Archives of Mechanics • Archiwum Mechaniki Stosowanej • 30, 3, pp. 243–257, Warszawa 1978

# On equations of net shells of revolution subjected to rotationally-symmetric loads

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THE EQUILIBRIUM equations for the net shells of revolution in a rotationally-symmetric state of loading are derived in a new unknown form (comp. [1]). The procedure is the same as in deriving the Reissner's or Meissner's equations for full-walled shells of revolution.

W pracy wyprowadzono równania równowagi obrotowych powłok siatkowych w obrotowo-symetrycznym stanie obciążenia — w innej od dotychczas znanej postaci (por. [1]). Postępowano przy tym tak samo jak przy wyprowadzaniu równań H. Reissnera lub E. Meissnera dla pełnościennych powłok obrotowych.

В работе выведены уравнения равновесия сетчатых оболочек вращения в вращательно-симметричном нагруженном состоянии, но в другом, чем известный до сих пор, виде (ср. [1]). При этом поступается таким самым образом, как при выводе уравнений Г. Рейсснера или Э. Мейсснера для полностенных оболочек вращения.

## Introduction

IN A THEORY of net shells of revolution which are in a rotationally-symmetric state we have three differential equilibrium equations containing six unknown static quantities (comp. [1]). In this paper, following Reissner's or Meissner's procedure of deriving equations for full-walled shells of revolution, we shall obtain a different form of equilibrium conditions for three unknown variables.

## 1. Basic equations and relations of the net shell theory

We shall deal with a surface system built of rigidly-joined rods in hinges and perforated shell. A detailed explanation of the assumptions as well as the equations, relations and symbols used in this section may be found in a monograph [1].

Let $\pi$ be a surface segment covered by a net shell parametrized by means of the coordinate system $x^1$, $x^2$. We shall restrict our considerations to the surface on which two discrete families of curves $(\Delta)$ $(\Delta = \mathrm{I}, \mathrm{II})$ are given. We shall assume these curves to be, according to the assumptions of the theory, the axes of elements from which the shell is constructed. The points of intersection of both family curves form the nodes of the system. Let $a_{KL}$, $b_{KL}$, $e_{KL}$ denote the components of the first and second quadratic forms of the surface and components of the Ricci's bivector, respectively, while $t^K_{(\Delta)}$, $\tilde{t}^K_{(\Delta)}$ are the components of the versors tangent and normal to the family curves $(\Delta)$, $(\Delta = \mathrm{I}, \mathrm{II};$ $K, L = 1, 2)$ (comp. Fig. 1), correspondingly.

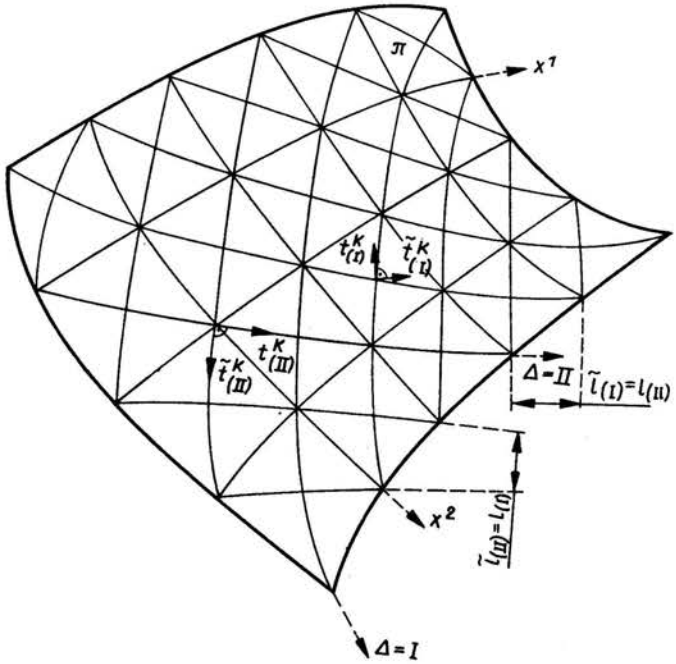

FIG. 1.

The system of equilibrium equations has the form

$$
\begin{aligned}
p^{KN}|_K - b^N_K p^K + b^N &= 0,\\
m^K|_K + e_{KL} p^{KL} + b_{KL} m^{KL} + h &= 0,\\
p^K|_K + b_{KL} p^{KL} + b &= 0,\\
m^{KN}|_K - b^N_K m^K + e^N{}_K p^K + h^N &= 0,
\end{aligned} \tag{1.1}
$$

where $b^N$, $b$ are the tangent and normal components of the vector forces, respectively, while $h^N$ denotes the $h$-components of the vector of moments of external load.

The "internal forces" $p^{KN}, p^K, m^{KN}, m^K$ are determined by means of the formulae

$$
\begin{aligned}
p^{KN} &= \sum_{\Delta} p^{KN}_{(\Delta)} = \sum_{\Delta} t^K_{(\Delta)} (t^N_{(\Delta)} P_{(\Delta)} + \tilde{t}^N_{(\Delta)} \tilde{P}_{(\Delta)}) \tilde{l}^{-1}_{(\Delta)},\\
m^{KN} &= \sum_{\Delta} m^{KN}_{(\Delta)} = \sum_{\Delta} t^K_{(\Delta)} (t^N_{(\Delta)} M_{(\Delta)} + \tilde{t}^N_{(\Delta)} \tilde{M}_{(\Delta)}) \tilde{l}^{-1}_{(\Delta)},\\
p^K &= \sum_{\Delta} p^K_{(\Delta)} = \sum_{\Delta} t^K_{(\Delta)} \check{P}_{(\Delta)} \tilde{l}^{-1}_{(\Delta)},\\
m^K &= \sum_{\Delta} m^K_{(\Delta)} = \sum_{\Delta} t^K_{(\Delta)} \check{M}_{(\Delta)} \tilde{l}^{-1}_{(\Delta)},
\end{aligned} \tag{1.2}
$$

where $\tilde{l}_{(\Delta)}$ denotes a distance between the family curves ($\Delta$). The quantities $P_{(\Delta)}$, $\tilde{P}_{(\Delta)}$, $\check{P}_{(\Delta)}$ and $M_{(\Delta)}$, $\tilde{M}_{(\Delta)}$, $\check{M}_{(\Delta)}$, appearing in the formulae (1.2), are real components of the force and moment vectors, respectively, existing in a cross-section situated at a half-length of the net shell element (Fig. 2).

The geometrical relations have the form

$$\gamma_{KS} = u_S|_K - b_{SK}u + e_{SK}v, \qquad \varkappa_{KS} = v_S|_K - b_{SK}v,$$
(1.3)
$$\gamma_K = u|_K + b_K^L u_L + e_{KL}v^L, \qquad \varkappa_K = v|_K + b_K^L v_L,$$

where $u^K$, $u$ are the components of the linear displacement vector and $v^k$, $v$ denote the components of the vector of infinitesimal rotations in the nodes of the net system. In

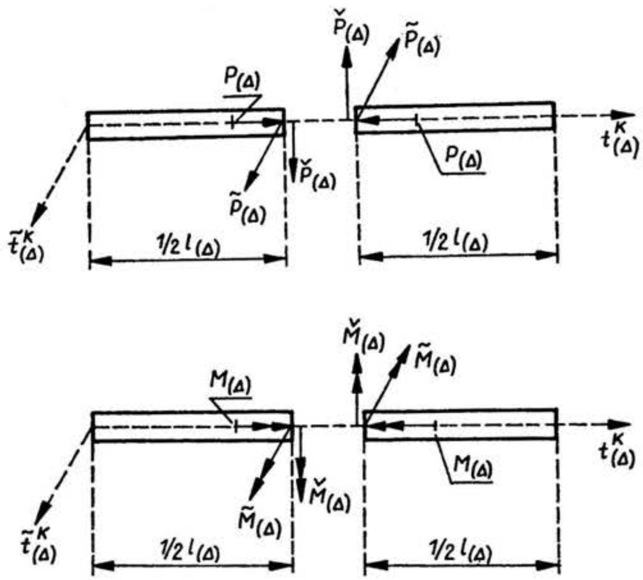

FIG. 2.

agreement with the theory all quantities are continuous and sufficiently regular functions of the variables $x^1$, $x^2$ and have physical interpretation in the corresponding points of the net of the family curves $(\Delta)$.

We shall still use the constitutive equations

$$p^{KL}_{(\Delta)} = A^{KLMN}_{(\Delta)}\gamma_{MN}, \qquad m^{KL}_{(\Delta)} = C^{KLMN}_{(\Delta)}\varkappa_{MN},$$
(1.4)
$$p^{K}_{(\Delta)} = A^{KL}_{(\Delta)}\gamma_L, \qquad m^{K}_{(\Delta)} = C^{KL}_{(\Delta)}\varkappa_L,$$

where $A^{KLMN}_{(\Delta)}$, $C^{KLMN}_{(\Delta)}$, $A^{KL}_{(\Delta)}$, $C^{KL}_{(\Delta)}$ denote the tensors of rigidity of the net shell elements of the family $(\Delta)$. For shells composed of rods we assume them in the form

$$A^{KLMN}_{(\Delta)} = t^K_{(\Delta)} t^M_{(\Delta)} (t^L_{(\Delta)} t^N_{(\Delta)} R_{(\Delta)} + \tilde{t}^L_{(\Delta)} \tilde{t}^N_{(\Delta)} \tilde{R}_{(\Delta)}),$$
(1.5)
$$C^{KLMN}_{(\Delta)} = t^K_{(\Delta)} t^M_{(\Delta)} (t^L_{(\Delta)} t^N_{(\Delta)} S_{(\Delta)} + \tilde{t}^L_{(\Delta)} \tilde{t}^N_{(\Delta)} \tilde{S}_{(\Delta)}),$$
$$A^{KL}_{(\Delta)} = t^K_{(\Delta)} t^L_{(\Delta)} \check{R}_{(\Delta)}, \qquad C^{KL}_{(\Delta)} = t^K_{(\Delta)} t^L_{(\Delta)} \check{S}_{(\Delta)},$$

where

$$(1.6)\qquad \begin{aligned} R_{(\Delta)} &= \frac{E_{(\Delta)}A_{(\Delta)}}{\tilde{l}_{(\Delta)}}, & \tilde{R}_{(\Delta)} &= \frac{12E_{(\Delta)}\check{J}_{(\Delta)}}{\tilde{l}_{(\Delta)}l^2_{(\Delta)}}, & \check{R}_{(\Delta)} &= \frac{12E_{(\Delta)}\tilde{J}_{(\Delta)}}{\tilde{l}_{(\Delta)}l^2_{(\Delta)}}, \\ S_{(\Delta)} &= \frac{c_{(\Delta)}}{\tilde{l}_{(\Delta)}}, & \tilde{S}_{(\Delta)} &= \frac{E_{(\Delta)}\tilde{J}_{(\Delta)}}{\tilde{l}_{(\Delta)}}, & \check{S}_{(\Delta)} &= \frac{E_{(\Delta)}\check{J}_{(\Delta)}}{\tilde{l}_{(\Delta)}}. \end{aligned}$$

Here $\tilde{J}_{(\Delta)}$ and $\check{J}_{(\Delta)}$ are the basic central moments of inertia of the cross-section of the shell element with respect to the axes tangent and normal to the surface $\pi$; $A_{(\Delta)}$ is a cross-

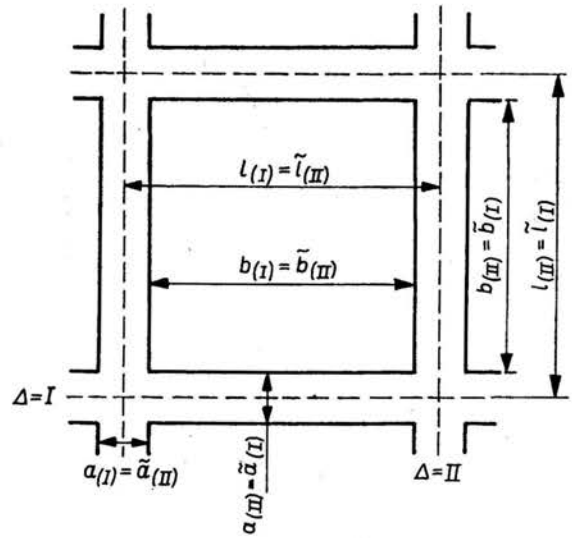


FIG. 3.

section area; $E_{(\Delta)}$ — the Young modulus and $c_{(\Delta)}$ is the rigidity of torsion of the cross-section for the family rods $(\Delta)$ $(\Delta = \text{I, II})$.

In the case of the perforated shell (comp. Fig. 3) the formulae (1.5) will take the form

$$(1.7)\qquad \begin{aligned} A^{KLMN}_{(\Delta)} &= t^K_{(\Delta)}t^L_{(\Delta)}\sum_{\Lambda} t^M_{(\Lambda)}t^N_{(\Lambda)}R_{(\Delta)(\Lambda)} + t^K_{(\Delta)}\tilde{t}^L_{(\Delta)}t^M_{(\Delta)}\tilde{t}^N_{(\Delta)}\tilde{R}_{(\Delta)}, \\ C^{KLMN}_{(\Delta)} &= t^K_{(\Delta)}\tilde{t}^L_{(\Delta)}\sum_{\Lambda} t^M_{(\Lambda)}\tilde{t}^N_{(\Lambda)}S_{(\Delta)(\Lambda)} + t^K_{(\Delta)}t^L_{(\Delta)}t^M_{(\Delta)}t^N_{(\Delta)}\tilde{S}_{(\Delta)}, \\ A^{KL}_{(\Delta)} &= t^K_{(\Delta)}t^L_{(\Delta)}\check{R}_{(\Delta)}, \quad C^{KL}_{(\Delta)} = t^K_{(\Delta)}t^L_{(\Delta)}\check{S}_{(\Delta)}, \end{aligned}$$

where

$$(1.8)\qquad \begin{aligned} [R_{(\Delta)(\Lambda)}] &= \frac{\delta}{1-\nu_{(\mathrm{I})}\nu_{(\mathrm{II})}}\begin{bmatrix}\tilde{E}_{(\mathrm{I})} & \nu_{(\mathrm{I})}\tilde{E}_{(\mathrm{I})} \\ \nu_{(\mathrm{II})}\tilde{E}_{(\mathrm{II})} & \tilde{E}_{(\mathrm{II})}\end{bmatrix}, \quad S_{(\Delta)(\Lambda)} = \frac{\delta^2}{12}R_{(\Delta)(\Lambda)}, \\ \tilde{R}_{(\Delta)} &= \frac{\delta\tilde{a}^2_{(\Delta)}\tilde{E}_{(\Delta)}}{b^2_{(\Delta)}+2(1+\nu)\tilde{a}^2_{(\Delta)}}, \quad \check{R}_{(\Delta)} = \frac{\delta^3\tilde{E}_{(\Delta)}}{b^2_{(\Delta)}}, \\ \tilde{S}_{(\Delta)} &= \frac{k_{(\Delta)}\delta^3\tilde{E}_{(\Delta)}}{2(1+\nu)}, \quad \check{S}_{(\Delta)} = \frac{\delta\tilde{a}^2_{(\Delta)}\tilde{E}_{(\Delta)}l_{(\Delta)}}{12b_{(\Delta)}}, \end{aligned}$$

with

$$\tilde{E}_{(\Delta)} = E\frac{\tilde{a}_{(\Delta)}}{\tilde{l}_{(\Delta)}}, \quad \nu_{(\Delta)} = \nu\frac{a_{(\Delta)}}{l_{(\Delta)}} \qquad (\Delta = \mathrm{I}, \mathrm{II}), \tag{1.9}$$

$E$ denotes the Young modulus, $\nu$ — the Poisson ratio, $\delta$ is the constant thickness of the shell and $k_{(\Delta)}$ is a numerical coefficient dependent on the ratio $\tilde{a}_{(\Delta)}\delta^{-1}$ (comp. for example [2]).

## 2. Equations of the net shells of revolution in a rotationally-symmetric state

Let $x^1 = \vartheta$, $x^2 = \varphi$, $R_1$ and $R_2$ the radii of curvatures in parallel and meridian directions (Fig. 4). The following relations hold:

$$\begin{gathered} a_{11} = R_0^2, \quad a_{22} = R_2^2, \quad a_{12} = a_{21} = a^{12} = a^{21} = 0, \quad a^{11} = \frac{1}{R_0^2}, \\ a^{22} = \frac{1}{R_2^2}, \quad b_{11} = R_1\sin^2\varphi, \quad b_{22} = R_2, \quad b_1^1 = \frac{1}{R_1}, \quad b_2^2 = \frac{1}{R_2}, \\ b_{12} = b_{21} = 0, \quad e_{12} = -e_{21} = R_0R_2, \quad e^1{}_2 = -e_2{}^1 = \frac{R_2}{R_0}, \\ b_1^2 = b_2^1 = 0, \quad e_1{}^2 = -e^2{}_1 = \frac{R_0}{R_2}. \end{gathered} \tag{2.1}$$

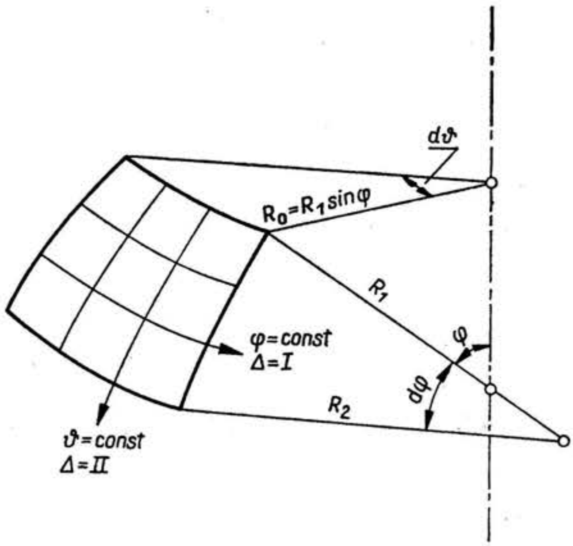


FIG. 4.

In the assumed system of coordinates $\vartheta$, $\varphi$ we evaluate the Christoffel symbols of the second kind

$$\begin{Bmatrix} 1 \\ 2\ 1 \end{Bmatrix} = \frac{R_2\cos\varphi}{R_0}, \quad \begin{Bmatrix} 2 \\ 1\ 1 \end{Bmatrix} = -\frac{R_0\cos\varphi}{R_2}, \quad \begin{Bmatrix} 2 \\ 2\ 2 \end{Bmatrix} = \frac{1}{R_2}\frac{dR_2}{d\varphi}. \tag{2.2}$$

The remaining Christoffel symbols equal zero. Besides, we use the relations

$$(2.3)\qquad \begin{Bmatrix} K \\ K\ 2 \end{Bmatrix} = \frac{1}{R_0R_2}\frac{d}{d\varphi}(R_0R_2), \qquad \begin{Bmatrix} K \\ K\ 1 \end{Bmatrix} = 0, \qquad \frac{dR_0}{d\varphi} = R_2\cos\varphi.$$

The formula $(2.3)_3$ has been derived from the Codazzi-Mainardi equations. Assume that a shell is in a rotationally-symmetric state, i.e. all quantities are functions of the angle $\varphi$ only $(d/d\vartheta(\ldots) = 0)$. Apart from that we restrict our considerations to the case in which (comp. page 244)

$$(2.4)\qquad b^1 \equiv h \equiv h^2 \equiv 0.$$

Inserting Eq. (2.4) into Eqs. (1.1) and assuming corresponding boundary conditions we obtain

$$(2.5)\qquad p^{12} \equiv p^{21} \equiv p^1 \equiv m^{11} \equiv m^{22} \equiv m^2 \equiv 0.$$

From Eqs. (2.5), (1.2)–(1.4) we find

$$(2.6)\qquad u_1 \equiv v \equiv v_2 \equiv 0.$$

After using the relations (2.1)–(2.3) three among the six equilibrium equations (1.1), which are not identities, assume the form

$$(2.7)\qquad \begin{aligned} &\frac{dp^{22}}{d\varphi} + \frac{1}{R_0R_2}\frac{d}{d\varphi}(R_0R_2)p^{22} + \frac{1}{R_2}\frac{dR_2}{d\varphi}p^{22} - \frac{R_0\cos\varphi}{R_2}p^{11} - \frac{1}{R_2}p^2 + b^2 = 0,\\ &\frac{dp^2}{d\varphi} + \frac{1}{R_0R_2}\frac{d}{d\varphi}(R_0R_2)p^2 + R_1p^{11}\sin^2\varphi + R_2p^{22} + b = 0,\\ &\frac{dm^{21}}{d\varphi} + \frac{1}{R_0R_2}\frac{d}{d\varphi}(R_0R_2)m^{21} + \frac{R_2}{R_0}p^2 + \frac{R_2\cos\varphi}{R_0}(m^{12} + m^{21}) - \frac{1}{R_1}m^1 + h^1 = 0. \end{aligned}$$

Introducing the physical quantities

$$(2.8)\qquad \begin{aligned} &N_1 = R_0^2p^{11}, \quad N_2 = R_2^2p^{22}, \quad Q = R_2p^2, \quad q_2 = R_2b^2,\\ &q = b, \quad M_1 = -R_0R_2m^{12}, \quad M_2 = R_0R_2m^{21}, \quad M = R_0m^1, \quad m = R_0h^1 \end{aligned}$$

to the system of equations (2.7), we have

$$(2.9)\qquad \begin{aligned} &\frac{d}{d\varphi}(R_0N_2) - R_2\cos\varphi N_1 - R_0Q + R_0R_2q_2 = 0,\\ &\frac{d}{d\varphi}(R_0Q) + R_2\sin\varphi N_1 + R_0N_2 + R_0R_2q = 0,\\ &\frac{d}{d\varphi}(R_0M_2) - R_2\cos\varphi M_1 - R_2\sin\varphi M + R_0R_2Q + R_0R_2m = 0. \end{aligned}$$

The non-zero components of the state of strain, after using Eqs. (2.1)–(2.3) and (2.6) may be written in the form

$$(2.10)\qquad \begin{aligned} &\gamma_{11} = R_0(v\cos\varphi - w\sin\varphi), \quad \gamma_{22} = R_2\left(\frac{dv}{d\varphi} - w\right),\\ &\gamma_2 = R_2(\chi - \theta), \quad \chi = \frac{1}{R_2}\left(\frac{dw}{d\varphi} + v\right), \quad \varkappa_1 = \theta\sin\varphi,\\ &\varkappa_{12} = -R_2\theta\cos\varphi, \quad \varkappa_{21} = R_0\frac{d\theta}{d\varphi}, \end{aligned}$$

where the following notations were introduced:

$$w = u, \quad v = u^2 R_2 = \frac{u_2}{R_2}, \quad \theta = v^1 R_0 = \frac{v_1}{R_0}. \tag{2.11}$$

Consider the net of the curves ($\Delta$) on the rotational surface composed of the family of parallels ($\Delta$ = I) and meridians ($\Delta$ = II). Then the following relations hold (comp. [1], page 29):

$$\begin{aligned} &t^1_{(I)} = \frac{1}{R_0}, \quad t^2_{(I)} = 0, \quad \tilde{t}^1_{(I)} = 0, \quad \tilde{t}^2_{(I)} = \frac{1}{R_2}, \\ &t^1_{(II)} = 0, \quad t^2_{(II)} = \frac{1}{R_2}, \quad \tilde{t}^1_{(II)} = -\frac{1}{R_0}, \quad \tilde{t}^2_{(II)} = 0. \end{aligned} \tag{2.12}$$

Using Eqs. (2.10)–(2.12), (2.8) and (1.2), (1.4)–(1.7) the constitutive equations assume the form:

for rod shells

$$\begin{aligned} &N_1 = \frac{R_{(I)}}{R_1}(v\operatorname{ctg}\varphi - w), \quad N_2 = \frac{R_{(II)}}{R_2}\left(\frac{dv}{d\varphi} - w\right), \quad Q = \check{R}_{(II)}(\chi - \theta), \\ &M_1 = \frac{\tilde{S}_{(I)}}{R_1}\theta\operatorname{ctg}\varphi, \quad M_2 = \frac{\tilde{S}_{(II)}}{R_2}\frac{d\theta}{d\varphi}, \quad M = \frac{\check{S}_{(I)}}{R_1}\theta, \end{aligned} \tag{2.13}$$

and perforated shells

$$\begin{aligned} N_1 &= C_{(I)}\left[\frac{v\operatorname{ctg}\varphi - w}{R_1} + \frac{\nu_{(I)}}{R_2}\left(\frac{dv}{d\varphi} - w\right)\right], \\ N_2 &= C_{(II)}\left[\frac{\nu_{(II)}}{R_1}(v\operatorname{ctg}\varphi - w) + \frac{1}{R_2}\left(\frac{dv}{d\varphi} - w\right)\right], \\ M_1 &= D_{(I)}\left[\frac{\theta\operatorname{ctg}\varphi}{R_1} + \frac{\nu_{(I)}}{R_2}\frac{d\theta}{d\varphi}\right], \\ M_2 &= D_{(II)}\left[\frac{\nu_{(II)}}{R_1}\theta\operatorname{ctg}\varphi + \frac{1}{R_2}\frac{d\theta}{d\varphi}\right], \\ Q &= \check{R}_{(II)}(\chi - \theta), \quad M = \frac{\check{S}_{(I)}}{R_1}\theta, \end{aligned} \tag{2.14}$$

where

$$C_{(\Delta)} = \frac{\delta\tilde{E}_{(\Delta)}}{1 - \nu_{(I)}\nu_{(II)}}, \quad D_{(\Delta)} = \frac{C_{(\Delta)}\delta^2}{12}. \tag{2.15}$$

After solving the given boundary-value problem, i.e. after evaluating the static quantities $N_1, N_2, Q, M_1, M_2, M$ and the geometrical quantities $w, v, \theta$, the real forces and moments appearing in a middle section of the net shell element are determined from the relations (1.2). Taking into account Eqs. (2.8) and (2.12), we have

$$
(2.16)\qquad \begin{aligned} P_{(I)} &= N_1 \tilde{l}_{(I)}, & P_{(II)} &= N_2 \tilde{l}_{(II)}, & \check{P}_{(II)} &= Q\tilde{l}_{(II)},\\ \tilde{M}_{(I)} &= -M_1 \tilde{l}_{(I)}, & \tilde{M}_{(II)} &= -\tilde{M}_2 l_{(II)}, & \check{M}_{(I)} &= M\tilde{l}_{(I)}. \end{aligned}
$$

It should be noted that the quantities (2.11) and (2.16) have a physical meaning in the determined points of the surface $\pi$. Therefore, posing the boundary condition in a manner which is usually assumed in mechanics is, to some degree, not exact. However, on account of the dense domain of the net elements one may assume that the above fact has small influence on the solution.

Let us now transform the first two equilibrium equations (2.9) to a different form which will be more convenient in further considerations. We eliminate $N_1$ from Eqs. $(2.9)_{1,2}$ and the resulting equation integrate with respect to

$$
(2.17)\qquad R_0 N_2 \sin\varphi + R_0 Q \cos\varphi = -P^*,
$$

where

$$
(2.18)\qquad P^* = -(N_2 \sin\varphi + Q\cos\varphi)R_0|_{\varphi=\bar{\varphi}} + \int_{\bar{\varphi}}^{\varphi} R_0 R_2 (q_2 \sin\alpha + q\cos\alpha)\, d\alpha.
$$

From Eqs. (2.17) and $(2.9)_2$ one obtains

$$
(2.19)\qquad \begin{aligned} N_2 &= -Q\operatorname{ctg}\varphi - \frac{P^*}{R_1 \sin^2\varphi},\\ N_1 &= -\frac{1}{R_2}\frac{d}{d\varphi}(R_1 Q) + \frac{P^*}{R_2 \sin^2\varphi} - R_1 q. \end{aligned}
$$

Next we reduce the system of equilibrium equations (2.19), $(2.9)_3$ and the relations (2.13) or (2.14) to the system of three differential equations containing the unknown variables $(\theta, v)$ of geometrical kind and one unknown of static kind — $Q$. We assume that the loads $q_2, q, m$ are arbitrarily distributed along the meridian. We shall consider the rod shell and the perforated shell.

## 3. System of differential equations for rod shells

Using the relation $(2.13)_{1,2}$ and $(2.10)_4$ we find

$$
(3.1)\qquad \begin{aligned} \frac{dv}{d\varphi} - v\operatorname{ctg}\varphi &= \frac{N_2 R_2}{R_{(II)}} - \frac{N_1 R_1}{R_{(I)}},\\ \frac{d}{d\varphi}\left(\frac{N_1 R_1}{R_{(I)}}\right) &= \left(\frac{dv}{d\varphi} - v\operatorname{ctg}\varphi\right)\operatorname{ctg}\varphi - R_2\chi,\\ R_2(\chi-\theta) &= -\frac{d}{d\varphi}\left(\frac{N_1 R_1}{R_{(I)}}\right) + \left(\frac{N_2 R_2}{R_{(II)}} - \frac{N_1 R_1}{R_{(I)}}\right)\operatorname{ctg}\varphi - R_2\theta. \end{aligned}
$$

On the basis of Eqs. $(3.1)_{1,3}$, after applying Eqs. (2.19) and $(2.13)_3$, we obtain two differential equations

$$\frac{dv}{d\varphi} - v\,\mathrm{ctg}\,\varphi = \frac{1}{R_{(I)}}\left[\frac{R_1}{R_2}\frac{dU}{d\varphi} - \frac{R_{(I)}R_2}{R_{(II)}R_1}U\,\mathrm{ctg}\,\varphi - \left(\frac{R_{(I)}R_2}{R_{(II)}R_1} + \frac{R_1}{R_2}\right)\frac{P^*}{\sin^2\varphi} + R_1^2 q\right],$$

(3.2)

$$\frac{d}{d\varphi}\left(\frac{1}{R_{(I)}}\frac{R_1}{R_2}\frac{dU}{d\varphi}\right) + \frac{1}{R_{(I)}}\frac{R_1}{R_2}\frac{dU}{d\varphi}\mathrm{ctg}\,\varphi - \frac{1}{R_{(II)}}\frac{R_2}{R_1}U\,\mathrm{ctg}^2\varphi - \frac{1}{\check{R}_{(II)}}\frac{R_2}{R_1}U - R_2\theta = \left(\frac{R_1}{R_{(I)}R_2} + \frac{R_2}{R_{(II)}R_1}\right)\frac{P^*}{\sin^2\varphi}\mathrm{ctg}\,\varphi + \frac{d}{d\varphi}\left(\frac{R_1}{R_{(I)}R_2}\frac{P^*}{\sin^2\varphi}\right) - \frac{d}{d\varphi}\left(\frac{1}{R_{(I)}}R_1^2 q\right) - \frac{1}{R_{(I)}}R_1^2 q\,\mathrm{ctg}\,\varphi,$$

where

$$U = R_1 Q. \tag{3.3}$$

Inserting Eqs. $(2.13)_{4-6}$ into Eq. $(2.9)_3$ we obtain the third differential equation

$$\frac{d}{d\varphi}\left(\tilde{S}_{(II)}\frac{R_1}{R_2}\frac{d\theta}{d\varphi}\right) + \tilde{S}_{(II)}\frac{R_1}{R_2}\frac{d\theta}{d\varphi}\mathrm{ctg}\,\varphi - \tilde{S}_{(I)}\frac{R_2}{R_1}\theta\,\mathrm{ctg}^2\varphi - \check{S}_{(I)}\frac{R_2}{R_1}\theta + R_2 U = -R_1 R_2 m. \tag{3.4}$$

The system of equations $(3.2)_2$, (3.4) may be written in a simpler form

(3.5)

$$L^*\left(\frac{1}{R_{(I)}}, \frac{1}{R_{(II)}}, \frac{1}{\check{R}_{(II)}};\ U\right) - R_2\theta = F^*,$$

$$L^*(\tilde{S}_{(II)}, \tilde{S}_{(I)}, \check{S}_{(I)}; \theta) + R_2 U = -R_1 R_2 m,$$

where $L^*$ is the following ordinary differential operator with variable coefficients

$$L^*[\alpha, \beta, \gamma; (\ldots)] = \frac{d}{d\varphi}\left[\alpha\frac{R_1}{R_2}\frac{d(\ldots)}{d\varphi}\right] + \alpha\frac{R_1}{R_2}\frac{d(\ldots)}{d\varphi}\mathrm{ctg}\,\varphi - \beta\frac{R_2}{R_1}(\ldots)\mathrm{ctg}^2\varphi - \gamma\frac{R_2}{R_1}(\ldots), \tag{3.6}$$

while $F^*$ is

$$F^* = \frac{d}{d\varphi}\left(\frac{1}{R_{(I)}}\frac{R_1}{R_2}\frac{P^*}{\sin^2\varphi}\right) + \left(\frac{R_1}{R_{(I)}R_2} + \frac{R_2}{R_{(II)}R_1}\right)\frac{P^*}{\sin^2\varphi}\mathrm{ctg}\,\varphi - \frac{d}{d\varphi}\left(\frac{1}{R_{(I)}}R_1^2 q\right) - \frac{1}{R_{(I)}}R_1^2 q\,\mathrm{ctg}\,\varphi. \tag{3.7}$$

The system of equations (3.5) and $(3.2)_1$ constitutes a complete set of differential equations of the problem under consideration. After its solution one may, on the basis of

Eqs. (3.3), (2.19), $(2.13)_{4-6}$, evaluate $Q, N_1, N_2, M_1, M_2$ and $M$. The geometrical quantity $w$ is determined from Eq. $(2.13)_1$

$$w = v\,\mathrm{ctg}\varphi - \frac{R_1 N_1}{R_{(1)}}. \tag{3.8}$$

In the following we shall derive the basic differential equations and relations for two technologically important cases, namely for the spherical shell and conical shell. Making use of a corresponding limit transition we shall obtain the equations and relations for a circular grid.

### Spherical shell

In this case we have $R_1 = R_2 = R = \mathrm{const}$. We write the system of equations $(3.2)_1$, (3.5) in the form

$$\frac{dv}{d\varphi} - v\,\mathrm{ctg}\varphi = \frac{R}{R_{(1)}}\left[\frac{dQ}{d\varphi} - \frac{R_{(1)}}{R_{(11)}}Q\,\mathrm{ctg}\varphi + \left(1+\frac{R_{(1)}}{R_{(11)}}\right)\frac{P_1^*}{\sin^2\varphi} + Rq\right],$$

$$L_1\left(\frac{1}{R_{(1)}}, \frac{1}{R_{(11)}}, \frac{1}{\check{R}_{(11)}}; Q\right) - \theta = F_1^*, \tag{3.9}$$

$$L_1(\tilde{S}_{(11)}, \tilde{S}_{(1)}, \check{S}_{(1)}; \theta) + R^2 Q = -R^2 m,$$

where

$$L_1[\alpha, \beta, \gamma; (\ldots)] = \frac{d}{d\varphi}\left[\alpha\frac{d(\ldots)}{d\varphi}\right] + \alpha\frac{d(\ldots)}{d\varphi}\mathrm{ctg}\varphi - \beta(\ldots)\mathrm{ctg}^2\varphi - \gamma(\ldots),$$

$$F_1^* = \frac{d}{d\varphi}\left(\frac{1}{R_{(1)}}\frac{P_1^*}{\sin^2\varphi}\right) + \left(\frac{1}{R_{(1)}} + \frac{1}{R_{(11)}}\right)\frac{P_1^*}{\sin^2\varphi}\mathrm{ctg}\varphi - R\frac{d}{d\varphi}\left(\frac{q}{R_{(1)}}\right) - \frac{R}{R_{(1)}}q\,\mathrm{ctg}\varphi, \tag{3.10}$$

with

$$P_1^* = \frac{P^*}{R} = -(N_2\sin^2\varphi + Q\sin\varphi\cos\varphi)|_{\varphi=\bar{\varphi}} + R\int_{\bar{\varphi}}^{\varphi}(q_2\sin^2\alpha + q\sin\alpha\cos\alpha)\,d\alpha. \tag{3.11}$$

The form of the relations (2.19), $(2.13)_{4-6}$, (3.8) essentially does not simplify.

### Conical shell

The conical shell constitutes a degenerated case since $\varphi = \mathrm{const}$, and $R_2 = \infty$. Let us introduce a new independent variable $y$ which denotes a distance of an arbitrary point lying on the cone generator from the apex of the cone. Then the following relations hold:

$$R_2\,d\varphi = dy, \quad R_1 = y\,\mathrm{ctg}\varphi. \tag{3.12}$$

Taking into account Eq. (3.12) we transform the equations and relations (2.9), (2.13), (2.17), (2.19), $(3.2)_1$ and (3.5) to the form

$$\frac{d}{dy}(yN_2)-N_1+yq_2 = 0,$$

(3.13)
$$\frac{d}{dy}(yQ)+N_1\operatorname{tg}\varphi+yq = 0,$$

$$\frac{d}{dy}(yM_2)-M_1-M\operatorname{tg}\varphi+yQ+ym = 0;$$

(3.14)
$$N_1 = \frac{R_{(I)}}{y}(v-w\operatorname{tg}\varphi), \qquad N_2 = R_{(II)}\frac{dv}{dy}, \qquad Q = \check{R}_{(II)}\left(\frac{dw}{dy}-\theta\right),$$
$$M_1 = \frac{\tilde{S}_{(I)}}{y}\theta, \qquad M_2 = \tilde{S}_{(II)}\frac{d\theta}{dy}, \qquad M = \frac{\check{S}_{(I)}}{y}\theta\operatorname{tg}\varphi;$$

(3.15)
$$N_1 = -\frac{d}{dy}(yQ)\operatorname{ctg}\varphi-yq\operatorname{ctg}\varphi,$$
$$N_2 = -Q\operatorname{ctg}\varphi-\frac{P_2^*}{y\sin\varphi\cos\varphi};$$

(3.16)
$$P_2^* = -\bar{y}(N_2\sin\varphi\cos\varphi+Q\cos^2\varphi)|_{y=\bar{y}}+\int_{\bar{y}}^{y} t(q_2\sin\varphi\cos\varphi+q\cos^2\varphi)\,dt;$$

$$\frac{dv}{dy} = -\frac{1}{R_{(II)}}\left(Q+\frac{P_2^*}{y\cos^2\varphi}\right)\operatorname{ctg}\varphi,$$

(3.17)
$$L_2\left[\frac{1}{R_{(I)}}, \frac{1}{R_{(II)}}, \frac{1}{\check{R}_{(II)}}; (yQ)\right]-\theta\operatorname{tg}^2\varphi = F_2^*,$$
$$L_2[\tilde{S}_{(II)}, \tilde{S}_{(I)}, \check{S}_{(I)}; \theta]+yQ = -ym,$$

where

(3.18)
$$L_2[\alpha,\beta,\gamma;(\ldots)] = \frac{d}{dy}\left[\alpha y\frac{d(\ldots)}{dy}\right]-\frac{\beta}{y}(\ldots)-\frac{\gamma}{y}(\ldots)\operatorname{tg}^2\varphi,$$
$$F_2^* = \frac{1}{R_{(II)}}\frac{1}{y}\frac{P_2^*}{\cos^2\varphi}-\frac{d}{dy}\left(\frac{y^2q}{R_{(I)}}\right).$$

A certain limit case may be of great interest. Tending in the equations and expressions (3.13)–(3.17) with $\varphi$ to zero, we obtain in the limit the corresponding equations and relations for a circular grid. Equations $(3.13)_2$, (3.15), $(3.17)_{1,2}$ lead to the equilibrium condition

(3.19)
$$\frac{d}{dy}(yQ)+yq = 0.$$

Equation $(3.17)_3$ takes the form

(3.20)
$$\frac{d}{dy}\left(\tilde{S}_{(II)}\frac{d\theta}{dy}\right)-\frac{\tilde{S}_{(I)}}{y}\theta+yQ = -ym.$$

This equation describes the plate state of the grid. The shield state equations may be found by substituting the expressions $(3.14)_{1,2}$ to $(3.13)_1$:

$$\frac{d}{dy}\left(R_{(\mathrm{II})}\frac{dv}{dy}\right)-\frac{R_{(\mathrm{I})}}{y}v=-yq_2. \tag{3.21}$$

**4. Differential equations for perforated shells**

From the formulae $(2.14)_{1,2}$ we calculate

$$\begin{aligned}
&v\operatorname{ctg}\varphi-w=\frac{R_1}{\delta}\left(\frac{N_1}{\tilde{E}_{(\mathrm{I})}}-\frac{\nu_{(\mathrm{I})}N_2}{\tilde{E}_{(\mathrm{II})}}\right),\\
&\frac{dv}{d\varphi}-w=\frac{R_2}{\delta}\left(\frac{N_2}{\tilde{E}_{(\mathrm{II})}}-\frac{\nu_{(\mathrm{II})}N_1}{\tilde{E}_{(\mathrm{I})}}\right),\\
&\frac{dv}{d\varphi}-v\operatorname{ctg}\varphi=\frac{R_2}{\delta}\left(\frac{N_2}{\tilde{E}_{(\mathrm{II})}}-\frac{\nu_{(\mathrm{II})}N_1}{\tilde{E}_{(\mathrm{I})}}\right)-\frac{R_1}{\delta}\left(\frac{N_1}{\tilde{E}_{(\mathrm{I})}}-\frac{\nu_{(\mathrm{I})}N_2}{\tilde{E}_{(\mathrm{II})}}\right),\\
&\left(\frac{dv}{d\varphi}-v\operatorname{ctg}\varphi\right)\operatorname{ctg}\varphi-R_2\chi=\frac{d}{d\varphi}\left(\frac{R_1N_1}{\delta\tilde{E}_{(\mathrm{I})}}\right)-\frac{d}{d\varphi}\left(\frac{R_1\nu_{(\mathrm{I})}N_2}{\delta\tilde{E}_{(\mathrm{II})}}\right).
\end{aligned} \tag{4.1}$$

On the basis of Eqs. $(4.1)_{3,4}$ and (2.19), $(2.14)_5$ we obtain the differential equations

$$\begin{aligned}
&\frac{dv}{d\varphi}-v\operatorname{ctg}\varphi=\frac{1}{\delta}\Bigg[\frac{\nu_{(\mathrm{II})}}{\tilde{E}_{(\mathrm{I})}}\frac{dU}{d\varphi}+\frac{1}{\tilde{E}_{(\mathrm{I})}}\frac{R_1}{R_2}\frac{dU}{d\varphi}-\frac{1}{\tilde{E}_{(\mathrm{II})}}\left(\frac{R_2}{R_1}+\nu_{(\mathrm{I})}\right)U\operatorname{ctg}\varphi\\
&\quad-\frac{1}{\tilde{E}_{(\mathrm{II})}}\left(\frac{R_2}{R_1}+\nu_{(\mathrm{I})}\right)\frac{P^*}{\sin^2\varphi}-\frac{1}{\tilde{E}_{(\mathrm{I})}}\left(\frac{R_1}{R_2}+\nu_{(\mathrm{II})}\right)\frac{P^*}{\sin^2\varphi}+\frac{R_2}{\tilde{E}_{(\mathrm{I})}}\left(\nu_{(\mathrm{II})}+\frac{R_1}{R_2}\right)R_1q\Bigg],\\
&\frac{d}{d\varphi}\left(\frac{R_1}{R_2}\frac{1}{\tilde{E}_{(\mathrm{I})}}\frac{dU}{d\varphi}\right)-\frac{d}{d\varphi}\left(\frac{\nu_{(\mathrm{I})}}{\tilde{E}_{(\mathrm{II})}}U\operatorname{ctg}\varphi\right)+\frac{\nu_{(\mathrm{II})}}{\tilde{E}_{(\mathrm{I})}}\frac{dU}{d\varphi}\operatorname{ctg}\varphi+\frac{1}{\tilde{E}_{(\mathrm{I})}}\frac{R_1}{R_2}\frac{dU}{d\varphi}\operatorname{ctg}\varphi\\
&\quad-\frac{1}{\tilde{E}_{(\mathrm{II})}}\left(\frac{R_2}{R_1}+\nu_{(\mathrm{I})}\right)U\operatorname{ctg}^2\varphi-\frac{R_2}{R_1}\frac{\delta}{\check{R}_{(\mathrm{II})}}U-R_2\delta\theta=\frac{d}{d\varphi}\left(\frac{R_1}{R_2}\frac{1}{\tilde{E}_{(\mathrm{I})}}\frac{P^*}{\sin^2\varphi}\right)\\
&\quad+\frac{d}{d\varphi}\left(\frac{\nu_{(\mathrm{I})}}{\tilde{E}_{(\mathrm{II})}}\frac{P^*}{\sin^2\varphi}\right)+\left[\frac{1}{\tilde{E}_{(\mathrm{II})}}\left(\frac{R_2}{R_1}+\nu_{(\mathrm{I})}\right)+\frac{1}{\tilde{E}_{(\mathrm{I})}}\left(\frac{R_1}{R_2}+\nu_{(\mathrm{II})}\right)\right]\frac{P^*}{\sin^2\varphi}\operatorname{ctg}\varphi\\
&\quad-\frac{d}{d\varphi}\left(\frac{1}{\tilde{E}_{(\mathrm{I})}}R_1^2q\right)-\frac{1}{\tilde{E}_{(\mathrm{I})}}(R_2\nu_{(\mathrm{II})}+R_1)R_1q\operatorname{ctg}\varphi,
\end{aligned} \tag{4.2}$$

where

$$U=R_1Q. \tag{4.3}$$

After substituting Eqs. $(2.14)_{3-6}$ to the equilibrium equation $(2.9)_3$ we may find the third differential equation

$$\begin{aligned}
&\frac{d}{d\varphi}\left(D_{(\mathrm{II})}\frac{R_1}{R_2}\frac{d\theta}{d\varphi}\right)+\frac{d}{d\varphi}(D_{(\mathrm{II})}\nu_{(\mathrm{II})}\theta\operatorname{ctg}\varphi)+D_{(\mathrm{II})}\frac{R_1}{R_2}\frac{d\theta}{d\varphi}\operatorname{ctg}\varphi+D_{(\mathrm{II})}\nu_{(\mathrm{II})}\theta\operatorname{ctg}^2\varphi\\
&\quad-D_{(\mathrm{I})}\nu_{(\mathrm{I})}\frac{d\theta}{d\varphi}\operatorname{ctg}\varphi-D_{(\mathrm{I})}\frac{R_2}{R_1}\theta\operatorname{ctg}^2\varphi-\frac{R_2}{R_1}\check{S}_{(\mathrm{I})}\theta+R_2U=-R_1R_2m.
\end{aligned} \tag{4.4}$$

The system of equations $(4.2)_2$, (4.4) may be written in a compact form

$$(4.5)\quad \begin{aligned} &L^*\left(\frac{1}{\tilde{E}_{(I)}}, -\frac{\nu_{(I)}}{\tilde{E}_{(II)}}, \frac{\nu_{(II)}}{\tilde{E}_{(I)}}, \frac{1}{\tilde{E}_{(II)}}, \frac{\delta}{\check{R}_{(II)}}; U\right) - R_2\,\delta\theta = F^*,\\ &L^*(D_{(II)}, D_{(II)}\nu_{(II)}, -D_{(I)}\nu_{(I)}, D_{(I)}, \check{S}_{(I)}; \theta) + R_2 U = -R_1 R_2 m, \end{aligned}$$

where

$$(4.6)\quad L^*[\bar{\alpha}, \bar{\beta}, \bar{\gamma}, \bar{\delta}, \bar{\varepsilon}; (\dots)] = \frac{d}{d\varphi}\left[\bar{\alpha}\frac{R_1}{R_2}\frac{d(\dots)}{d\varphi}\right] + \frac{d}{d\varphi}[(\dots)\bar{\beta}]\operatorname{ctg}\varphi + \left(\bar{\alpha}\frac{R_1}{R_2} + \bar{\gamma}\right)\frac{d(\dots)}{d\varphi}\operatorname{ctg}\varphi - \frac{R_2}{R_1}\bar{\delta}(\dots)\operatorname{ctg}^2\varphi - \left(\bar{\beta} + \bar{\varepsilon}\frac{R_2}{R_1}\right)(\dots)$$

denotes an ordinary differential operator with variable coefficients and where $F^*$ is

$$(4.7)\quad F^* = \frac{d}{d\varphi}\left(\frac{R_1}{R_2}\frac{1}{\tilde{E}_{(I)}}\frac{P^*}{\sin^2\varphi}\right) + \frac{d}{d\varphi}\left(\frac{\nu_{(I)}}{\tilde{E}_{(II)}}\frac{P^*}{\sin^2\varphi}\right) - \frac{d}{d\varphi}\left(\frac{1}{\tilde{E}_{(I)}}R_1^2 q\right) + \left[\frac{1}{\tilde{E}_{(II)}}\left(\frac{R_2}{R_1} + \nu_{(I)}\right) + \frac{1}{\tilde{E}_{(I)}}\left(\frac{R_1}{R_2} + \nu_{(II)}\right)\right]\frac{P^*}{\sin^2\varphi}\operatorname{ctg}\varphi - \frac{R_2}{\tilde{E}_{(I)}}\left(\frac{R_1}{R_2} + \nu_{(II)}\right)R_1 q\operatorname{ctg}\varphi.$$

After solving the system of equations (4.5), $(4.2)_1$ we find $U$, $\theta$, $v$. Then, applying Eqs. (4.3), (2.19), $(2.14)_{3-6}$ we evaluate $Q, N_1, N_2, M_1, M_2, M$. The magnitude of $w$ is determined from Eq. $(4.1)_1$

$$(4.8)\quad w = v\operatorname{ctg}\varphi - \frac{R_1}{\delta}\left(\frac{N_1}{\tilde{E}_{(I)}} - \frac{\nu_{(I)}N_2}{\tilde{E}_{(II)}}\right).$$

It should be stressed out that under proper choice of $\bar{\alpha}$, $\bar{\beta}$, $\bar{\gamma}$, $\bar{\delta}$, $\bar{\varepsilon}$ and the remaining coefficients characterizing rigidity of structure the equations and relations derived above are valid for certain full-walled orthotropic shells of revolution.

We shall still derive the basic differential equations for two technologically important cases, i.e. for spherical and conical shells. Making use of the proper limit transition we shall obtain the equations and expressions for a circular perforated plate.

**Spherical shell**

In this case we have $R_1 = R_2 = R = \text{const}$. We write the differential equations $(4.2)_1$, (4.5) in the form

$$(4.9)\quad \begin{aligned} &\frac{dv}{d\varphi} - v\operatorname{ctg}\varphi = \frac{R}{\delta}\left[\frac{1+\nu_{(II)}}{\tilde{E}_{(I)}}\frac{dQ}{d\varphi} - \frac{1+\nu_{(I)}}{\tilde{E}_{(II)}}Q\operatorname{ctg}\varphi - \left(\frac{1+\nu_{(I)}}{\tilde{E}_{(II)}} + \frac{1+\nu_{(II)}}{\tilde{E}_{(I)}}\right)\frac{P_1^*}{\sin^2\varphi} + \frac{1+\nu_{(II)}}{\tilde{E}_{(I)}}Rq\right],\\ &L_1\left(\frac{1}{\tilde{E}_{(I)}}, -\frac{\nu_{(I)}}{\tilde{E}_{(II)}}, \frac{1+\nu_{(II)}}{\tilde{E}_{(I)}}, \frac{1}{\tilde{E}_{(II)}}, \frac{\delta}{\check{R}_{(II)}} - \frac{\nu_{(I)}}{\tilde{E}_{(II)}}; Q\right) - \delta\theta = F_1^*,\\ &L_1(D_{(II)}, D_{(II)}\nu_{(II)}, D_{(II)} - \nu_{(I)}D_{(I)}, D_{(I)}, \check{S}_{(I)} + D_{(II)}\nu_{(II)}; \theta) + R^2 Q = -R^2 m, \end{aligned}$$

where

$$L_1[\bar{\alpha}, \bar{\beta}, \bar{\gamma}, \bar{\delta}, \bar{\varepsilon}; (\ldots)] = \frac{d}{d\varphi}\left[\bar{\alpha}\frac{d(\ldots)}{d\varphi}\right] + \frac{d}{d\varphi}[(\ldots)\bar{\beta}]\mathrm{ctg}\varphi + \bar{\gamma}\frac{d(\ldots)}{d\varphi}\mathrm{ctg}\varphi - \bar{\delta}(\ldots)\mathrm{ctg}^2\varphi - \bar{\varepsilon}(\ldots),$$

(4.10)

$$F_1^* = \frac{d}{d\varphi}\left[\left(\frac{1}{\tilde{E}_{(I)}} + \frac{\nu_{(I)}}{\tilde{E}_{(II)}}\right)\frac{P_1^*}{\sin^2\varphi}\right] + \left(\frac{1+\nu_{(I)}}{\tilde{E}_{(II)}} + \frac{1+\nu_{(II)}}{\tilde{E}_{(I)}}\right)\frac{P_1^*}{\sin^2\varphi}\mathrm{ctg}\varphi - R\frac{d}{d\varphi}\left(\frac{1}{\tilde{E}_{(I)}}q\right) - \frac{R}{\tilde{E}_{(I)}}(1+\nu_{(II)})q\,\mathrm{ctg}\varphi,$$

and where $P_1^*$ is determined by Eq. (3.11).

The form of the relations (2.19), $(2.14)_{3,4,6}$, (4.8) essentially does not simplify.

**Conical shell**

In this case we have $\varphi = \mathrm{const}$, $R_2 = \infty$. Let us introduce a new variable $y$ as the distance of an arbitrary point lying on the generator of the cone from its apex. Then the following relations hold:

$$dy = R_2 d\varphi, \quad R_1 = y\,\mathrm{ctg}\varphi. \tag{4.11}$$

Taking into account Eq. (4.11) we transform the equations and relations (2.14), $(4.2)_1$, (4.5) to the form

(4.12)

$$N_1 = C_{(I)}\left(\frac{v - w\,\mathrm{tg}\varphi}{y} + \nu_{(I)}\frac{dv}{dy}\right), \quad N_2 = C_{(II)}\left(\nu_{(II)}\frac{v - w\,\mathrm{tg}\varphi}{y} + \frac{dv}{dy}\right),$$

$$M_1 = D_{(I)}\left(\frac{\theta}{y} + \nu_{(I)}\frac{d\theta}{dy}\right), \quad M_2 = D_{(II)}\left(\frac{\nu_{(II)}}{y}\theta + \frac{d\theta}{dy}\right),$$

$$M = \frac{\check{S}_{(I)}}{y}\theta\,\mathrm{tg}\varphi, \quad Q = \check{R}_{(II)}\left(\frac{dw}{dy} - \theta\right);$$

(4.13)

$$\frac{dv}{dy} = \frac{1}{\delta}\left[\frac{\nu_{(II)}}{\tilde{E}_{(I)}}\frac{d(yQ)}{dy} - \frac{Q}{\tilde{E}_{(II)}} - \frac{P_2^*}{y\tilde{E}_{(II)}\cos^2\varphi} + \frac{\nu_{(II)}}{\tilde{E}_{(I)}}yq\right]\mathrm{ctg}\varphi,$$

$$L_2\left(\frac{1}{\tilde{E}_{(I)}}, -\frac{\nu_{(I)}}{\tilde{E}_{(II)}}, \frac{\nu_{(II)}}{\tilde{E}_{(I)}}, \frac{1}{\tilde{E}_{(II)}}, \frac{\delta}{\check{R}_{(II)}}; yQ\right) - \delta\theta\,\mathrm{tg}^2\varphi = F_2^*,$$

$$L_2(D_{(II)}, D_{(II)}\nu_{(II)}, -D_{(I)}\nu_{(I)}, D_{(I)}, \check{S}_{(I)}; \theta) + yQ = -ym,$$

where

(4.14)

$$L_2\left[\bar{\alpha}, \bar{\beta}, \bar{\gamma}, \bar{\delta}, \bar{\varepsilon}; (\ldots)\right] = \frac{d}{dy}\left[\bar{\alpha}y\frac{d(\ldots)}{dy}\right] + \frac{d}{dy}[\bar{\beta}(\ldots)] + \bar{\gamma}\frac{d(\ldots)}{dy} - \frac{\bar{\delta}}{y}(\ldots) - \frac{\bar{\varepsilon}(\ldots)}{y}\mathrm{tg}^2\varphi,$$

$$F_2^* = \frac{d}{dy}\left(\frac{\nu_{(I)}}{\tilde{E}_{(II)}}\frac{P_2^*}{\cos^2\varphi}\right) + \frac{1}{\tilde{E}_{(II)}}\frac{P_2^*}{y\cos^2\varphi} - \frac{d}{dy}\left(\frac{1}{\tilde{E}_{(I)}}y^2q\right) - \frac{\nu_{(II)}}{\tilde{E}_{(I)}}yq.$$

$P_2^*$ is determined by means of the formula (3.16).

Similarly as in Sect. 3 of this paper we perform in the equations and relations (4.12)–(4.14) and (3.13), (3.15) the limit transition tending with $\varphi$ to zero. In this way we shall obtain the corresponding equations and relations for a perforated circular plate. The relations $(3.13)_2$, (3.15), $(4.13)_{1,2}$ lead to the equilibrium condition

$$Q = -\frac{1}{y}\int yq\,dy. \tag{4.15}$$

On the basis of Eq. $(4.13)_3$ we obtain the equation describing a plate state

$$\frac{d}{dy}\left(D_{(II)}y\frac{d\theta}{dy}\right)+\frac{d}{dy}(D_{(II)}\nu_{(II)}\theta)-D_{(I)}\nu_{(I)}\frac{d\theta}{dy}-\frac{D_{(I)}}{y}\theta+yQ = -ym. \tag{4.16}$$

According to Eq. $(4.12)_5$ $M \equiv 0$. Using Eq. $(4.12)_6$ we evaluate the quantity $w$

$$w = \int\left(\frac{Q}{\check{R}_{(II)}}+\theta\right)dy. \tag{4.17}$$

Substituting Eq. $(4.12)_{1,2}$ to the equilibrium equation $(3.13)_1$ we find the equation describing the shield state problem

$$\frac{d}{dy}\left(C_{(II)}y\frac{dv}{dy}\right)+\frac{d}{dy}(C_{(II)}\nu_{(II)}v)-C_{(I)}\nu_{(I)}\frac{dv}{dy}-\frac{C_{(I)}}{y}v = -yq_2. \tag{4.18}$$

The practical significance of the equations and relations derived will be demonstrated in the next papers in which the solution of some static boundary-value problems for rotationally-symmetric net shells will be presented.

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*Received June* 14, 1977.