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and the adjustment to the GHG  
limiting policy. Preliminary results  
from the four sector optimization  
model of the Polish economy**

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Technological conversion and the adjustment to the GHG limiting policy. Preliminary results  
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Abstract

Model is designed as a decision supporting tool in performing the policy of technology conversion in Poland necessary to meet increasingly stricter future limits of the GHG emissions. Model consists of four production sectors producing respectively: material inputs and services, energy inputs and services, non-energy consumption goods and investment goods. Each sector chooses between two technologies: the recently used one, economically more effective (without emission limits), and the cleaner but more expensive one. Consumption sector consists of the households sector and public sector. Each production sector consumes both material inputs and energy. Consumption sector consumes the energy and non-energy consumption goods. Outcomes of policy are assessed by analyzing the results of optimization of the discounted macroeconomic consumption

**1. Introduction**

The aim of the model is to create a decision supporting tool for the macroeconomic policy fostering technology conversion necessary to meet limits of the GHG emissions. The model presented in this paper is an extension of the three-production sector model developed last year in Systems Research Institute of PAS. This extension was necessary because the energy sector remains the main contributor to the total CO<sub>2</sub> emissions.

Model consists of four production sectors producing: M – material inputs, E – energy inputs, C – consumption goods and services and I – investment goods. Consuming sector consists of households and public sector. Consumption in this model consists both of the very consumption goods and services, as well as energy. Beside the national output and consumption the foreign exchange is also accounted for: supply of each production sector is supported by import, and the demand for each kind of goods and services consists of both foreign and national demand.

Each production sector is characterized by the following parameters: the capital productivity rate, unit emission, intensity of the material consumption, intensity of the energy consumption, capital depreciation rate Consumption sector is characterized by constant shares

of the expenditures on the proper consumption goods and energy. In each sector the following decisions are being made:

- division of income between the consumption and investment
- division of investment between two technologies
- utilization of the production capacity in each technology
- export and import

The overall goal is maximization of the discounted consumption rate in the infinite time horizon accounting for the maximum duration of the process of the technological conversion.

The linear optimization problem has 2376 variables and 6470 constraints.

Initial values of the variables and the values of parameters were estimated on the basis of Input–Output Table at Basic Prices in 2010, Central Statistical Office, Poland, by reaggregation of the original input-output table.

### 1. Macroeconomic model formulation

In this section the following notation of numbering the model parameters is used. The letter  $i = M, E, C, I$ , is used to denote the sector, the letter  $j = 1, 2$ , to denote technology, and the letter  $t = 1, \dots, T$ , to denote the year. The numbering of years starts with the year 2005, so  $t = 1$  corresponds to the year 2006.

**Technology of production.** Each technology of production in any sector is described by the following set of parameters in  $i$ -th sector,  $i = M, E, C, I$ ; in  $j$ -th technology,  $j=1, 2$ ; in year  $t$ ,  $t=1, \dots, T$ :

$\gamma_{ijt}$  - productivity of fixed assets, it is assumed that the technical progress increases the productivity of the fixed assets by a constant ratio in each year:

$$\gamma_{ijt} = \gamma_{ijt_0} (1 + r_\gamma)^{t-t_0};$$

where  $\gamma_{ijt_0}$  denotes productivity of the fixed assets in the year  $t_0$ , and  $r_\gamma$  denotes the growth rate of the productivity of the fixed assets;

$\delta_{ij}$  - depreciation rate of fixed assets;

$\alpha_{ij}$  - use of goods produced in sector  $M$  in producing the unit of the gross product of the  $i$ -th sector;

$\beta_{ij}$  - use of goods produced in sector  $E$  in producing the unit of the gross product of the  $i$ -th sector;

$\mu_j$  - unit emission.

**Potential gross output.** Potential gross output  $Q_{ijt}$  produced by  $i$ -th sector using  $j$ -th technology in year  $t$  is described by the Harrods production function:

$$Q_{ijt} = \gamma_{ijt} K_{ijt}, \quad i = M, E, C, I; j = 1, 2; t = 1, \dots, T, \quad (1)$$

where  $K_{ijt}$  stands for stock of the fixed assets in  $i$ -th sector and  $j$ -th technology at the beginning of year  $t$ . In this paper, the potential gross output (1) will be also called the production capacity of the  $j$ -th technology in  $i$ -th sector in year  $t$ .

**Actual gross output.** Actual gross output  $X_{ijt}$  may be smaller due to the fact that production capacity may not be fully used:

$$X_{ijt} = \lambda_{ijt} Q_{ijt}, \quad i = M, E, C, I; j = 1, 2; t = 1, \dots, T, \quad (2)$$

where  $\lambda_{ijt}$  stands for the coefficient of the production capacity utilization in  $i$ -th sector,  $i = M, C, I$ , in  $j$ -th technology,  $j = 1, 2$ ; in year  $t$ , assuming values from the range  $[0;1]$ . In particular,  $\lambda_{ijt} = 0$  indicates fully idle capital and  $\lambda_{ijt} = 1$  represents full utilization of the production capacity. Total actual output of  $i$ -th sector is the sum of outputs produced using both technologies:

$$X_{it} = X_{i1t} + X_{i2t}, \quad i = M, E, C, I; t = 1, \dots, T. \quad (3)$$

**Stock of the fixed assets.** Stock of the fixed assets  $K_{ijt}$  in  $i$ -th sector is given by the standard relationship:

$$K_{ijt} = K_{ijt-1} + I_{it-1} + \delta_j K_{ijt-1}, \quad i = M, E, C, I; j = 1, 2; t = 1, \dots, T, \quad (4)$$

where  $I_{ijt}$  denotes investment in  $i$ -th sector and the term  $\delta_j K_{ijt-1}$  denotes depreciation of the capital in  $i$ -th sector. One year lag between the investment and its contribution to the stock of fixed assets determining production capacity is assumed for simplicity.

**Emission.** Production of  $i$ -th sector using  $j$ -th technology causes the emissions  $EM_{ijt}$  of GHG:

$$EM_{ijt} = \mu_{ij} X_{ijt}, \quad i = M, E, C, I; j = 1, 2; t = 1, \dots, T. \quad (5)$$

The emission  $EM_{it}$  of the  $i$ -th sector equals:

$$EM_{it} = EM_{i1t} + EM_{i2t}, \quad i = M, E, C, I, j=1, 2; t=1, \dots, T, \quad (6)$$

and the total emission  $EM_t$  is given by the following expression:

$$EM_t = EM_{Mt} + EM_{Et} + EM_{Ct} + EM_{It} \quad t=1, \dots, T. \quad (7)$$

It is also assumed that there exist market equilibria in all three markets.

**Balance of the sector  $M$ .** The demand for goods and services produced by sector  $M$ , i.e. their consumption in all sectors and the export equals the domestic supply increased by the import:

$$\begin{aligned} \alpha_{M1}X_{M1t} + \alpha_{M2}X_{M2t} + \alpha_{E1}X_{E1t} + \alpha_{E2}X_{E2t} + \alpha_{C1}X_{C1t} + \alpha_{C2}X_{C2t} + \alpha_{I1}X_{I1t} + \alpha_{I2}X_{I2t} + EXP_{Mt} = \\ = X_{M1t} + X_{M2t} + IMP_{Mt} \\ , t=1, \dots, T; \end{aligned} \quad (8)$$

where  $\alpha_{ij}X_{ijt}$  denotes consumption of goods and services from sector  $M$  in  $i$ -th sector, produced using  $j$ -th technology, in the year  $t$ , and  $EXP_{Mt}$  denotes the export of goods produced by sector  $M$ , while  $IMP_{Mt}$  stands for the import of goods produced by sector  $M$  in the year  $t$ .

**Balance of the sector  $I$ .** Demand for the goods and services supplied by sector  $I$ , being the sum of domestic demand and the balance of the foreign trade in goods and services in  $I$ , equals domestic supply of these goods in all sectors:

$$I_{M1t} + I_{M2t} + I_{E1t} + I_{E2t} + I_{C1t} + I_{C2t} + I_{I1t} + I_{I2t} + EXP_{It} = X_{I1t} + X_{I2t} + IMP_{It}; \quad t=1, \dots, T; \quad (9)$$

where  $EXP_{It}$  denotes the export of goods produced in sector  $I$  while  $IMP_{It}$  stands for import of investment goods in year  $t$ .

**Total income.** Total income  $Y_t$  from sectors  $M$ ,  $E$ ,  $C$  and  $I$  is given by the following expression:

$$\begin{aligned} Y_t = (1 - \alpha_{M1} - \beta_{M1})X_{M1t} + (1 - \alpha_{M2} - \beta_{M2})X_{M2t} + (1 - \alpha_{E1} - \beta_{E1})X_{E1t} + (1 - \alpha_{E2} - \beta_{E2})X_{E2t} + \\ + (1 - \alpha_{C1} - \beta_{C1})X_{C1t} + (1 - \alpha_{C2} - \beta_{C2})X_{C2t} + (1 - \alpha_{I1} - \beta_{I1})X_{I1t} + (1 - \alpha_{I2} - \beta_{I2})X_{I2t} \\ t=1, \dots, T. \end{aligned} \quad (10)$$

**Disposable income.** Disposable income  $Y_t^d$  is given as:

$$Y_t^d = Y_t - rD_{t-1}, \quad t=1, \dots, T. \quad (11)$$

where  $rD_{t-1}$  is the payment of the debt (if  $D_{t-1}$  is positive, or an income from foreign assets if  $D_{t-1}$  is negative);  $r$  denotes the interest rate, while  $D_t$  stands for the debt at the beginning of year  $t$ .

**Consumption demand.** National consumption demand  $CD_t$  is given by the following expression:

$$CD_t = Y_t^d - I_t, \quad t=1, \dots, T. \quad (12)$$

where the total investment in all sectors  $I_t$  equals:

$$I_t = I_{M1t} + I_{M2t} + I_{E1t} + I_{E2t} + I_{C1t} + I_{C2t} + I_{I1t} + I_{I2t}. \quad (13)$$

**Balance of the sector C.** Total consumption  $CD_t$  demand is divided in fixed proportions into the demand for the consumption products proper  $CC_t$  and demand for energy  $CE_t$ :

$$CD_t = CC_t + CE_t = (1 - \varepsilon)CD_t + \varepsilon CD_t, \quad (14)$$

where fixed coefficient  $\varepsilon$ ,  $\varepsilon \in (0,1)$ , determines the share of the energy expenditures in consumption expenditures of households:

$$CE_t = \varepsilon CD_t. \quad (15)$$

The balance of the sector C is given by the following relationship:

$$CC_t + EXP_{Ct} = X_{C1t} + X_{C2t} + IMP_{Ct}, \quad t=1, \dots, T. \quad (16)$$

**Balance of the sector E.** Balance of the sector  $E$  is given by the following relationship:

$$\begin{aligned} \beta_{M1}X_{M1t} + \beta_{M2}X_{M2t} + \beta_{E1}X_{E1t} + \beta_{E2}X_{E2t} + \beta_{C1}X_{C1t} + \beta_{C2}X_{C2t} + \beta_{I1}X_{I1t} + \beta_{I2}X_{I2t} + CE_t + EXP_{Et} = \\ = X_{E1t} + X_{E2t} + IMP_{Et} \end{aligned}$$

where the left-hand side represents the total demand for energy containing production demand from sectors  $M$ ,  $E$ ,  $C$  and  $I$ , as well as demand for energy from the households and the energy exports, while the right-hand side represents supply including output using both technologies and the energy import.

**Discounted consumption.** A country pursues maximization of the consumption in the long run, which in this model is represented by the discounted value of the future flow of consumption (present value at the time point  $t=t_0$ ):

$$PVC = \sum_{i=0}^{\infty} \frac{CD_{t_0+i}}{(1+r_d)^i}, \quad (17)$$

where  $r_d$  denotes the discounting rate and  $CD_{0+i}$ , denote future consumption rates in years  $t_0+i$ ;  $i=0, 1, 2, \dots$ .

**Number of the committed emission permits.** The number of the committed emission permits is modeled by a trajectory of an assumed form in time, dependent on the number of the permits  $N_{t_d}$  in the destination year  $t_d$ :

$$N_t = f_N(t, N_{t_d}), \quad t=1, \dots, T. \quad (18)$$

**Net result of the trade in the emission permits.** In each year the trade in the emission permits gives the following net result  $V_t$ :

$$V_t = p_t(N_t - EM_t), \quad (19)$$

where  $p_t$  stands for the permission price in year  $t$  and  $N_t$  is the number of the committed emission permits. In the case of an excess in the emission permits,

$$N_t - EM_t > 0,$$

a country sells the surplus of the emission permits at price  $p_t$ , while in the case of deficit, this is when

$$N_t - EM_t < 0,$$

a country has to buy the lacking amount of emission permits at price  $p_t$ . Prices  $p_t$  are determined exogenously; they are set in an international GHG permit market.

**Debt.** Debt  $D_t$  is defined by the following relationship:

$$D_t = D_{t-1} + (IMP_{Mt} + IMP_{Et} + IMP_{Ct} + IMP_{It}) - (EXP_{Mt} + EXP_{Et} + EXP_{Ct} + EXP_{It}) - V_t, \quad t=1, \dots, T. \quad (20)$$

Note that the debt can be positive or negative; net import increases the debt while the trade surplus decreases it. Note also that interest on the debt affects the disposable income, as described by equation (11). Foreign debt is interpreted in this paper as a result of trade in the emission permits as well as products  $M$ ,  $E$ ,  $C$  and  $I$ . By assuming initial value  $D_0 = 0$ , we will attribute changed structure of the foreign trade to the process of technology conversion.

**Decision variables.** The decision variables are: actual gross output from each technology in every sector, investment in each technology in every sector, import and export of each sector:

$$\begin{aligned} & X_{M1t}, X_{M2t}, X_{E1t}, X_{E2t}, X_{C1t}, X_{C2t}, X_{I1t}, X_{I2t}, I_{M1t}, I_{M2t}, I_{E1t}, I_{E2t}, I_{C1t}, I_{C2t}, I_{I1t}, I_{I2t}, \\ & EXP_{M1t}, EXP_{M2t}, EXP_{E1t}, EXP_{E2t}, EXP_{C1t}, EXP_{C2t}, EXP_{I1t}, EXP_{I2t}, \\ & IMP_{M1t}, IMP_{M2t}, IMP_{E1t}, IMP_{E2t}, IMP_{C1t}, IMP_{C2t}, IMP_{I1t}, IMP_{I2t}. \end{aligned}$$



**Constraints.** The model includes the following inequality constraints. The outputs and investment outlays are non-negative:

$$\begin{aligned} X_{M1t}, X_{M2t}, X_{E1t}, X_{E2t}, X_{C1t}, X_{C2t}, X_{I1t}, X_{I2t}, I_{M1t}, I_{M2t}, I_{E1t}, I_{E2t}, I_{C1t}, I_{C2t}, I_{I1t}, I_{I2t}, \\ EXP_{M1t}, EXP_{M2t}, EXP_{E1t}, EXP_{E2t}, EXP_{C1t}, EXP_{C2t}, EXP_{I1t}, EXP_{I2t}, \\ IMP_{M1t}, IMP_{M2t}, IMP_{E1t}, IMP_{E2t}, IMP_{C1t}, IMP_{C2t}, IMP_{I1t}, IMP_{I2t} \geq 0. \end{aligned} \quad (21)$$

The following constraints make the technological conversion socially and politically feasible.

The constraint:

$$I_t \leq \sigma_{I1Y} Y_t \quad (22)$$

prevents too high investment rates; coefficient  $\sigma_{I1Y}$  denotes the highest acceptable investment

rate. In each sector the constraints:

$$\begin{aligned} \sigma_{IMPjX} \leq \frac{IMP_t}{X_t} \leq \sigma_{IMPjX} \quad j = M, E, C, I; \\ \sigma_{EXPjX} \leq \frac{EXP_t}{X_t} \leq \sigma_{EXPjX} \quad j = M, E, C, I; \end{aligned} \quad (23)$$

imposes maximum share of import and export respectively, in the national supply of the given product, where coefficients  $\sigma_{IMPjX}$  and  $\sigma_{EXPjX}$ ,  $j = M, E, C, I$ ; denote maximum share of import and export respectively in given product in its national gross output. Another two sets of constraints:

$$-r_{Ij}^- \leq \frac{I_{jt} - I_{j,t-1}}{I_{j,t-1}} \leq r_{Ij}^+, \quad j = M, E, C, I; \quad (24)$$

and

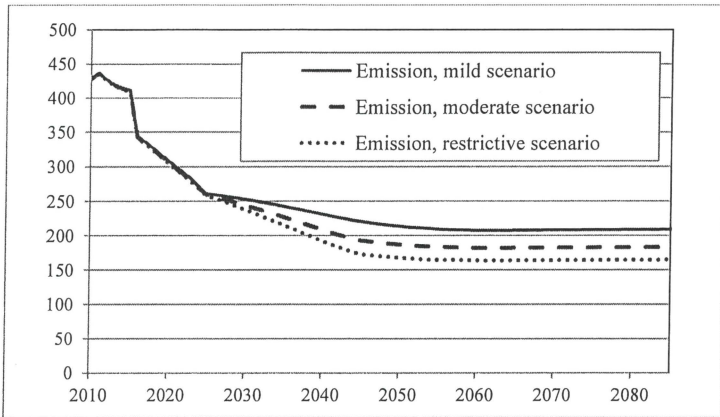
$$-r_{cons}^- \leq \frac{C_t - C_{t-1}}{C_{t-1}} \leq r_{cons}^+ \quad j = M, E, C, I; \quad (25)$$

limit relative increases and decreases of investments in sectors and total consumption, respectively, where parameters  $r_{Ij}^-$  and  $r_{Ij}^+$  stand for the lowest and highest admissible rate of increase of the investment in technology  $j$ ,  $j = M, E, C, I$ ; while  $r_{cons}^-$  and  $r_{cons}^+$  denote the lowest and highest admissible rate of the consumption change.

The end-point constraint included into the model requires that the debt from year 2080 and beyond should be equal to zero,  $D_t = 0$ ,  $t = 2080, 2081, \dots$ ; determining completion of the process of adjustment till year 2080.

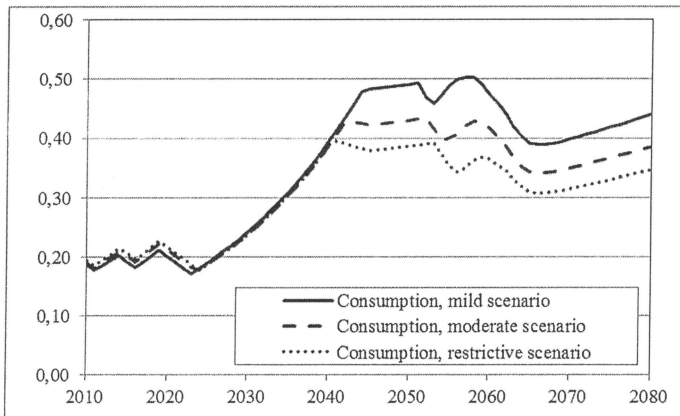
## 2. Preliminary results

Results presented in this section are preliminary in such a sense that they require verification as to the initial values of variables and values of parameters.



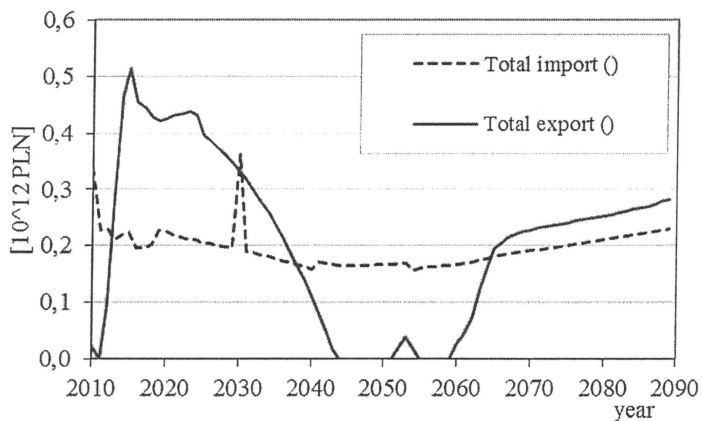
Source: own computations

Fig. 1. Stylized trajectories of the emission limits, million ton CO<sub>2</sub>.



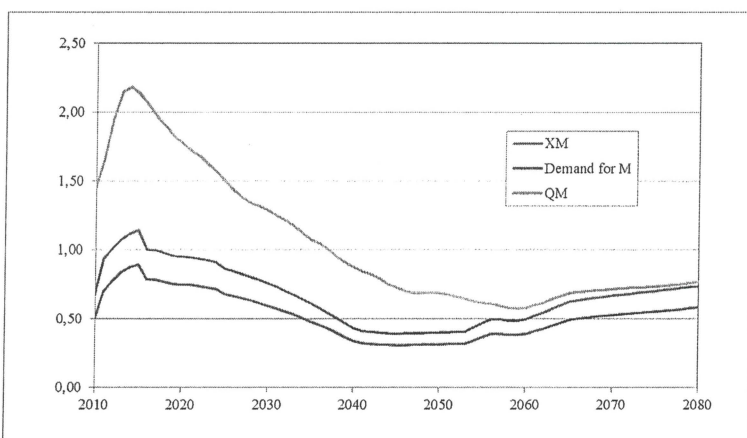
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Fig. 2. Stylized trajectories of the emission limits, million ton CO<sub>2</sub>.



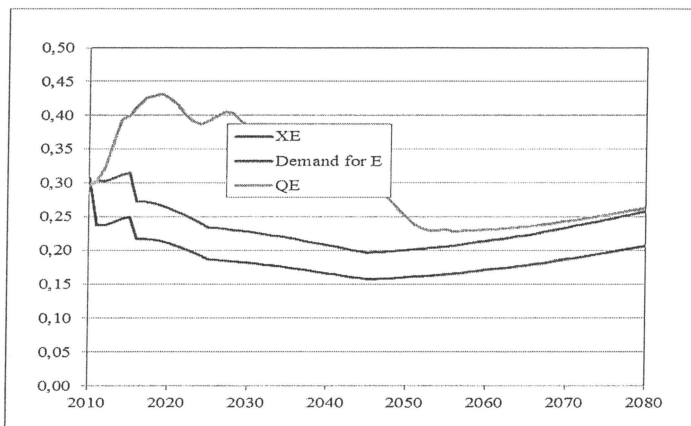
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Fig. 3. Total Import and export. Restrictive scenario.



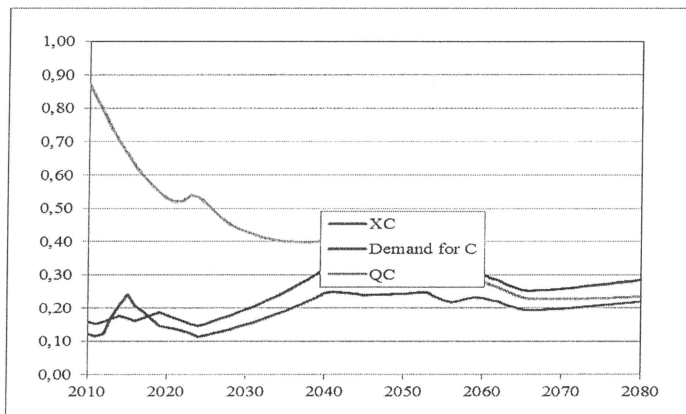
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Fig. 4. Production capacity QM, demand and actual gross output XM in sector M. Restrictive scenario.



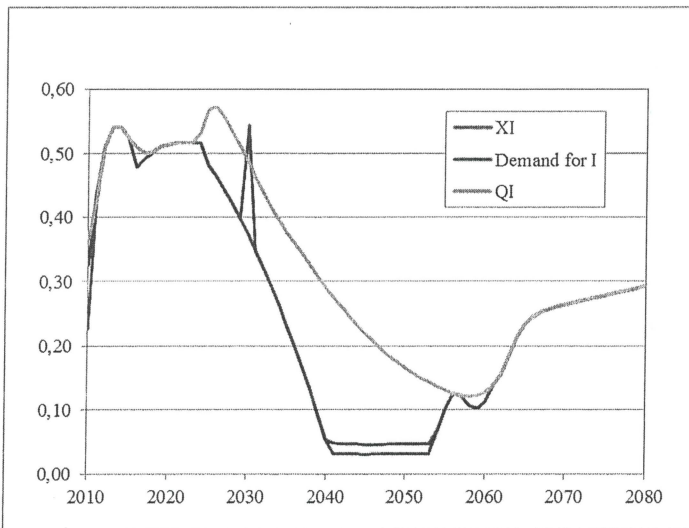
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Fig. 5. Production capacity QE, demand and actual gross output XE in sector E. Restrictive scenario.



Source: own computations

Fig. 6. Production capacity QC, demand and actual gross output XC in sector C. Restrictive scenario.



Source: own computations

Fig. 7. Production capacity QI, demand and actual gross output XI in sector I. Restrictive scenario.

#### References

Input–Output Table at Basic Prices in 2010, Central Statistical Office, Poland.

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