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Political Representation: Perspectives from Fuzzy Systems Theory

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Abstract

The theory of fuzzy sets has been applied to social choice primarily in the contexts where one is given a set of individual fuzzy preference relations and the aim is to find a non-fuzzy choice set of winners or best alternatives. In this article we discuss the problem of composing multi-member deliberative bodies starting again from a set of individual fuzzy preference relations. We outline methods of aggregating these relations into a measure of how well each candidate represents each voter in terms of the latter's preferences. Our main goal is to show how the considerations discussed in the context of individual non-fuzzy complete and transitive preference relations can be extended into the domain of fuzzy preference relations.

1 Introduction

The theory of voting is now an established field of research. Its main constituent is the social choice theory with its somewhat discontinuous history originating in the years preceding the Great Revolution of 1789 in France [McLean and Urken 1995]. The early advocates of the social choice theory were primarily interested in single-winner elections or - in the case of Marquis de Condorcet - in maximizing the probability of a correct decision, both problems that are actively being studied today. The problems related to the election of multi-member bodies were not discussed at length by the pioneers of social choice theory. Indeed, the problems of representative institutions have up to the present been discussed largely separately from the principles of electing presidents, chairpersons etc. Over the past decades several new ideas about representation have, however, emerged. The aim of this paper is to discuss these ideas and extend them to a new domain, viz. to collective decision making with fuzzy preference relations.

The next section outlines and evaluates the new ideas of representation. The section following this is devoted to introducing the basic constituents of fuzzy social choice. Thereafter, we discuss distance minimizing and misrepresentation minimizing social choices rules. We then define several social choice rules based on fuzzy individual preferences relations. The final section concludes the discussion.

2 What is good representation?

If democracy is rule by the people, representative democracy must be rule by the representatives of the people. But not all systems where a group persons declares themselves the representatives of the others, qualify as representative ones in the deeper sense of the term. To say that a body of persons of represents a larger body of people requires that the values, tastes, opinions, attitudes of the latter are somehow reflected in the activities of the former. In the words in Rogowski (1981, cited by Chamberlin and Courant 1983, 719):

> A person, A, is represented in some matter by another person, B, to the extent that B's actions in the matter reflect what might be called A's ideal preferences – the choices that A would make if A were ideally informed, ideally expert, and ideally clear about his own interests.

One might read this as suggesting that a representative body at its best consists of the people itself. This guarantees the presence of every opinion held by someone among the people to be present in the representative body. But this means that the representative body is no smaller than the body it is supposed to represent, clearly an unpracticable arrangement.

Since the very rationale of representative institutions requires them to be reasonably small in size, the customary arrangement is one where the electorate (people) chooses a set of candidates to represent itself. The process whereby the set of candidates is formed vary a great deal from election to another, from a country to another and from a historical time-period to another. Once this set is given, it is the task of the voters in democratic systems to choose the representatives using some legally sanctioned method of election. Not all systems end up with particularly "representative" outcomes. To wit, suppose that a k-member body is to elected so that every voter has k votes at his/her disposal. These he/she can distribute among precisely k candidates, i.e. he/she picks those k candidates each of whom gets one vote from him/her. The k largest vote-getters are elected. Obviously,this system may lead to grossly unrepresentative outcome since any group of voters comprising more than 50% of the voters may dictate the composition of the elected body. Thus, nearly half of the electorate may end up with no representation at all.

A straight-forward – if somewhat impractical - remedy to this problem is the method suggested by Tullock (1967). It is a one person - one vote system yielding a representative body that consists of those candidates that have been vote for by at least one voter. Assuming that each relevant opinion is represented by some candidate, the method assures the election of a truly representative body in the sense of not excluding any relevant opinion. Yet, the method has not found real world applications. An obvious reason is that the size of the elected body may vary widely from one election to another. Moreover, if there are no restrictions with regard to the number of candidates, the elected body may be enormous. Finally, the composition of the elected body in no way reflects the distribution of opinions among the electorate. In response to the last mentioned shortcoming, Tullock suggests that the body resort to weighted voting whereby the voting weight of each representative is equal to the number of votes he/she received in the election of the body.

Despite its prima facie plausibility, this system of weighted voting does not guarantee fair representation if one aims at securing influence over voting outcomes roughly proportional to their support in the electorate. This has been one of the standard motivations of the extensive literature on voting power indices (see Banzhaf 1965; Felsenthal and Machover 1998). Relative vote distribution is a notoriously poor proxy of the influence over voting outcomes.

Chamberlin and Courant (1983, 721) impose the following requirements on representation. A committee member represents a voter to the extent that

1. the committee member "makes present" the voter's opinions in the deliberations that take place within the committee,

2. the committee member is similarly responsive to various kinds of arguments presented in those deliberations as the voter, and

3. the committee member votes in the same way as the voter should the latter be present in the committee.

Clearly, representation of any voter by any member of the committee is matter of degree and can never be perfect in the sense that all three requirements are satisfied. In fact, the degree of representation for any (member, voter)-pair can be determined only *ex post*, i.e. once the committee's term has expired. This does not the problem of electing a representative committee. There are basically two approaches to this election problem. Firstly, one may ask the voters to vote for their favorite committee, that is, signal their preferences regarding all conceivable committees. The choice of the committee would then be determined on the basis of the ballots cast. This is what Chamberlin and Courant call the preferences-over-committees approach. Secondly, the voters could vote their favorite representative or provide a ranking over candidates. This is the preferences-over-candidates approach. Since representation is mainly about getting one's opinions heard in the committee proceedings rather than influencing the outcomes of the committee decision making, the latter approach seems more appropriate.

Given a preference profile of the voters over candidates and the size of the committee, say k, Chamberlin and Courant's proposal for determining the optimal committee composition is equivalent to the following. For each possible committee, compute the number of individuals whose most preferred candidate is present in the committee. Denote this number by n_1. It can obviously be any number between 0 and n, the total number of voters. Then count the number of voters whose first or second preference candidate is present in the committee and denote this by n_2. Continue in this manner until all ranks $1, \ldots, k$ have been considered. Obviously, $n_k = n$. Let now the set of all k-member committees be C^k with elements $c_1, c_2, \ldots, c_s$. The value $C(c_i) = \sum_j^k n_j$, for each $i = 1, \ldots, s$ is the indicator of the representativeness of a committee: the higher the value, the better represented are the voters. Clearly, $k \times n$ is the maximum attainable value and is associated with a committee where each voter's first ranked candidate is present. Similarly, 0 is the minimum value of $C(c_i)$. This "worst possible" committee has the distinction that no voter ranks any committee member higher than $k + 1$th in his/her ranking.

It turns out that if one maximizes $C(c_i)$ over all possible k-member committees, one – in a specific way to be explained shortly – maximizes the sum of the Borda scores of committee members. The most representative one-member committee is one consisting of the candidate with the maximum Borda score. A maximally representative k-member committee, on the other hand, is determined by a modified Borda count. Define each voter's representative as the committee member getting the largest number of Borda points from that voter, i.e. the member ranked highest in the voter's ranking over candidates. Thus, each voter has a representative in each committee.

Now, let $B(c_i)$ denote the sum over voters of the Borda points given to their representative in the committee c_i. The most representative committee is then defined as $c = arg\, max_i B(c_i)$, i.e. the committee where the sum of the Borda points given by each voter to his/her representative is maximal. This is, indeed, a modified Borda count since each voter gives only one score, viz. that of his/her representative.

Chamberlin and Courant show that the method for constructing the most representative committee described above can be given another interpretation, viz. the most representative committee minimizes the number of objections raised by the voters against various committees. By objection these authors refer to a situation where, given a committee c_i not including candidate x, voter j would be better represented by a committee including x in the sense that x is preferred by this voter to every member of c_i. Now, summing the objections of all voters for each committee gives a index of misrepresentation for the committee. Choosing the committee with the minimum index value amounts to choosing the most representative committee. Thus, the modified Borda count yields the misrepresentation-minimizing committee.

Monroe (1995) outlines a similar approach to optimal representation. Its basic concept is also the amount of misrepresentation. However, this concept is applied to pairs consisting of committee members and voters. Consider a committee C and electorate N. For each pair (j, l) where $j \in C$ and $l \in N$, let μ_{jl} be the amount of misrepresentation related to l being represented by j. It is reasonable to set $\mu_{jl} = 0$ if k is top-ranked in l's preferences. In searching for the pure fully proportional representation Monroe embarks upon finding a set of k representatives,each representing an equally-sized group of voters (constituency), so that the total misrepresentation – the sum over voters of the misrepresentations of all committee members – is minimal. He suggests a procedure which firstly generates all possible $\binom{n}{k}$ committees of k members. For each committee one then assigns each voter to the representative that represents him/her best. Since this typically leads to committees consisting of members with constituencies of different size, one proceeds by moving voters from one constituency to another so that eventually each constituency has equally many voters. As a criterion in moving voters is the the difference between their misrepresentation in the source and target constituencies: the smaller the difference, the more likely is the voter to be transferred.

For large n and k the procedure is extremely tedious.[1] Potthoff and Brams (1998) suggest a simplification that essentially turns the committee formation problem into an integer programming one. Let μ_{ij} be the misrepresentation

[1]There is also some ambiguity as to how one should proceed in transferring voters from candidates (constituencies) to another.

value of candidate i to voter j. Define x_i for $i = 1, \ldots, k$ so that it is 1 if i is present in the committee and 0, otherwise. Furthermore, we define $x_{ij} = 1$ if candidate i is assigned to voter j, that is, if i represents j in the committee. Otherwise $x_{ij} = 0$. The objective function we aim at minimizing now becomes:

$$z = \sum_i \sum_j \mu_{ij}.$$

In other words, we minimize the sum of misrepresentations associated with the committee members. In the spirit of Monroe, Potthoff and Brams impose the following constraints:

$$\sum_i x_i = k \quad (1)$$

$$\sum_i x_{ij} = 1, \forall i \quad (2)$$

$$-\frac{n}{m} x_i + \sum_j x_{ij} = 0, \forall i \quad (3)$$

(1) states that the committee consists on k candidates. (2) says that each voter be represented by only one candidate,and (3) amounts to the requirement that each committee member represents an equal number of voters. In Monroe's system, $\mu_{ij} = k - 1 - b_{ij}$ where b_{ij} is the number of Borda points given by j to candidate i.

3 Representation of Fuzzy Preferences

We shall now extend the idea of fully proportional representation to the domain of fuzzy individual preferences. We assume that the committee to be formed is a group of candidates as in the preceding. In contrast to the preceding, however, the voters have fuzzy preference relations over the candidates. We denote the set of candidates by K. With m alternatives these preferences can be represented by $m \times m$ matrices where entry $(i, j) \in [0, 1]$ for $i \neq j$ and $i, j = 1, \ldots, m$. We denote this entry by r_{ij}. It indicates the degree in which the alternative i represented by the row is preferred to the alternative j represented by the column. In many contexts it is plausible to assume that the fuzzy preferences are reciprocal, that is, $r_{ij} = 1 - r_{ji}$, for all alternatives i and j. In reciprocal fuzzy relations it is natural to interpret $r_{ij} > 0.5$ as indicating strict preference of i over j with the strength of the preference reaching its maximum at $r_{ij} = 1$. Similarly, $r_{ij} < 0.5$ indicates a preference of j over i and $r_{ij} = r_{ji} = 0.5$ indifference between the two. We hasten to add that the reciprocal fuzzy relations are by no means universally adopted by the scholarly community (see e.g. Barrett et al. 1990). Very little of what is being said in the following assumes reciprocity of the preference relations. Whenever this assumption is made, it will be pointed out.

3.1 Maximizing Representation

Consider now the concept of representation in the context of fuzzy individual preference relations. Voter i's preference relation over candidates can be presented as:

$$\begin{array}{cccc} - & r^i_{12} & \dots & r^i_{1k} \\ r^i_{21} & - & \dots & r^i_{2k} \\ \dots & \dots & \dots & \dots \\ r^i_{k1} & r^i_{k2} & \dots & - \end{array}$$

Consider now voter i and a committee c_t consisting of k candidates as required. We are now primarily interested in finding the members of c_t that best represent i. Denote the set of these representatives by $B(i, c_t)$. Several plausible ways of finding the best representatives can be envisioned:

1. $B^i_{\text{sum}}(c_t) = \{j \in c_t | \sum_l r_{jl} \geq \sum_l r_{ql}, \forall q \in c_t\}$,
2. $B^i_{\text{min}}(c_t) = \{j \in c_t | \min_l \ r_{jl} \geq \min_l r_{ql}, \forall l \in K, \forall q \in c_t\}$,
3. $B^i_h(c_t) = \{j \in c_t | h(j) \geq h(q), \forall q \in c_t\}$ where $h(j) = p\,(\max_l r_{jl}) + (1 - p)(\min_l r_{jl})$,
4. $B^i_{\text{cop}}(c_t) = \{j \in c_t | cop(j) \geq cop(q), \forall q \in c_t\}$ where $cop(j) = |\{l \in c_t | r_{jl} > r_{lj}, \forall l \in K\}|$

The first one determines the best representatives on the basis of the sums of the preference degrees obtained by candidates in all pairwise comparisons. This method is very much in the spirit of the Borda count. The second method looks at the minimum preference degree of each candidate when compared with all others and picks the candidate with the largest minimum. It is a variant of the min-max method in social choice theory. The third method is a version of Hurwicz's rule which maximizes the weighted sum of the smallest and largest preference degrees (Milnor 1954). The fourth method is motivated by Copeland's rule in social choice theory. The Copeland winner is the candidate that defeats more candidates than any other candidate. In the setting of fuzzy preference relation $cop(j)$ is the number of candidates in c_s that are less preferred to j than j is preferred to them. In reciprocal preference matrices, $cop(j)$ is simply the number of entries larger than 0.5 on the j'th row.

Each of these methods singles out the best representatives of every voter in any given committee. Since each of the methods is based on a score, we can define a ranking of candidates in accordance with those scores. From the point of view of representation more important is, however, the ranking over committees ensuing from these methods. The most straightforward way to accomplish this is to define the score of committee c_t as follows:

$$S_t = \sum_{i \in N} \sum_{a \in c_t} \sum_{j \in K} r^i_{aj}.$$

Thus, the score of a committee is the sum of values given by voters to each of its members. The values, in turn, are the sums of preference degrees in all pairwise comparisons. This method is a variation of the Borda count. The most representative committee RC^B would then be:

$$RC^B = \{c_i \in C^k | S_i \geq S_j, \forall c_j \in C^k\}.$$

Although the Chamberlin-Courant approach is very close to the Borda count as well, the above method is not its most plausible fuzzy counterpart. Rather than summing the preference degrees over alternatives and voters, the Chamberlin-Courant approach sums the Borda scores of each voter's representative in any given committee. First we define

$$r^i_j = \sum_{q \in K} r^i_{jq}.$$

Then, for each committee c_t we define:

$$V_{it} = max_{j \in c_t} r^i_j.$$

This can be viewed as the value of the committee c_t to voter i as reflected by the value i assigns to his/her representative in c_t.

Now, the most representative committee in the sense of Chamberlin-Courant is:

$$RC^{CC}_{\text{sum}} = \{c_j \in C^k \mid \sum_i V_{ij} \geq \sum_i V_{iq}, \forall c_q \in C^k, i \in N, j \in K\}.$$

The RC_{Sum}^{CC} committee thus defined is based on the summation of preference degrees in individual preference matrices. In analogous manner one can define the most representative committee in the min-max sense. Let $\underline{r_j^i} = min_{q \in K} r_{jq}$. Now define, for each committee c_t and each voter i:

$$V'_{it} = max_{j \in c_t} \underline{r_j^i}.$$

Then the most representative committee in the min-max sense is:

$$RC_{\text{min}}^{\prime CC} = \{c_j \in C^k \mid \sum_i V'ij \geq \sum_i V'_{iq}, \forall c_q \in C^k\}.$$

The $RC_{\text{min}}^{\prime CC}$ differs from the previous committee in using the min-max calculus to determine each voter's representative. In a way, $RC_{\text{min}}^{\prime CC}$ mixes two kinds of maximands: the "utilitarian"and "Rawlsian". The former maximizes the average utility, while the latter maximizes the utility of the worst-off individual (Rawls 1971).

A purely Rawlsian committee can also be envisioned. This is obtained as follows:

$$RC^R = \{c_j \in C^k \mid min_i V'_{ij} \geq min_i V'_{iq}, \forall c_q \in C^k\}.$$

In similar vein, one can define Hurwicz and Copeland committees, RC^H and RC^{Co}, respectively. For a fixed value of $p^i \in [0,1]$, let $r_j^{iH} = p^i(\max_q r_{jq}) + (1-p^i)(\min_q r_{jq})$ and $V_{it}^H = max_{j \in c_t} r_j^{iH}$. The set of most representative Hurwicz-type committees is, then:

$$RC^H = \{c_j \in C^k \mid \sum_i V_{ij}^H \geq V_{iq}^H, \forall c_q \in C^k\}.$$

Note that the value p^i is voter specific measure of his/her "optimism", i.e. the weight assigned to $\max_j r_{ij}^i$, i.e. the degree of preference assigned to each candidate in the comparison of its weakest competitor. Intuitively speaking the exclusive emphasis on strongest and weakest pairwise comparisons is somewhat questionable in voting contexts.

To define, the Copeland-type committee, let RC^{Co}, in turn, is based on the voters' value function $r_j^{iCo} = \mid \{q \in K \mid r_{jq} > r_{qj}\} \mid$ and the value function $V_{it}^{iCo} = max_{j \in c_t} r_j^{iCo}$. Now,

$$RC^{Co} = \{c_j \in C^k \mid \sum_i V_{ji}^{iCo} \geq \sum_i V_{qi}^{iCO}, \forall c_q \in C^k\}.$$

Of these four types of committees, the Rawlsian and Copeland types utilize the least amount of the voter preference information. The former looks at the minimal level preference of each candidate when compared with all others. The latter uses only the order information of preference degrees. Of course, if the aim is to economize on information usage, the very idea of resorting to fuzzy preference degrees loses much of its appeal.

3.2 Committees with Equal-Sized Constituencies

In the preceding we aimed at maximally representative committees. Now we approach the committee design problem from the point of view of minimizing misrepresentation, as suggested by Monroe. Again we assume that we are given, for each voter, a matrix of fuzzy preference over all candidates. Our task is to form a committee that minimizes the misrepresentation of voters. It will be recalled that Monroe's procedure has the following elements:

1. Every possible committee of k members is considered.
2. For each committee, the voter set N is partitioned into k equal sized groups, constituencies.
3. Each voter is first assigned to the candidate whose election would be accompanied with the smallest degree of misrepresentation to the voter.

The third stage calls typically some adjustments, i.e. transfers of voters from one candidate to another to obtain equal sized constituencies. With fuzzy preference relations, the first problem to be discussed is how to measure misrepresentation. Consider a situation where voter i prefers one candidate, say a_w, to a maximum degree to any other candidate. This would be indicated in i's preference matrix on w'th row so that it would then consist of straight 1's on all non-diagonal columns. Obviously then $\sum_j r^i_{wj} = k-1, j \neq w, j = 1,\ldots,k$. A natural measure of misrepresentation for i, if candidate a_v is the sole member of the committee is $m_{iv} = k - 1 - \sum_j r^i_{vj}$. In multi-member committees, the same measure is applied to the candidate that represents i in the committee. The best representative, in turn, can be determined as discussed in the preceding subsection. In the following we shall assume that i's best representative in c_t is determined as:

$$B^i_{\text{sum}}(c_t) = \{j \in c_t \mid \sum_l r^i_{jl} \geq \sum_l r^i_{ql}, \forall q \in c_t\} = \{j \in c_t \mid r^i_j \geq r^i_q, \forall q \in c_t\}.$$

Let $\max_{j \in c_t} r^i_j = g(i,t)$.

The degree of misrepresentation of committee c_t is then:

$$M_t = \sum_i (k-1) - g(i,t) = n(k-1) - \sum_i g(i,t).$$

However, Monroe suggests that the optimal committees be composed of candidates with equal-sized constituencies. This means that committees with the minimum value of M_t are not, in general, acceptable since each committee member is not necessarily the best representative of an equal number of voters. Hence, voters have to be "transferred" from one candidate to another. As a criterion for transfers Monroe suggests that those voters who suffer least from being associated with another committee member be transferred first. For example, suppose that in committee c_t voter 1's best representative is candidate a_j and voter 2's best representative is a_j as well. If $m_{1j} - m_{1l} > m_j - m_{2p}$ where a_l and a_p are the next-best representatives of 1 and 2, respectively, then voter 2 is transferred before voter 1. Unfortunately, Monroe does not give full details of the transfer procedure, but, as was pointed out above, Potthoff and Brams have transformed the procedure into an integer programming problem.

It turns out that the Potthoff-Brams procedure can be applied to the fuzzy preference representation problem as well.[2] The objective function is:

$$\min_t M_t = n(k-1) - \sum_i g(i,t).$$

The constraints, in turn, are exactly the same as those defined by Potthoff and Brams, i.e. (1)-(3) in the end of section 2.

4 Discussion

The problem of representation lies at the heart of democratic governance. There are basically two distinct views of how representative assemblies ought to be composed. The first one, particularly important in England and United States, stresses the local representation and clear lines of accountability. This view are expressed in the prevalence of single-member constituency system and first-past-the-post elections. The representative of the areal unit is simply the person who receives more votes than his/her competitors in elections. If the voters and candidates can be placed along a single main policy dimension (e.g. left–right, liberal–conservative) and certain not too implausible conditions regarding voter preferences hold, the chosen candidate can be

[2]In fact, Potthof and Brams extend their analysis to several voting systems including approval voting.

expected to be located near the median voter (Black 1958; Downs 1957). One can then argue that the constituency will be represented by a candidate whose view on the salient policy dimension is very close to the median voter's opinion. A case can be made that the winner's views do, indeed, best represent those of the electorate in the constituency. However, a glance at the election results in any English parliamentary election reveals that the composition of the parliament and the distribution of support for the parties competing may be dramatically different. Hence, an equally plausible case can be made that the single-member constituency system does not result in an assembly whose composition would reflect the prevailing voter opinions. This is the main justification of the second view concerning principles of composing assemblies: one ought to strive at nearly identical distribution of opinions in the electorate and in the assembly.

Single-member constituencies combined with plurality voting typically lead to two-party contests. In these, whichever candidate receives more votes than the other, defeats the latter in the binary comparison. This intuitive notion of winning has a prominent place in social choice and multi-criterion decision making. In the latter context, it lends itself to the interpretation that whichever of two alternatives performs better than another alternative on a majority of criteria, defeats the latter alternative. This kind of calculus may, of course, underly individual preferences over alternatives. But the individual may be interested in not just whether an alternative is superior to another by a majority of criteria, but also in finding out on how many criteria it is superior. This "deeper" interest can give rise to a fuzzy preference relation over alternatives. Of course, a fuzzy preference relation may be the result of other kinds of considerations.

The problem of optimal representation under fuzzy preferences resembles the problem of electing representative assemblies under various electoral systems. A voter's best representative might be one that "defeats" more contestants than any other candidate in the sense of having larger preference degrees in its favor in pairwise contests. This would amount ranking candidates according to their fuzzy Copeland scores. It is, however, questionable whether the notion of defeating has the same unambiguous meaning in individual fuzzy preference relations as in non-fuzzy preference tournaments, especially, if the fuzzy preference relation is non-reciprocal. For this reason, it may make more sense to consider the preference degrees in more detail in defining the degree of various candidates from a voter's point of view. The min-max calculus provides an alternative foundation for such a definition. Similarly, the Hurwicz-type representation calculus takes a closer look at the preference degrees. In our opinion, however, the sum-type definition of representation and misrepresentation is most appropriate to summarize the information contained in fuzzy individual preference relations. It is in the spirit of Borda count, but takes advantage of the additional information provided by the degrees of preference.

The sum-type definition of individuals' best representatives has the additional advantage of being fitting naturally together with the sum-type definition of a committee's degree of presentation or misrepresentation. We emphasize, however, that the sum-type definition of a committee's degree of representation or misrepresentation is compatible with any method used in aggregating individual fuzzy preference relations into a measure of how well various candidates represent the individual in question. As shown above, linear programming provides a useful tool for finding representative committees once the the misrepresentation measure is given.

References

[Banzhaf 1965] Banzhaf, J. (1965), Weighted Voting Doesn't Work: A Mathematical Analysis, *Rutgers Law Review 19*, 317-343.

[Barrett et al.] Barrett, C. R., Pattanaik, P. and Salles, M. (1990), On Choosing Rationally when Preferences Are Fuzzy, *Fuzzy Sets and Systems 34*, 197-212.

[Black 1958] Black, D., *Theory of Committees and Elections*, Cambridge: Cambridge University Press.

[Chamberlin and Courant] Chamberlin, J. and Courant, P. (1983), Representative Deliberations and Representative Decisions: Proportional Representation and the Borda Rule, *The American Political Science Review 77*, 718-733.

[Downs 1957] Downs, A. (1957), *An Economic Theory of Democracy*, New York: Harper & Row.

[Felsenthal and Machover] Felsenthal, D. and Machover, M. (1998), *The Measurement of Voting Power*, Cheltenham: Edward Elgar.

[McLean and Urken 1995] McLean, I. and Urken, A., (eds) (1995), *Classics of Social Choice*, Ann Arbor: The University of Michigan Press.

[Milnor 1954] Milnor, J. (1954), Games against Nature, in Thrall, R., Coombs, C. and Davis, R., eds, *Decision Processes*, New York: Wiley.

[Monroe 1995] Monroe, B. (1995), Fully Proportional Representation, *The American Political Science Review 89*, 925-940.

[Potthoff and Brams] Potthoff, R. and Brams, S. (1998), Proportional Representation: Broadening the Options, *Journal of Theoretical Politics 10*, 147-178.

[Rogowski] Rogowski, R. (1981), Representation in Political Theory and Law, *Ethics 91*, 395-430.

[Saari 1995] Saari, D. (1995), *Basic Geometry of Voting*, Berlin-Heidelberg-New York: Springer Verlag.

[Saari 2006] Saari, D. (2006), Which Is Better: The Condorcet or Borda Winner?, *Social Choice and Welfare*, forthcoming.

[Tullock 1967] Tullock, G. (1967), *Toward a Mathematics of Politics*, Ann Arbor: University of Michigan Press.