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Abstract

We show how the basic Bellman and Zadeh's (1970) model of multistage decision making (control) in a fuzzy environment can be extended to account for human perceptions concerning its basic elements, i.e. the fuzzy constraints and fuzzy goals, by introducing objective (related more to measurements) and subjective (related more to perceptions) fuzzy constraints and fuzzy goals. To illustrate the extended, perception based model, we present a fuzzy socioeconomic sustainable regional development model initiated by Kacprzyk and Straszak (1984), and further developed by Kacprzyk (1997), Kacprzyk, Romero and Gomide (1999), etc. The model may be viewed as as example of how fuzzy logic, or – more generally – the computing with words paradigm, can help devise new more human consistent, perception based models.

Keywords: multistage decision making (control) under fuzziness, fuzzy dynamic programming, computing with words, perceptions, socio-economic planning, regional development.

1 Introduction

Traditionally, computing involves basically the manipulation of numbers that are, in a natural way, supplied by (objective and precise) measurements. Humans, however, evaluate and assess virtually all aspects chracterizing reality around them not by means of measurements but by employing perceptions. Though a pivital role of perceptions has been recognized for a long time in various domains of science, no formal, "computational" approach to deal with perceptions has been proposed.

Computing with words seems to be the first constructive yet simple enough attempt to devise a formal apparatus to calculate with perception. Its point of departure is natural: humans employ mostly words in computing and reasoning, arriving at conclusions expressed as words from premises expressed in a natural language or having the form of mental perceptions. As used by humans, words have fuzzy denotations, and the same applies to the role of words in computing with words.

Computing with words, as a general paradigm, has been proved to be successful in many areas, and the the best source of various approaches is here Zadeh and Kacprzyk's (1999) volumes. In this paper, we present its use in extending – by making it possible to reflect and express perceptions – Bellman and Zadeh's (1970) general approach to decision making in a fuzzy environment that is a framework for all fuzzy decision making, optimization, control, etc. models, and presumably the most widely used general fuzzy approach [cf. Kacprzyk's (1983, 1997) books]. Then, we will show as an example its use in a sustainable regional development planning model developed over the years by Kacprzyk and Straszak (1984), Kacprzyk (1997), Kacprzyk, Romero and Gomide (1999), etc.

2 Extending Bellman and Zadeh's approach to decision making and control under fuzziness

In Bellman and Zadeh's (1970) model, if $X = \{x\}$ is some set of possible *options* (alternatives, variants, choices, decisions, ...), then the *fuzzy goal* is defined as a fuzzy set G in X, characterized by its membership function $\mu_G : X \longrightarrow [0,1]$ such that $\mu_G(x) \in [0,1]$ specifies the grade of membership of a particular option $x \in X$ in the fuzzy goal G, and the *fuzzy constraint* is similarly defined as a fuzzy set C in the set of options X, characterized by $\mu_C : X \longrightarrow [0,1]$ such that $\mu_C(x) \in [0,1]$ specifies the grade of membership of a particular option $x \in X$ in the fuzzy constraint C.

The general problem formulation is: "Attain G and satisfy C" which leads to the *fuzzy decision*

$$\mu_D(x) = \mu_G(x) \wedge \mu_C(x), \qquad \text{for each } x \in X \tag{1}$$

where "$\wedge$" is the minimum that may be replaced by another appropriate operation (e.g., a t-norm).

The *maximizing decision* is defined as an $x^* \in X$ such that

$$\mu_D(x^*) = \max_{x \in X} \mu_D(x) \qquad (2)$$

The human factor is crucial in reality, and this implies that the satisfaction of constraints and attainment of goals have both an objective and subjective aspect. The Bellman and Zadeh's (1970) framework can therefore be extended by introducing: an *objective fuzzy goal* $\mu_{G_o}(x)$, a *subjective fuzzy goal* $\mu_{G_s}(x)$, an *objective fuzzy constraint* $\mu_{C_o}(x)$, and a *subjective fuzzy constraint* $\mu_{C_s}(x)$.

We wish to "Attain $[G_o$ and $G_s]$ and satisfy $[C_o$ and $C_s]$" which leads to the fuzzy decision

$$\mu_D(x) = [\mu_{G_o}(x) \wedge \mu_{G_s}(x)] \wedge [\mu_{C_o}(x) \wedge \mu_{C_s}(x)], \qquad \text{for each } x \in X \qquad (3)$$

and the *maximizing*, or *optimal* decision is defined as in (2).

This framework can be extended to handle multiple fuzzy constraints and fuzzy goals, and also fuzzy constraints and fuzzy goals defined in different spaces [cf. Kacprzyk (1997)]. Namely, if we have: $n_o > 1$ objective fuzzy goals – $G_o^1, \ldots, G_o^{n_o}$ defined in Y, $n_s > 1$ subjective fuzzy goals – $G_s^1, \ldots, G_s^{n_s}$ defined in Y, $m_o > 1$ objective fuzzy constraints – $C_o^1, \ldots, C_o^{m_o}$ defined in X, $m_s > 1$ subjective fuzzy constraints – $C_s^1, \ldots, C_s^{m_s}$ defined in X, and a function $f : X \longrightarrow Y$, $y = f(x)$, then

$$\begin{aligned} \mu_D(x) &= \\ &= (\mu_{G_o^1}[f(x)] \wedge \cdots \wedge \mu_{G_o^{n_o}}[f(x)]) \wedge (\mu_{G_s^1}[f(x)] \wedge \cdots \wedge \mu_{G_s^{n_s}}[f(x)]) \wedge \\ &\quad \wedge [\mu_{C_o^1}(x) \wedge \cdots \wedge \mu_{C_o^{m_o}}(x)] \wedge [\mu_{C_s^1}(x) \wedge \cdots \wedge \mu_{C_s^{m_s}}(x)] \wedge \\ &\quad \wedge [\mu_{C_s^1}(x) \wedge \cdots \wedge \mu_{C_s^{m_s}}(x)], \qquad \text{for each } x \in X \end{aligned} \qquad (4)$$

and the *maximizing decision* is defined as (2), i.e. $\mu_D(x^*) = \max_{x \in X} \mu_D(x)$.

3 Extending multistage decision making (control) in Bellman and Zadeh's setting

The *control process* proceeds basically as follows. The decision (control) space is $U = \{u\} = \{c_1, \ldots, c_m\}$, the state (output) space is $X = \{x\} = \{s_1, \ldots, s_n\}$, and both are finite. We start from an initial state $x_0 \in X$, apply a decision (control) $u_0 \in U$, which is subjected to a fuzzy constraint $\mu_{C^0}(u_0)$, and attain a state $x_1 \in X$ via a known state transition equation of the system under control S; a fuzzy goal $\mu_{G^1}(x_1)$ is imposed on x_1. Next, we apply u_1, subjected to $\mu_{C^1}(u_1)$, and attain x_2, subjected to $\mu_{G^2}(x_2)$, etc.

The (deterministic) system under control is described by a *state transition equation*

$$x_{t+1} = f(x_t, u_t), \qquad t = 0, 1, \ldots \qquad (5)$$

where $x_t, x_{t+1} \in X = \{s_1, \ldots, s_n\}$ are the states at t and $t+1$, respectively, and $u_t \in U = \{c_1, \ldots, c_m\}$ is the decision (control) at t.

At t, $t = 0, 1, \ldots$, $u_t \in U$ is subjected to a *fuzzy constraint* $\mu_{C^t}(u_t)$, and on $x_{t+1} \in X$ a *fuzzy goal* is imposed, $\mu_{G^{t+1}}(x_{t+1})$. The fixed and specified in advance *initial state* is $x_0 \in X$, and the *termination time* (planning horizon), $N \in \{1, 2, \ldots\}$, is finite, and fixed and specified in advance.

The *performance* of the particular decision making (control) stage t, $t = 0, 1, \ldots, N-1$, is given by

$$v_t = \mu_{C^t}(u_t) \wedge \mu_{G^{t+1}}(x_{t+1}) = \mu_{C^t}(u_t) \wedge \mu_{G^{t+1}}[f(x_t, u_t)] \tag{6}$$

while the *performance* of the whole multistage decision making (control) process is given by the fuzzy decision

$$\begin{aligned} \mu_D(u_0, \ldots, u_{N-1} \mid x_0) &= v_0 \wedge v_1 \wedge \ldots \wedge v_{N-1} = \\ &= [\mu_{C^0}(u_0) \wedge \mu_{G^1}(x_1)] \wedge \ldots \wedge [\mu_{C^{N-1}}(u_{N-1}) \wedge \mu_{G^N}(x_N)] \end{aligned} \tag{7}$$

The problem is to find an optimal sequence of decisions (controls) $u_0^*, \ldots, u_{N-1}^*$ such that

$$\mu_D(u_0^*, \ldots, u_{N-1}^* \mid x_0) = \max_{u_0, \ldots, u_{N-1} \in U} \mu_D(u_0, \ldots, u_{N-1} \mid x_0) \tag{8}$$

Kacprzyk's (1997a) book provides and wide coverage of various aspects and extensions to this basic formulation.

In case of an extention proposed in this paper and outlined in Section 2 in which the objective and subjective fuzzy constraints and fuzzy goals are assumed, we have, at each $t = 0, 1, \ldots, N-1$: an objective fuzzy constraint $\mu_{C_t^t}(u_t)$ and a subjective fuzzy constraint $\mu_{C_s^t}(u_t)$, and an objective fuzzy goal $\mu_{G_o^{t+1}}(u_{t+1})$ and a subjective fuzzy constraint $\mu_{G_s^{t+1}}(u_{t+1})$.

The (extended) performance of the particular stage t, $t = 0, 1, \ldots, N-1$, is then given by

$$\overline{v}_t = [\mu_{C_o^t}(u_t) \wedge \mu_{C_s^t}(u_t)] \wedge [\mu_{G_o^t}(x_t) \wedge \mu_{G_s^t}(x_t)] \tag{9}$$

which can be schematically shown as in Figure 1.

The (extended) performance of the whole multistage decision making (control) process is then given by the fuzzy decision

$$\begin{aligned} \mu_{\overline{D}}(u_0, \ldots, u_{N-1} \mid tx_0) &= \overline{v}_0 \wedge \overline{v}_1 \wedge \ldots \wedge \overline{v}_{N-1} = \\ &= \{[\mu_{C_o^0}(u_0) \wedge \mu_{C_s^0}(u_0)] \wedge [\mu_{G_o^1}(x_1) \wedge \mu_{G_s^1}(x_1)]\} \wedge \ldots \\ &= \wedge \{[\mu_{C_o^{N-1}}(u_{N-1}) \wedge \mu_{C_s^{N-1}}(u_{N-1})] \wedge [\mu_{G_o^N}(x_N) \wedge \mu_{G_s^N}(x_N)]\} \end{aligned} \tag{10}$$

and we seek again an $u_0^*, \ldots, u_{N-1}^*$ such that

$$\mu_{\overline{D}}(u_0^*, \ldots, u_{N-1}^* \mid x_0) = \max_{u_0, \ldots, u_{N-1} \in U} \mu_{\overline{D}}(u_0, \ldots, u_{N-1} \mid x_0) \tag{11}$$

There is an extremely relevant aspect related to the subjective fuzzy constraints and fuzzy goals. Consider subjective fuzzy goals in which this is presumably much more pronounced than in subjective fuzzy constraints. Namely,

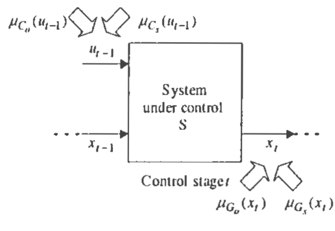

Figure 1: Evaluation of (extended) performance of decision making (control) stage t

it often happens that the (subjective) human satisfaction resulting from the attainment of some level of x_{t+1} – exemplified by a value of a life qnality index in Section 4 – depends not only on the "objectively attained" value but on how this value looks like in comparision with the past, future prospects, etc. For simplicity, let us concentrate on he past only.

The trajectory of the multistage decision making (control) process from $t = 0$ to a currect stage $t = k$ is

$$H_k = (x_0, u_0, C_o^0, C_s^0, x_1, G_o^1, G_s^1, \ldots, u_{k-1}, C_o^{k-1}, C_s^{k-1}, x_k, G_o^k, G_s^k) \qquad (12)$$

that is, it involves all aspects of what has happended in terms of decisions aplied, states attained, and objective and subjective opinions of how well the fuzzy constraints have been satisfied and fuzzy goals attained. However, it is often sufficient to take into account the reduced trajectory

$$h_k = (x_{k-2}, u_{k-2}, C_o^{k-2}, C_s^{k-2}, x_{k-1}, G_o^{k-1}, G_s^{k-1}, u_{k-1}, C_o^{k-1}, C_s^{k-1}, x_k, G_o^k, G_s^k) \qquad (13)$$

which only takes into account the current, $t = k$, and previous stage, $t = k - 1$. Let us assume this reduced trajectory.

A further simplification is that with a trajectory, or reduced trajectory, an evaluation function is associated, $E : S(H_k) \longrightarrow [0, 1]$ or $e : S(h_k) \longrightarrow [0, 1]$, where $S(H_k)$ and $S(h_k)$ are the sets of trajectories and reduced trajectories, respectively, such that $E(H_k) \in [0, 1]$ and $e(h_k) \in [0, 1]$ denote the satisfaction of the past development, from 1 for full satisfaction to 0 for full dissatisfaction, through all intermediate values.

The subjective fuzzy constraints and huzzy goals are now:

- when the (reduced) trajectory is accounted for

$$\begin{cases} \mu_{C_o^k}(u_k \mid h_k) & \text{and} \quad \mu_{C_s^k}(u_k \mid h_k) \\ \mu_{G_o^{k+1}}(x_{k+1} \mid h_k) & \text{and} \quad \mu_{G_s^{k+1}}(x_{k+1} \mid h_k) \end{cases} \qquad (14)$$

- when the evaluation of the (reduced) trajectory is accounted for

$$\begin{cases} \mu_{C_o^k}[u_k \mid E(h_k)] & \text{and} \quad \mu_{C_s^k}[u_k \mid E(h_k)] \\ \mu_{G_o^{k+1}}[x_{k+1} \mid E(h_k)] & \text{and} \quad \mu_{G_s^{k+1}}[x_{k+1} \mid E(h_k)] \end{cases} \tag{15}$$

Problem (8) can be solved using the following two basic traditional techniques: dynamic programming, and branch-and-bound, and also using the two new ones: a neural network, and a genetic algorithm. We will only briefly show the use of dynamic programming, and refer the redear for an extensive coverage on this and other solution techniques to Kacprzyk's (1997a) book.

First, we rewrite (8) as to find $u_0^*, \ldots, u_{N-1}^*$ such that

$$\begin{aligned} &\mu_D(u_0^*, \ldots, u_{N-1} \mid x_0) = \\ &= \max_{u_0,\ldots,u_{N-1}} [\mu_{C^0}(u_0) \wedge \mu_{G^1}(x_1) \wedge \ldots \\ &\ldots \wedge \mu_{C^{N-1}}(u_{N-1}) \wedge \mu_{G^N}(f(x_{N-1}, u_{N-1}))] \end{aligned} \tag{16}$$

and then, since

$$\mu_{C^{N-1}}(u_{N-1}) \wedge \mu_{G^N}(f(x_{N-1}, u_{N-1}))$$

depends only on u_{N-1}, then the maximization with respect to $u_0, \ldots, u_{N-1}$ in (16) can be split into:

- the maximization with respect to $u_0, \ldots, u_{N-2}$, and
- the maximization with respect to u_{N-1},

written as

$$\begin{aligned} &\mu_D(u_0^*, \ldots, u_{N-1}^* \mid x_0) = \\ &= \max_{u_0,\ldots,u_{N-2}} \{\mu_{C^0}(u_0) \wedge \mu_{G^1}(x_1) \wedge \ldots \\ &\ldots \wedge \mu_{C^{N-2}}(u_{N-2}) \wedge \mu_{G^{N-1}}(x_{N-1}) \wedge \\ &\wedge \max_{u_{N-1}}[\mu_{C^{N-1}}(u_{N-1}) \wedge \mu_{G^N}(f(x_{N-1}, u_{N-1}))]\} \end{aligned} \tag{17}$$

which may be continued for u_{N-2}, u_{N-3}, etc.

This backward iteration leads to the following set of fuzzy dynamic programming recurrence equations:

$$\begin{cases} \mu_{\overline{G}^{N-i}}(x_{N-i}) = \\ \quad = \max_{u_{N-i}}[\mu_{C^{N-i}}(u_{N-i}) \wedge \mu_{G^{N-i}}(x_{N-i}) \wedge \mu_{\overline{G}^{N-i+1}}(x_{N-i+1})] \\ x_{N-i+1} = f(x_{N-i}, u_{N-i}); \qquad i = 0, 1, \ldots, N \end{cases} \tag{18}$$

where $\mu_{\overline{G}^{N-i}}(x_{N-i})$ is viewed as a fuzzy goal at control stage $t = N - i$ induced by the fuzzy goal at $t = N - i + 1$, $i = 0, 1, \ldots, N$; $\mu_{\overline{G}^N}(x_N) = \mu_{G^N}(x_N)$.

The $u_0, \ldots, u_{N-1}$ sought is given by the successive maximizing values of u_{N-i}, $i = 1, \ldots, N$ in (18) which are obtained as fnnctions of x_{N-i}, i.e. as an *optimal policy*, $a_{N-i} : X \longrightarrow U$, such that $u_{N-i} = a_{N-i}(x_{N-i})$.

It easy to notice that if we use the subjective fuzzy constraints and fuzzy goals to extend the above fuzzy dynamic programming model, then the very idea of dynamic programming, i.e. the use of backward iteration represented by the recurrence equations (18), prohibits the use of subjective fuzzy constraints and subjective fuzzy goals being function of the trajectory, or any evaluation of the trajectory, as both of them are somehow calculated on the basis of outcomes of control stages prior to those which have been accounted for so far while proceedings with backward iteration. Therefore, if we intend to employ fuzzy dynamic programming, as in this paper, we can only use the subjective fuzzy constraints and goals depending on the current value of decision (control) applied and state attained. The involvement of subjective fuzzy constraints and goals depending on the trajectory or its evaluation needs another approach as, e.g., the use of a genetic algorithm [cf. Kacprzyk (1996, 1997b, 1998)] or a neural network based approach by Francelin, Gomide and Kacprzyk (1995, 2001, 2002), Kacprzyk, and Francelin and Gomide (1998).

Therefore, by involving the line of reasoning (16)–(18), using the objective and subjective fuzzy constraints and fuzzy goals: $\mu_{C_o^{N-i}}(u_{N-i})$ and $\mu_{C_s^{N-i}}(u_{N-i})$, and $\mu_{G_o^{N-i+1}}(x_{N-i+1})$ and $\mu_{G_s^{N-i+1}}(x_{N-i+1})$, for $i = 1, 2, \ldots, N$, we arrive at the following set of (extended) dynamic programming recurrent equations:

$$\begin{cases} \mu_{\overline{G}^{N-i}}(x_{N-i}) = \\ \qquad = \max_{u_{N-i}} \{[\mu_{C_o^{N-i}}(u_{N-i}) \wedge \mu_{C_s^{N-i}}(u_{N-i})] \wedge \\ \qquad\qquad [\mu_{G_o^{N-i}}(x_{N-i}) \wedge \mu_{G_s^{N-i}}(x_{N-i}) \wedge \mu_{\overline{G}^{N-i+1}}(x_{N-i+1})]\} \\ x_{N-i+1} = f(x_{N-i}, u_{N-i}); \qquad i = 0, 1, \ldots, N \end{cases} \tag{19}$$

4 Sustainable socioeconomic regional development planning under fuzziness

Regional development is a problem of general importance but difficult to formalize and solve as it involves various aspects (political, economic, social, environmental, technological, etc.), different parties (inhabitants, authorities of different levels, formal and informal groups, etc.), and many entities that are difficult to precisely single out, define and quantify. To overcome these difficulties, the use of a fuzzy model was Kacprzyk and Straszak (1982a, b,1984), and then extended by Kacprzyk (1997a), and Kacprzyk, Francelin and Gomide (1998). They consider a (rural) region plagued by severe difficulties mainly related to a poor *life quality* perceived. Hence, life quality (or perception therof) should be improved, by some (mostly external) funds (investments) whose amount and their temporal distribution should be found. We will show now how the extended, perception based model developed above can be employed.

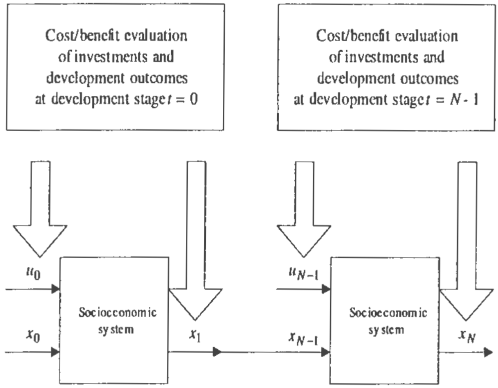

Figure 2: Essential elements of socioeconomic regional development

4.1 A multistage fuzzy decision making model of regional develoment planning

For our purposes the essence of socioeconomic regional development may be depicted as in Figure 2. The region is represented by a socioeconomic dynamic system under control whose state at development (planning) stage $t-1$, X_{t-1}, is characterized by a set of relevant socioeconomic life quality indicators. Then, the decision (investment), at $t-1$, u_{t-1}, changes X_{t-1} to X_t; $t = 1, \ldots, N$, and N is a finite, fixed and specified planning horizon.

The assessment of a planning stage t, $t = 1, \ldots, N$, is performed by accounting for both the "goodness" of the u_{t-1} applied (i.e. costs), and the "goodness" of the X_t attained (i.e. benefits); the former has to do with how well some constraints are satisfied, and the latter with how well some goals are attained. We will involve a subjective assessment for the attainment of fuzzy goals only.

First, the socioeconomic system is represented as in Figure 3. Its state (output) X_t is equated with a *life quality index* that consists of the following seven *life quality indicators* (i.e. $X_t = [x_t^1, \ldots, x_t^7]$):

- x_t^1 – economic quality (e.g., wages, salaries, income, ...),
- x_t^2 – environmental quality,
- x_t^3 – housing quality,
- x_t^4 – health service quality,
- x_t^5 – infrastructure quality,

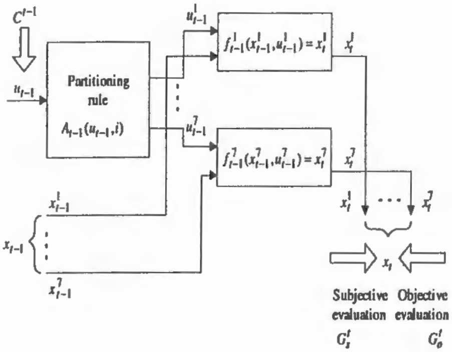

Figure 3: Basic elements of the socioeconomic system under control

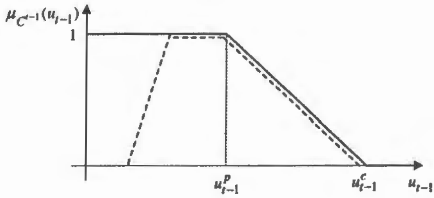

Figure 4: Fuzzy constraints on investment u_{t-1}

- x_t^6 – work opportunity,
- x_t^7 – leisure time opportunity,

The decision at $t-1$, u_{t-1} is investment, and we impose on u_{t-1} a fuzzy constraint $\mu_{C^{t-1}}(u_{t-1})$ in a piecewise linear form as shown in Figure 4 to be read as follows. The investment may be fully utilized up to u_{t-1}^p, hence $\mu_{C^{t-1}}(u_{t-1}) = 1$ for $0 < u_{t-1} < u_{t-1}^p$. However, this is usually insufficient and some additional contingency investment is needed, maximally up to u_{t-1}^c (the more the worse, of course). The fuzzy constraints are often as shown in the dotted line in Figure 4 in that too low a use of available investments should also be avoided, for "political" reasons.

The $t-1$, u_{t-1} is partitioned into $u_{t-1}^1, \ldots, u_{t-1}^7$, devoted to improve the respective life quality indicators, but we will assume here that this rule is fixed.

The temporal evolution of the particular life quality indicators is governed

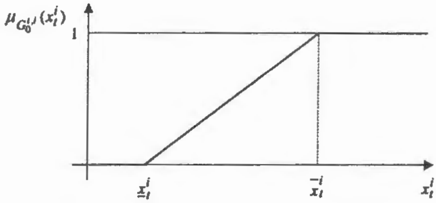

Figure 5: Objective fuzzy subgoal

by the state transition equation

$$x_t^i = f_{t-1}^i(x_{t-1}^i, u_{t-1}^i), \qquad i = 1, \ldots, 7; t = 1, \ldots, N \tag{20}$$

which may be derived by, e.g., using experts' opinions, past experience, mathematical models, etc.

The evaluation of development takes into account how well some predetermined goals are fulfilled, i.e. *effectiveness*, then be related to the investment spent, i.e. *efficiency* – cf. Kacprzyk's (1997a) book.

The effectiveness of regional development involves two aspects: the effectiveness of a particular development stage, and the effectiveness of the whole development.

The effectiveness of a particular development stage has both an objective and subjective aspect. The objective evaluation is basically the determination of how well the fuzzy constraints are fulfilled, and fuzzy goals are attained. The objective fuzzy goals concern desired values of the life quality indicators, i.e. concern objective entities; however, goal attainment is not clear-cut, and a fuzzy goal should rather be used.

For each life quality indicator at $t = 1, \ldots, N$, x_t^i, we define an *objective fuzzy subgoal* $G_o^{t,i}$ characterized by $\mu_{G_o^{t,i}}(x_t^i)$ as shown in Figure 5 to be read as follows: $G_o^{t,i}$ is fully satisfied for $x_t^i \geq \overline{x}_t^u$, where $\overline{x}_o^i$ is some *aspiration level* for the indicator x_t^i; therefore, $\mu_{G_o^{t,i}}(x_t^i) = 1$, for $x_t^i \geq \overline{x}_t^i$. Less preferable are $\underline{x}_t^i < x_t^i < \overline{x}_t^i$ for which $0 < \mu_{G_o^{t,i}}(x_t^i) < 1$, and $x_t^i \,\square\, \underline{x}_t^i$ are assumed to be impossible, hence $\mu_{G_o^{t,i}}(x_t^i) = 0$. Notice that an objective fuzzy (sub)goal may be relatively easily determined by experts by specifying two values only, $\underline{xt}_t^i$ and $\overline{x}_t^i$.

The objective evaluation of the life quality index at t, $X_t = [x_t^1, \ldots, x_t^7]$, is obtained by the aggregation of partial assessments of the particular life quality indicators, i.e.

$$\mu_{G_o^t}(X_t) = \mu_{G_o^{t,1}}(x_t^1) \wedge \ldots \wedge \mu_{G_o^{t,7}}(x_t^7) \tag{21}$$

and "$\wedge$" may be replaced here and later on by another suitable operation as, e.g., a t-norm [cf. Kacprzyk (1997a)] but this will not be considered here.

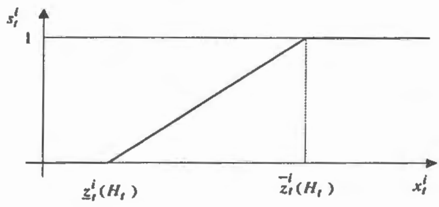

Figure 6: Partial social satisfaction

Basically, the use of "$\wedge$" (minimum) reflects a pessimistic, safety-first attitude, and a lack of substitutability (i.e. that a low value of one life quality indicator cannot be compensated by a higher value of another), which is often adequate.

Finally, note that the objective evaluation concerns more the authorities than the inhabitants by somehow "mechanically" checking the values of life quality indicators attained against some desired predetermined levels. The inhabitants' assessment of the "goodness" of development concerns in fact the (perception of) *social satisfaction* resulting from the life quality index attained. This is clearly subjective. The attained value of a particular life quality indicator at t, x_t^i, implies its corresponding partial social satisfaction s_t^i derived as in Figure 6, and its interpretation is basically as for the objective evaluation shown in Figure 5.

In general, both $\underline{z}_t^i$ and $\overline{z}_t^i$ may be functions of the trajectory (history) of development [cf. (12)]

$$H_t = [(X_1, s_1, \mu_{G_o^1}(X_1), \mu_{G_s^1}(s_1)), \ldots, (X_t, s_t, \mu_{G_o^t}(s_t), \mu_{G_s^t}(s_t))]$$

where $s_k = [s_k^1, \ldots, s_k^7], k = 1, \ldots, t$, is the social satisfaction resulting from X_k. Basically, if H_t is encouraging, then the inhabitants may become more demanding, and $\underline{z}_t^i(H_t)$ and $\overline{z}_t^i(H_t)$ may move up. On the other hand, if H_t is discouraging, then $\underline{z}_t^i(H_t)$ and $\overline{z}_t^i(H_t)$ may move down (cf. Kacprzyk, 1983, 1997a). Very often, however, one can limit the analysis to the reduced trajectory [cf. 13)]. This important aspect, discussed in Section 3, will not be considered here.

The social satisfaction at t is now

$$s_t = s_t^1 \wedge \ldots \wedge s_t^7 \tag{22}$$

where "$\wedge$" again reflects a pessimistic, safety-first attitude, and a lack of substitutability.

The social satisfaction s_t is subjected to a subjective fuzzy goal $\mu_{G_s^t}(s_t)$ which is meant similarly as its objective counterpart shown in Figure 5.

The effectiveness of t is meant as a relation of what has been attained (the life quality indices and their respective social satisfactions) to what has been "paid for" (the respective investments), i.e. is a benefit–cost relationship. Formally, the (fuzzy) effectiveness of stage t is expressed as

$$\mu_{E^t}(u_{t-1}, X_t, s_t) = \mu_{C^{t-1}}(u_{t-1}) \wedge \mu_{G_o^t}(X_t) \wedge \mu_{G_s^t}(s_t) \tag{23}$$

and the aggregation reflects the nature of a compromise between the interests of the authorities (for whom the fuzzy constraints and the objective fuzzy goal matter), and those of the inhabitants (for whom the subjective fuzzy goal, and to some extent the objective fuzzy goal, matter); the minimum reflects a safety-first attitude, hence a "more just" compromise.

Then, the effectiveness measures of the particular $t = 1, \ldots, N$, $\mu_{E^t}(u_{t-1}, X_t, s_t)$ given by (23), are aggregated to yield the fuzzy effectiveness measure for the whole development

$$\mu_E(H_N) = \mu_{E^1}(u_0, X_1, s_1) \wedge \ldots \wedge \mu_{E^N}(u_{N-1}, X_N, s_n) \tag{24}$$

The fuzzy decision is

$$\begin{aligned} &\mu_D(u_0, \ldots, u_{N-1} \mid X_0, B_N) = \\ &= [\mu_{C^0}(u_0) \wedge \mu_{G_o^1}(X_1) \wedge \mu_{G_s^1}(s_1)] \wedge \ldots \\ &\quad \ldots \wedge [\mu_{C^{N-1}}(u_{N-1}) \wedge \mu_{G_o^N}(X_N) \wedge \mu_{G_s^N}(s_N)] \end{aligned} \tag{25}$$

and it expresses some crucial compromises between, e.g.:

- the fuzzy constraints and (objective and subjective) fuzzy goals,
- the interests of the authorities and inhabitants, etc.

The problem is now to find an optimal sequence of controls (investments) $u_0^*, \ldots, u_{N-1}^*$ such that (under a given policy B_N; the optimization of policy is a separate problem which will not be cosnidered here):

$$\begin{aligned} &\mu_D(u_0^*, \ldots, u_{N-1}^* \mid X_0, B_N) = \\ &= \max_{u_0, \ldots, u_{u_{N-1}}} \{[\mu_{C^0}(u_0) \wedge \mu_{G_o^1}(X_1) \wedge \mu_{G_s^1}(s_1)] \wedge \ldots \\ &\quad \ldots \wedge [\mu_{C^{N-1}}(u_{N-1}) \wedge \mu_{G_o^N}(X_N) \wedge \mu_{G_s^N}(s_N)]\} \end{aligned} \tag{26}$$

For illustration we will show a simple example [cf. Kacprzyk (1997a)].

Example 1 The region, predominantly agricultural, has a population of ca. 120,000 inhabitants, and its arable land is ca. 450,000 acres. For simplicity, the region's development will be considered over the next 3 development stages (years, for simplicity). The life quality index consists of the four life quality indicators:

- x_t^{I} – average subsidies in US\$ per acre (per year),
- x_t^{II} – sanitation expenditures (water and sewage) in US\$ per capita (per year),

- x_t^{III} – health care expenditures in US$ per capita (per year), and
- x_t^{IV} – expenditures for paved roads (new roads and maintenance of the existing ones) in US$ (per year).

Suppose now that the investments are partitioned into parts devoted to the improvement of the above life quality indicators due to the fixed partitioning rule $A_{t-1}(u_{t-1}, i)$: 5% for subsidies, 25% for sanitation, 45% for health care, and 25% for infrastructure.

Let the initial, at $t = 0$, values of the life quality indicators be:

$$x_0^{\mathrm{I}} = 0.5 \qquad x_0^{\mathrm{II}} = 15 \qquad x_0^{\mathrm{III}} = 27 \qquad x_0^{\mathrm{IV}} = 1,700,000$$

For clarity, we will only take into account the following two *scenarios* (policies):

- Policy 1: $u_0 = \$8,000,000 \quad u_1 = \$8,000,000 \quad u_2 = \$8,000,000$
- Policy 2: $u_0 = \$7,500,000 \quad u_1 = \$8,000,000 \quad u_2 = \$8,500,000$

Under Policy 1 and Policy 2, the values of the life quality indicators attained are:

Policy 1:	Year(t)	u_t	x_t^{I}	x_t^{II}	x_t^{III}	x_t^{IV}
	0	\$8,000,000				
	1	\$8,000,000	0.88	16.7	30	\$2,000,000
	2	\$8,000,000	0.88	16.7	30	\$2,000,000
	3		0.88	16.7	30	\$2,000,000

Policy 2:	Year(t)	u_t	x_t^{I}	x_t^{II}	x_t^{III}	x_t^{IV}
	0	\$7,500,000				
	1	\$8,000,000	0.83	15.6	28.1	\$1,875,000
	2	\$8,500,000	0.88	16.7	30	\$2,000,000
	3		0.94	17.7	31.9	\$2,125,000

For the evaluation of the above two development trajectories, for simplicity and readability we will only take into account the *effectiveness* of development, and the objective evaluation only. The consecutive fuzzy constraints and objective fuzzy subgoals are assumed piecewise linear, i.e. their definition requires two values only (cf. Figure 4, and Figure 5): the aspiration level (i.e. the fully acceptable value) and the lowest (or highest) possible (still acceptable) value) which are:

$$
\begin{array}{llllll}
t & & & & & \\
0 & C^0: & u_0^p = \$7,500,000 & & & \\
 & & u_0^c = \$8,500,000 & & & \\
1 & C^1: & u_1^p = \$7,750,000 & & & \\
 & & u_1^c = \$9,000,000 & G_o^{1,\mathrm{I}}: \underline{x}_1^{\mathrm{I}} = 0.6 & & \overline{x}_1^{\mathrm{I}} = 0.85 \\
 & & & G_o^{1,\mathrm{II}}: \underline{x}_1^{\mathrm{II}} = 14 & & \overline{x}_1^{\mathrm{II}} = 16 \\
 & & & G_o^{1,\mathrm{III}}: \underline{x}_1^{\mathrm{III}} = 27 & & \overline{x}_1^{\mathrm{III}} = 29 \\
 & & & G_o^{1,\mathrm{IV}}: \underline{x}_1^{\mathrm{IV}} = \$1,800,000 & & \overline{x}_1^{\mathrm{IV}} = \$1,900,000 \\
2 & C^2: & u_2^p = \$8,000,000 & & & \\
 & & u_1^c = \$10,000,000 & G_o^{2,\mathrm{I}}: \underline{x}_2^{\mathrm{I}} = 0.7 & & \overline{x}_1^{\mathrm{I}} = 0.9 \\
 & & & G_o^{2,\mathrm{II}}: \underline{x}_2^{\mathrm{II}} = 15 & & \overline{x}_1^{\mathrm{II}} = 17 \\
 & & & G_o^{2,\mathrm{III}}: \underline{x}_2^{\mathrm{III}} = 28 & & \overline{x}_1^{\mathrm{III}} = 30 \\
 & & & G_o^{2,\mathrm{IV}}: \underline{x}_2^{\mathrm{IV}} = \$1,900,000 & & \overline{x}_2^{\mathrm{IV}} = \$2,000,000 \\
3 & & & G_o^{3,\mathrm{I}}: \underline{x}_3^{\mathrm{I}} = 0.75 & & \overline{x}_3^{\mathrm{I}} = 1 \\
 & & & G_o^{3,\mathrm{II}}: \underline{x}_3^{\mathrm{II}} = 16 & & \overline{x}_1^{\mathrm{II}} = 18.5 \\
 & & & G_o^{3,\mathrm{III}}: \underline{x}_3^{\mathrm{III}} = 29 & & \overline{x}_1^{\mathrm{III}} = 31 \\
 & & & G_o^{3,\mathrm{IV}}: \underline{x}_3^{\mathrm{IV}} = \$1,950,000 & & \overline{x}_3^{\mathrm{IV}} = \$2,100,000
\end{array}
$$

Using the "$\wedge$" to reflect a safety-first attitude, which is clearly preferable in the situation considered (a rural region plagued by the aging of the society, out-migration to neighboring urban areas, economic decay, etc.), the evaluation of the two investment policies is:

- Policy 1

$$
\begin{aligned}
&\mu_D(\$8,000,000; \$8,000,000; \$8,000,000 \mid .) = \\
&= \mu_{C^0}(\$8,000,000) \wedge (\mu_{G_o^{1,\mathrm{I}}}(0.88) \wedge \\
&\quad \wedge \mu_{G_o^{1,\mathrm{II}}}(16.7) \wedge \mu_{G_o^{1,\mathrm{III}}}(30) \wedge \mu_{G_o^{1,\mathrm{IV}}}(\$2,000,000)) \wedge \\
&\quad \wedge \mu_{C^1}(\$8,000,000) \wedge (\mu_{G_o^{2,\mathrm{I}}}(0.88) \wedge \\
&\quad \wedge \mu_{G_o^{2,\mathrm{II}}}(16.7) \wedge \mu_{G_o^{2,\mathrm{III}}}(30) \wedge \mu_{G_o^{2,\mathrm{IV}}}(\$2,000,000)) \wedge \\
&\quad \wedge \mu_{C^2}(\$8,000,000) \wedge (\mu_{G_o^{3,\mathrm{I}}}(0.88) \wedge \\
&\quad \wedge \mu_{G_o^{3,\mathrm{II}}}(16.7) \wedge \mu_{G_o^{3,\mathrm{III}}}(30) \wedge \mu_{G_o^{3,\mathrm{IV}}}(\$2,000,000)) = \\
&= 0.5 \wedge (1 \wedge 1 \wedge 1 \wedge 1) \wedge 0.8 \wedge \\
&\quad \wedge (0.9 \wedge 0.85 \wedge 1 \wedge 1) \wedge 1 \wedge (0.52 \wedge 0.28 \wedge 0.5 \wedge 0.33) = \\
&= 0.5 \wedge 0.8 \wedge 0.28 = 0.28
\end{aligned}
$$

- Policy 2

$$
\begin{aligned}
&\mu_D(\$7,500,000; \$8,000,000; \$8,500,000 \mid .) = \\
&= \mu_{C^0}(\$7,500,000) \wedge (\mu_{G_o^{1,\mathrm{I}}}(0.83) \wedge
\end{aligned}
$$

$$
\begin{aligned}
& \wedge \mu_{G_o^{1,\mathrm{II}}}(15.6) \wedge \mu_{G_o^{1,\mathrm{III}}}(28.1) \wedge \mu_{G_o^{1,\mathrm{IV}}}(\$1,875,000)) \wedge \\
& \wedge \mu_{C^1}(\$8,000,000) \wedge (\mu_{G_o^{2,\mathrm{I}}}(0.88) \wedge \\
& \wedge \mu_{G_o^{2,\mathrm{II}}}(16.7) \wedge \mu_{G_o^{2,\mathrm{III}}}(30) \wedge \mu_{G_o^{2,\mathrm{IV}}}(\$2,000,000)) \wedge \\
& \wedge \mu_{C^2}(\$8,500,000) \wedge (\mu_{G_o^{3,\mathrm{I}}}(0.94) \wedge \\
& \wedge \mu_{G_o^{3,\mathrm{II}}}(17.7) \wedge \mu_{G_o^{3,\mathrm{III}}}(31.9) \wedge \mu_{G_o^{3,\mathrm{IV}}}(\$2,125,000)) = \\
= \;& 1 \wedge (0.92 \wedge 0.8 \wedge 0.55 \wedge 0.75) \wedge 0.8 \wedge \\
& \wedge (0.9 \wedge 0.85 \wedge 1 \wedge 1) \wedge 0.75 \wedge (0.76 \wedge 0.68 \wedge 1 \wedge 1) = \\
= \;& 0.55 \wedge 0.8 \wedge 0.68 = 0.55
\end{aligned}
$$

The second policy is therefore better.

5 Concluding remarks

We extended the basic Bellman and Zadeh's (1970) model of multistage decision making (control) in a fuzzy environment to include both objective and subjective evaluations of how well fuzzy constraints on decisions (controls) applied and fuzzy goals on states attained are satisfied. We discussed the solution by an extended fuzzy dynamic programming model, and showed the use of a neural network implementing fuzzy dynamic programming which by its inherent parallelism, makes it possible to procced with often time consuming computations in a parallel way. We considered an application for solving a socio-economic regional planning problem.

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