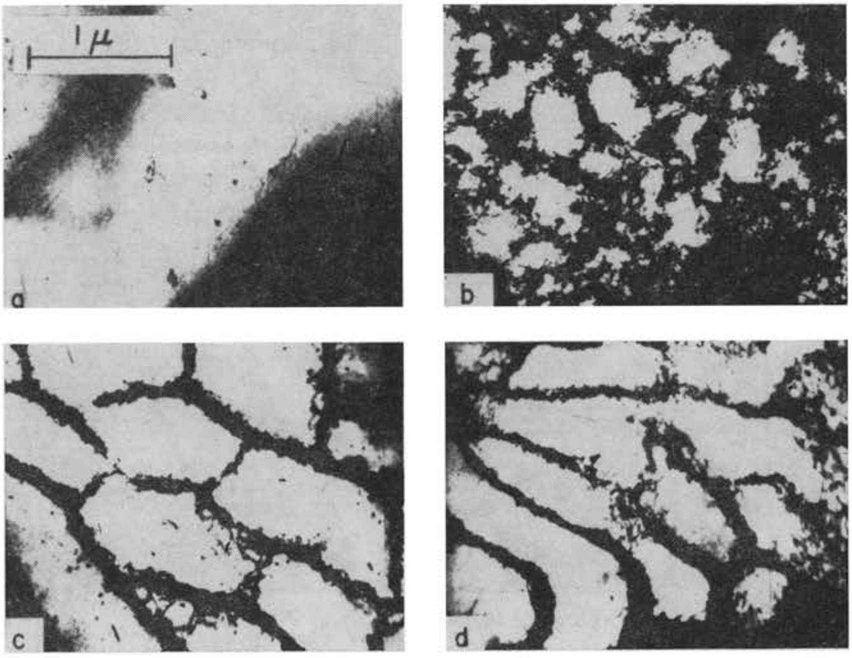

FIG. 6. Dislocation structures in copper before a) and b) and after cycling c) and d) [11].

(Fig. 6d). But the boundaries of cells composed of dislocations were narrower, more distinct and the greater was the number of loading cycles applied, the greater did the condensation of dislocations occur.

4. In Cu under initial cold work the significant condensation of cells and the increase of dislocation density occurred (Fig. 6b). Due to the action of cyclic stresses the cell structure and the density of dislocations changed to such a state which appeared in annealed specimens subjected to similar variable loading only (Fig. 6c, d). The history preceding

the process of fatigue had then no influence on the density and distribution of dislocations produced by variable loading under the condition that the plastic strain caused by these loads had a variable sign.

5. In the annealed and fatigued Cu–7.5% Al the dense planar strips of dislocations and the considerable amount of dislocations spreading between these strips appeared (Fig. 7c). At higher stresses the dislocations were distributed almost uniformly in planar sets. The number of tangles in cold worked and cycled material was greater than the number of tangles prior to fatigue. The number of tangles in the cold worked and cycled material was smaller than in a material annealed and cycled. In the cold worked and cycled material the density of interstrip dislocations was large, what was a consequence of the initial state (cold working).

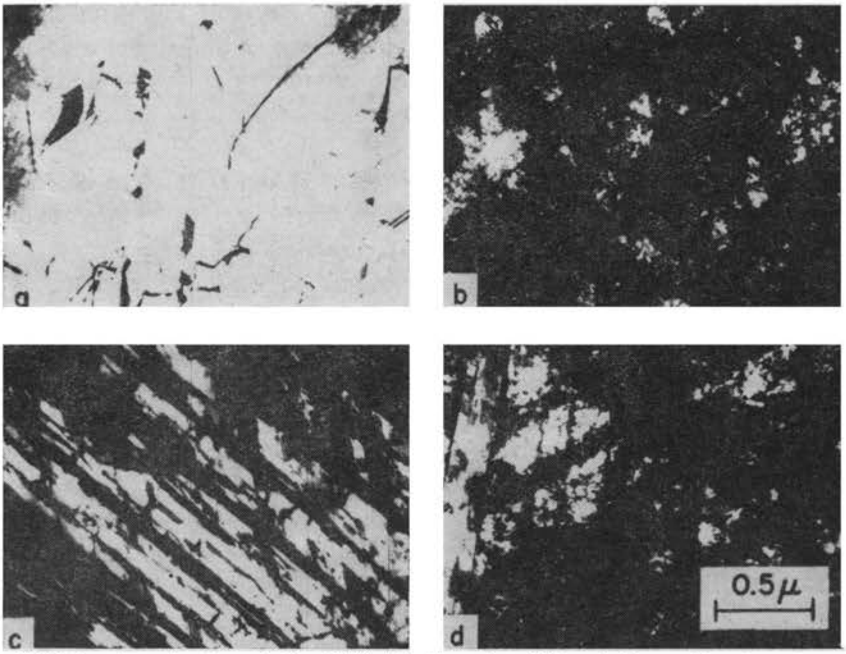

Fig. 7. Dislocation structures in Cu–7.5% Al before a) and b) and after cycling c) and d) [11].

6. In Cu–7.5% Al the history effect of the specimen was distinct, in particular at a lower amplitude of variable loads. The structure and density of dislocations in the cycled specimens after previous cold working or annealing differred essentially.

In the light of the considerations in Sect. 2–4 the discussed character of the hardening and softening curves and the differences in distributions of dislocations have found evident argumentation. We shall discuss these topics successively in the same order as we have illustrated the experimental results (points 1 to 6).

1. The fact that the shape of the cyclic stress-strain curve in Cu (Fig. 4) is independent of history is significant for two reasons.

Firstly, it indicates that the changes of dislocation densities responsible for the hardening or softening phenomenon exhibit really diffusive processes. The independence of these curves of the history is in agreement with the property discussed in the introduction

(Eqs. (1.6) and (1.7)) characteristic of the diffusion processes (and heat conduction) which claims that after a sufficiently long time (or corresponding number of cycles) a certain typical value and distribution of density of dislocations (or concentrations of other diffusive objects, or temperature) are established in the body and they depend solely on the value of the coefficients appearing in the equation of type (1.3), i.e. b, c and D but do not depend on the initial conditions.

Secondly, the results presented in Fig. 4 demonstrate that in Cu the value of the coefficient H (Eq. (1.5)) never changes under external factors, i.e. does not depend on the history. It also means that the coefficients b, c, D do not change because a change of b and c even at $b/c = \text{const}$ and $H = \text{const}$ would lead to the displacement of the hardening or softening curve to the left (accelerated process) or to the right (delayed process).

2. The similarity of the curves from Fig. 5 to the curves of Fig. 4 indicates that in Cu–7.5% Al cyclic hardening or softening is also caused by a diffusive change of dislocation density. But a qualitative dependence of the curves on the history indicates that in this material the history has influence on the value of the coefficient H, i.e. the coefficients b, c and D change in a different way. A change of the ratio b/c causes the growth or decay of the density of the state of saturation ϱ_{lim} (Eqs. (1.6) and (1.7)). The decrease of the coefficient c as consequence of cold working may be caused here by the increase of density of immobile dislocations under the influence of variable loads (see point 6).

3. In Cu due to the fact that dislocations move in 3-dimensional space they show the tendency to go away from the previously occupied positions. Thus the interiors of the 3-dimensional cells do not contain dislocations. The dislocations gather in 2-dimensional cell boundaries on which they can locally move. The cell boundaries appear probably at the initially existing boundaries of subgrains because of the presence of immobile dislocations. The cell boundaries may displace probably as a whole due to directional diffusion of the dislocation conglomerations (forming these boundaries).

4. In Cu, due to the fact that in 3-dimensional motions the dislocations there overpass more easy the barriers, the coefficient H (Eq. (1.5)) is independent of the actual dislocation density and β need not diminish to zero. This means that the coefficients b, c and D in the same way (e.q. by influence of β) depend on the actual dislocation density and are uniquely dependent only on the actual loads and temperature. Thanks to this fact the large density of dislocations arising during cold working—if it does not exceed the determined limits—does not lead to irreversible blocking of dislocations. These dislocations during the action of correspondingly smaller variable loads are then dismissed or annihilated. The mechanism of the process is such that a smaller coefficient b corresponds to the low variable loads. Thus, the density of saturation—in agreement with the formula (1.6)—decreases. The change of sign of plastic strain is necessary to cause 3-dimensional diffusion (displacements from a given point in 6 possible directions) and if it is not the case the effects of the process will not be obtained.

5. In Cu–7.5% Al the distribution of dislocations in the slip planes is more uniform, since in the incessant return of dislocations to their initial positions the distinct changes of their concentration state are not possible. The cold working makes a large number of blocked dislocations arise and during cycling the development of tangles is more difficult (smaller b) than in annealed material.

6. In connection with the state discussed in point 5 and in distinction to the situation existing in Cu (point 4), in alloy Cu–7.5% Al the distinct decrease of dislocation density under the influence of variable stresses cannot take place (due to greater blocking of the dislocations, $\beta \to 0$ and b, c and D depend on the actual dislocation density). The history dependence of the density and distribution of dislocations in this material is then fully justified.

6. Conclusions

From the above considerations it results that experimentally confirmed differences in the effects of displacements of atoms and dislocations do not contradict the possibility of describing the motion of dislocations and phenomena connected with these motions by means of diffusion equations, but on the contrary—finding a unique explanation—completly justify this possibility.

Thus it seems that there are no physically and mathematically justified reasons for not using diffusion equations in the case of dislocations. The agreement of the conclusions resulting from the analysis of the fatigue of metals performed using these equations in [1–3] and in the paper with the facts confirmed experimentally may constitute the additional argument supporting these ideas.

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